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Flexibility and Efficiency of New Analytical Method for Solving Systems of Linear and Nonlinear Differential Equations

Habibolla Latifizadeh[1], Esmail Hesameddini[2]
[1,2]Department of Mathematics, Faculty of Basic Sciences, Shiraz University of Technology, Shiraz, Iran
([1]h.latifi.62.math@sutech.ac.ir)

Abstract- In this paper, the Laplace Iteration Method (LIM) technique has been successfully applied to find approximate solution to the systems of linear and nonlinear equations specially coupled Schrodinger-KdV equations. The algorithm overcame the difficulty arising from the calculation of intricately nonlinear terms. Besides, it provided a simple way to ensure the convergence of series solution so that accurate enough approximations could be always obtained. Till now the LIM technique has been successfully applied to many nonlinear problems in science and engineering, all of which have verified the great potential and validity of the LIM technique in comparison with Variational Iteration Method (VIM) for strongly nonlinear problems in science and engineering.

Keywords- *Laplace Iteration Method, Analytical solution, systems of Partial Differential Equation.*

I. INTRODUCTION

It is very difficult to solve nonlinear problems, either numerically or theoretically; and even more difficult to establish a real model for nonlinear system problems. A great amount of assumption has to be made artificially or unnecessarily for making the practical engineering problems solvable, which leads to the loss of most important information. In this paper, a new kind of analytical method was proposed for the system of nonlinear problems which was called the Reconstruction of Variational Iteration Method. The similarities and differences with VIM and Homotopy Perturbation Method (HPM) [13] were pointed out and it was shown that it did not require any small parameter in an equation as the perturbation techniques do and not use a Lagrange multiplier. Moreover, it was demonstrated to solve a large class of linear and nonlinear system problems effectively, easily and accurately, using approximations which rapidly converge to accurate solutions. A broad class of analytical solutions methods and numerical solutions methods were used to handle these problems [1–3]. In [4] and [5], the characteristics method and the Riemann invariants method were used to handle systems of PDEs. Vandewalle and Piessens [6] implemented a method based on the combination of the wave form relaxation method and multigrid in order to solve nonlinear systems. Wazwaz [7] used the Adomian Decomposition Method (ADM) for handling some systems of PDEs and reaction–diffusion Brusselator model. Gu and Li [8] introduced the modified Adomian method that solve some systems of nonlinear differential equations (see also [9, 10]). Systems of nonlinear partial differential equations could be observed in many scientific models such as the propagation of shallow water waves [11, 15].

The aim of this work was to present an alternative approach based on LIM [12, 14-16] for finding series solutions to linear and nonlinear systems of PDEs. The applied method gave rapidly convergent successive approximations. As stated before, the aim was to obtain analytical solutions for system problems. Also, there was an attempt to confirm that the Laplace Iteration Method was powerful, efficient and promising in handling scientific and engineering problems. The efficiency and accuracy of LIM was demonstrated through several test examples. The LIM technique was completely independent from any small parameters. Besides, it provided a simple way to ensure the convergence of series solution so that accurate enough approximations could be always obtained. The LIM technique has been successfully applied to many nonlinear problems in science and engineering, all of which have verified the great potential and validity of the LIM technique for strongly nonlinear problems in science and engineering.

II. DESCRIPTION OF THE NEW METHOD

To illustrate the basic idea of the proposed method in [12, 14-16] for system of PDEs, the following non-homogeneous, non-linear system of PDEs was considered:

$$\begin{aligned}
&L_1u_1(x_1,\cdots,x_n)+N_1(u_1,\cdots,u_m)=f_1(x_1,\cdots,x_n),\\
&L_2u_2(x_1,\cdots,x_n)+N_2(u_1,\cdots,u_m)=f_2(x_1,\cdots,x_n),\\
&\vdots\qquad\qquad\vdots\qquad\qquad\vdots\\
&L_mu_m(x_1,\cdots,x_n)+N_m(u_1,\cdots,u_m)=f_m(x_1,\cdots,x_n),
\end{aligned}\tag{1}$$

Where L_i is a linear operator, N_i a nonlinear operator and $f_i(x_1,\cdots,x_n)$ is an inhomogeneous item for $i=1,\ldots,m$. Eq. (1) can be rewritten down as a correction function in the following way:

$$L_iu_i(x_1,\cdots,x_n)=\underbrace{f_i(x_1,\cdots,x_n)-N_i(u_1,\cdots,u_m)}_{R_i(u_1,\cdots,u_m)}$$

$$i=1,\ldots,m. \tag{2}$$

Therefore:

$$L_i u_i(x_1,\cdots,x_n) = R_i(u_1,\cdots,u_m), \qquad i = 1,\dots,m.$$

The Laplace Iteration Method assumed a series solution for u_i given by an infinite sum of components:

$$u_i(x_1,\cdots,x_n) = lim_{p\to\infty} u_i^p(x_1,\cdots,x_n) = lim_{p\to\infty} \sum_{j=0}^{p} v_i^j(x_1,\cdots,x_n), \qquad i = 1,\dots,m \tag{3}$$

in which u_i^n indicates the n-th approximation of u_i, where v_i^j is the j^{th} component of the solution of u_i and v_i^0 is the solution of $L_i u_i = 0$ along with the following initial conditions of the main problem:

$$v_i^1(x_1,\cdots,x_n) = \varphi_i(\ v_i^0),$$

$$v_i^{k+1}(x_1,\cdots,x_n) = \varphi_i\left(\sum_{j=0}^{k} v_i^j(x_1,\cdots,x_n)\right) - \sum_{j=0}^{k} v_i^j(x_1,\cdots,x_n), \qquad k \geq 1$$

in which $\varphi_i(v_i^k)$ is obtained as follows:

$$L_i \varphi_i(v_1^k,\cdots,v_m^k) = R_i(u_1,\cdots,u_m). \tag{4}$$

Using the homogenous initial condition, supposing that $L_i = \frac{\partial^q}{\partial (x_l)^q}$, therefore, taking Laplace transform to both sides of Eq. (4) in the usual way and using the homogenous initial conditions, the result can be obtained as following:

$$P_i(s).\Phi_i^k(x_1,\cdots,x_{l-1},s,x_{l+1},x_n) = \Re_i\left(v_i^k(x_1,\cdots,x_{l-1},s,x_{l+1},x_n)\right), \tag{5}$$

Where $\mathcal{L}\left[\varphi_i\left(v_1^k,\cdots,v_m^k\right)\right] = \Phi_i^k$, $P_i(s)$ is a polynomial with the degree of the highest derivative in Eq. (5). (The same as the highest order of the linear operator L_i). Thus,

$$\mathcal{L}[w] = \varpi, \quad \Psi(s) = \frac{1}{P(s)}, \quad \mathcal{L}[\psi_i(x_l)] = \Psi_i(s) \tag{6}$$

In Equations (4) and (5), the function $\Re_i\left(v_i^k(x_1,\cdots,x_{l-1},s,x_{l+1},x_n)\right)$ and $R_i(u_1,\cdots,u_m)$ are abbreviated as $\Re_i$ *and* R_i respectively. Hence, Eq. (5) is rewritten as:

$$\Phi_i^k(x_1,\cdots,x_{l-1},s,x_{l+1},x_n) = \Re_i\left((v_1^k,\cdots,v_m^k)(x_1,\cdots,x_{l-1},s,x_{l+1},x_n)\right).\Psi_i(s). \tag{7}$$

Now, by applying the inverse Laplace Transform to both sides of Eq. (7) and using the convolution theorem, the following relation can be presented:

$$\varphi_i(v_1^k,\cdots,v_m^k) = \int_0^{x_l} R_i\left((v_1^k,\cdots,v_m^k)(x_1,\cdots,x_{l-1},\tau,x_{l+1},x_n)\right).\psi_i(x_l-\tau)d\tau. \tag{8}$$

Therefore,

$$u_i^{p+1}(x_1,\cdots,x_n) = \sum_{j=0}^{p+1} v_i^j = u_i^0(x_1,\cdots,x_n) + \int_0^{x_l} R_i\left(u_i^p(x_1,\cdots,x_{l-1},\tau,x_{l+1},x_n)\right).\psi_i(x_l-\tau)d\tau, \quad i = 1,\dots,m. \tag{9}$$

After identifying the initial approximation of u_i^0, the remaining approximations u_i^p, $p > 0$ can be determined so that each term can be determined by previous terms and the approximation of iteration formula can be entirely evaluated. Consequently, the exact solution may be obtained by:

$$u_i = lim_{p\to\infty} u_i^p(x_1,\cdots,x_n) = lim_{p\to\infty} \sum_{i=0}^{p} v_i^j, \quad i = 1,\dots,m. \tag{10}$$

In an algorithmic form, the new presented method can be expressed and implemented the solutions as follows:

Algorithm. Let p be the iteration index, set a suitable value for the tolerance (Tol.)

Step 0: Choose a suitable $u_i^0(x_1,\cdots,x_n)$ so that $L_i\left(u_i^0(x_1,\cdots,x_n)\right) = 0$, for $i = 1,\dots,m$.

Step 1: Set $p = 0$.

Step 2: Use the calculated values of $u_i^p(x_1,\cdots,x_n)$ to compute $u_i^{p+1}(x_1,\cdots,x_n)$ from Eq. (2-9).

Step 3: Define $u_i^p(x_1,\cdots,x_n) \coloneqq u_i^{p+1}(x_1,\cdots,x_n)$ for $i = 1,\dots,m$.

Step 4: If $\text{Max}\left|u_i^p(x_1,\cdots,x_n) - u_i^{p-1}(x_1,\cdots,x_n)\right| < Tol$ stop, otherwise continue.

Step 5: Define $u_i^{p+1}(x_1,\cdots,x_n) \coloneqq u_i^p(x_1,\cdots,x_n)$ for $i = 1,\dots,m$.

Step 6: Set $p = p + 1$, and return to step **2**.

III. Capability of the Method in Solving Non-Linear Systems of P.D.Es

In this section the capability of the LIM through study of some well-known examples will be presented.

Example 1 : Considering the following system of P.D.E:

$$u_t + u + v_x w_y - w_x v_y = 0, \tag{11}$$

$$v_t - v + w_x u_y + w_y u_x = 0,$$

$$w_t - w + u_x v_y + u_y v_x = 0.$$

with the initial conditions:

$$u(x,0) = e^{x+y},$$

$$v(x,0) = e^{x-y},$$

$$w(x,0) = e^{y-x}.$$

At first, by use of (11) and optimal selection auxiliary linear operator the equation is represented as follows:

$$L_1 u(x,t) = u_t + u = w_x v_y - v_x w_y,$$

$$L_2 v(x,t) = v_t - v = w_x u_y + w_y u_x,$$

$$L_3 w(x,t) = w_t - w = u_x v_y + u_y v_x.$$

Therefore, $\varphi_i(v_i^k)$ is defined as:

$\varphi_1(v_1^k, v_2^k, v_3^k) = \int_0^t e^{\tau-t}\left((v_2^k)_y(v_3^k)_x - (v_3^k)_y(v_2^k)_x\right)d\tau$ (12)

$\varphi_2(v_1^k, v_2^k, v_3^k) = \int_0^t e^{t-\tau}\left((v_1^k)_x(v_3^k)_y + (v_1^k)_y(v_3^k)_x\right)d\tau,$

$\varphi_3(v_1^k, v_2^k, v_3^k) = \int_0^t e^{t-\tau}\left((v_1^k)_y(v_2^k)_x + (v_1^k)_x(v_2^k)_y\right)d\tau.$

Then, using Eq. (11), the LIM method formulae in t-direction for the calculation of the approximate solution of Eq. (12) can be readily obtained as:

$u_{n+1}(x,t) = \sum_{i=0}^{i} v_1^k(x,t) =$
$u_0(x,t) + \int_0^t e^{\tau-t}\left(\left(v_p\right)_y\left(w_p\right)_x - \left(v_p\right)_x\left(w_p\right)_y\right)d\tau,$ (13)

$v_{n+1}(x,t) = \sum_{i=0}^{i} v_2^k(x,t) =$
$v_0(x,t) + \int_0^t e^{t-\tau}\left(\left(u_p\right)_x\left(w_p\right)_y + \left(u_p\right)_y\left(w_p\right)_x\right)d\tau,$

$w_{n+1}(x,t) = \sum_{i=0}^{i} v_3^k(x,t) =$
$w_0(x,t) + \int_0^t e^{t-\tau}\left(\left(u_p\right)_x\left(v_p\right)_y + \left(u_p\right)_y\left(v_p\right)_x\right)d\tau,$

where the initial approximation must be satisfied by the following equations:

$L_1u(x,t) = 0, \quad u(x,0) = e^{x+y},$

$L_2v(x,t) = 0, \quad v(x,0) = e^{x-y},$

$L_3w(x,t) = 0, \quad u(x,0) = e^{y-x}.$

Therefore, based on inverse Laplace transformation of auxiliary linear operators in t-direction, consequently, the initial approximations can be indicated by:

$u_0(x,t) = v_1^0 = e^{x+y-t},$

$v_0(x,t) = v_2^0 = e^{x-y+t},$

$w_0(x,t) = v_2^0 = e^{y-x+t}.$

Accordingly, by Eq. (13), the higher order approximation of the exact solution can be obtained as follows:

$u_1(x,t) = \sum_{i=0}^{1} v_1^k(x,t) = 0,$

$v_2(x,t) = \sum_{i=0}^{2} v_2^k(x,t) = 0,$

$w_1(x,t) = \sum_{i=0}^{1} v_3^k(x,t) = 0,$

The remaining approximations $\mathrm{u_n} = 0, \mathrm{v_n} = 0, \mathrm{w_n} = 0$, $\mathrm{n} > 1$ can be completely determined such that each term will be determined using the previous term; thus, the exact solution is as follows:

$u(x,t) = lim_{p\to\infty} \sum_{i=0}^{p} v_1^k(x,t) = e^{x+y-t},$

$v(x,t) = lim_{p\to\infty} \sum_{i=0}^{p} v_2^k(x,t) = e^{x-y+t},$

$w(x,t) = lim_{p\to\infty} \sum_{i=0}^{p} v_3^k(x,t) = e^{y-x+t}.$

Example 2: We consider the following system of P.D.E:

$u_t - wu_x + u = 1,$ (14)

$w_t - uw_x - w = 1.$

Subjected to the initial conditions,

$u(x,0) = e^x, \qquad w(x,0) = e^{-x}.$

Here, linear operator is selected as $L_1u(x,t) = u_t$ and $L_2w(x,t) = w_t$. Therefore, $\varphi_i\left(v_i^k\right)$ is defined as:

$\varphi_1(v_1^k, v_2^k) = \int_0^t(v_2^k(v_1^k)_x - v_1^k + 1)d\tau,$ (15)

$\varphi_2(v_1^k, v_2^k) = \int_0^t(v_1^k(v_2^k)_x + v_2^k + 1)d\tau.$

Then, using Eq. (14), the LIM method formulae in t-direction for the calculation of the approximate solution of Eq. (15) can be readily obtained as:

$u_{n+1}(x,t) = \sum_{i=0}^{i} v_1^k(x,t) = u_0(x,t) + \int_0^t\left(w_p\left(u_p\right)_x - u_p + 1\right)d\tau,$ (16)

$w_{n+1}(x,t) = \sum_{i=0}^{i} v_2^k(x,t) = w_0(x,t) + \int_0^t\left(u_p\left(w_p\right)_x + w_p + 1\right)d\tau,$

where the initial approximation must be satisfied by the following equations:

$L_1u(x,t) = 0, \quad u(x,0) = e^x,$

$L_2w(x,t) = 0, \quad u(x,0) = e^{-x}.$

Therefore, it can be started by:

$u_0(x,t) = v_1^0 = e^x,$

$w_0(x,t) = v_2^0 = e^{-x}.$

Accordingly, by Eq. (16), the higher order approximation of the exact solution can be obtained in the following way:

$u_1(x,t) = \sum_{i=0}^{1} v_1^k(x,t) = (1-t)e^x,$

$w_1(x,t) = \sum_{i=0}^{1} v_2^k(x,t) = (1+t)e^{-x},$

$u_2(x,t) = \sum_{i=0}^{2} v_1^k(x,t) = \left(1 - t + \frac{t^2}{2!}\right)e^x,$

$w_2(x,t) = \sum_{i=0}^{2} v_2^k(x,t) = \left(1 + t + \frac{t^2}{2!}\right)e^{-x},$

$u_3(x,t) = \sum_{i=0}^{3} v_1^k(x,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!}\right)e^x,$

$w_3(x,t) = \sum_{i=0}^{3} v_2^k(x,t) = \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!}\right)e^{-x}.$

The remaining approximations $\mathrm{u_n}, \mathrm{w_n}, \mathrm{n} > 3$ can be completely determined such that each term is determined by the previous term; thus, the exact solution is given in the closed form as:

$u(x,t) = lim_{p\to\infty} \sum_{i=0}^{p} v_1^k(x,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right)e^x,$

$w(x,t) = lim_{p\to\infty} \sum_{i=0}^{p} v_2^k(x,t) = \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right)e^{-x},$

which again converges to the closed-form solutions,

$u(x,t) = e^{x-t} \qquad w(x,t) = e^{-x+t}.$

Example 3: Considering the following system of P.D.E:

$u_t + u_x - 2w = 0,$ (17)

$w_t + w_x + 2u = 0,$

With the initial conditions:

$u(x,0) = sin(x),$

$w(x,0) = cos(x).$

Here, auxiliary linear operator is selected as $L_1 u(x,t) = u_t$ and $L_2 w(x,t) = w_t$.

Therefore, $\varphi_i(v_i^k)$ is defined as:

$$\varphi_1(v_1^k, v_2^k) = \int_0^t (-(v_1^k)_x + 2v_2^k) d\tau, \quad (18)$$

$$\varphi_2(v_1^k, v_2^k) = \int_0^t (-(v_2^k)_x - 2v_1^k) d\tau.$$

Using Eq. (17), the LIM method formulae in t-direction for the calculation of the approximate solution of Eq. (18) can be readily obtained as:

$$u_{n+1}(x,t) = \sum_{i=0}^{i} v_1^k(x,t) = u_0(x,t) + \int_0^t \left((u_p)_x + 2w_p \right) d\tau, \quad (19)$$

$$w_{n+1}(x,t) = \sum_{i=0}^{i} v_2^k(x,t) = w_0(x,t) + \int_0^t \left(-(w_p)_x - 2u_p \right) d\tau,$$

Where the initial approximation must be satisfied by the following equations:

$L_1 u(x,t) = 0, \quad u(x,0) = sin(x),$

$L_2 w(x,t) = 0, \quad u(x,0) = cos(x).$

Therefore, we begin with:

$u_0(x,t) = v_1^0 = sin(x),$

$w_0(x,t) = v_2^0 = cos(x).$

Accordingly, by Eq. (19) the higher order approximation of the exact solution can be obtained

as follows:

$$u_1(x,t) = \sum_{i=0}^{1} v_1^k(x,t) = sin(x) + tcos(x),$$

$$w_1(x,t) = \sum_{i=0}^{1} v_2^k(x,t) = -tsin(x) + cos(x),$$

$$u_2(x,t) = \sum_{i=0}^{2} v_1^k(x,t) = sin(x) + t\,cos(x) - \frac{t^2}{2} sin(x),$$

$$w_2(x,t) = \sum_{i=0}^{2} v_2^k(x,t) = -tsin(x) + cos(x) - \frac{t^2}{2} cos(x).$$

$$\vdots$$

The remaining approximations $u_n, w_n, n > 3$ can be completely determined such that each term is determined by the previous term; thus, the exact solution is given in the closed form as:

$$u_p(x,t) = \sum_{i=0}^{p} v_1^k(x,t) = sin(x)\left(1 - \frac{t^2}{2!} + \cdots + \frac{t^{2p}}{(2p)!}\right) + cos(x)\left(t - \frac{t^3}{3!} + \cdots + \frac{t^{2p+1}}{(2p+1)!}\right),$$

$$w_p(x,t) = \sum_{i=0}^{p} v_2^k(x,t) = cos(x)\left(1 - \frac{t^2}{2!} + \cdots + \frac{t^{2p}}{(2p)!}\right) - sin(x)\left(t - \frac{t^3}{3!} + \cdots + \frac{t^{2p+1}}{(2p+1)!}\right),$$

which again converges to the closed-form solutions,

$u(x,t) = sin(x+t),$

$w(x,t) = cos(x+t).$

Example 4: We consider the following non-linear coupled Schrodinger-KdV equations [17]:

$$iu_t - (vu + u_{xx}) = 0, \quad i^2 = -1, \quad (20)$$

$$v_t + 6uv_x + v_{xxx} - (|u|^2)_x = 0, \quad i^2 = -1,$$

Subjected to the initial conditions:

$u(x,0) = 6\sqrt{2} e^{i\alpha x} k^2 sech^2(kx),$

$v(x,0) = \frac{\alpha + 16k^2}{3} - 16k^2 tanh^2(kx),$

where α and k are arbitrary constants.

In order to solve system (20) using LIM method, the auxiliary linear operator is selected as $L_t u(x,t) = u_t$ .Also Eq. (20) in an operator form can be written as:

$$L_t u(x,t) = iu_t = \overbrace{(vu + u_{xx})}^{w_1(u,v)}, \quad (21)$$

$$L_t v(x,t) = v_t = \overbrace{(-6uv_x - v_{xxx} + (|u|^2)_x)}^{w_2(u,v)}.$$

Therefore, $\varphi_k(v_i)$, for $k = 1,2$ is defined as:

$$\varphi_k(p_i) = \int_0^t w_k((u_i, v_i)(x,\tau)) d\tau \quad k = 1,2. \quad (22)$$

Then by using the Eq. (20), the LIM method formulae in t-direction for calculation of the analytical approximate solution of equation (22), can be readily obtained as follows:

$$u_{n+1}(x,t) = \sum_{i=0}^{n+1} p_i = u_0(x,t) - i \int_0^t \left[\frac{\partial^2 u_n(x,\gamma)}{\partial x^2} + v_n(x,\gamma) u_n(x,\gamma) \right] d\gamma, \quad (23)$$

$$v_{n+1}(x,t) = \sum_{i=0}^{n+1} q_i = u_0(x,t) + \int_0^t [-6u_n (v_n)_x(x,\gamma) - (v_n)_{xxx} + (|u_n(x,\gamma)|^2)_x] d\gamma,$$

In which the subscript n indicates the n^{th} approximation. Considering the given initial values, we can select:

$u_0(x,t) = p_0 = 6\sqrt{2} e^{i\alpha x} k^2 sech^2(kx),$

$v_0(x,t) = q_0 = \frac{\alpha + 16k^2}{3} - 16k^2 tanh^2(kx).$

By substituting this selection in (23) one obtain the following successive approximations:

$$u_1(x,y) = \sum_{i=0}^{1} p_i = \frac{-1}{cosh^4(kx)}\Big(2\sqrt{2}e^{i\alpha x}k^2(-3cosh^2(kx) - 3i\alpha^2 t\, cosh^2(kx) + +12\alpha\, t\, k\, sinh(kx)\, cosh(kx) - 20i\, tk^2\, cosh^2(kx) + 30i\, t\, k^2 + i\, t\, \alpha\, cosh^2(kx)\ )\Big),$$

(24)

⋮

$$v_1(x,y) = \sum_{i=0}^{1} q_i = \frac{-1}{3cosh^5(kx)}\big(\alpha\, cosh^5(kx) - 32\, k^2\, cosh^5(kx) + 48k^2\, cosh^3(kx) + 3456\sqrt{2}e^{i\alpha x}k^5 t\, sinh(kx) + 384t\, k^5\, sinh(kx)\, cosh^2(kx) - 1152\, t\, k^5\, sinh(kx) + 432\, i\, e^{2i\alpha x}k^4\alpha\, t\, cosh(kx) - 864\, e^{2i\alpha x}k^5\alpha\, t\, sinh(kx)\big).$$

⋮

And so on. In the same manner, the rest of components of the iteration formula can be obtained. To verify numerically whether the proposed LIM method leads to higher accuracy, we can evaluate the numerical solutions by using its n^{th} approximations. Also u_n the n^{th} approximation is accurate for quite low of n (n = 3). The obtained numerical result is summarized in Fig. 1 to Fig.13. From these results we can conclude that the method, LIM for system of the nonlinear Schrödinger equations, gives remarkable accuracy in comparison with its exact solution. The behavior of the solutions obtained by LIM is shown for a different values of times in comparison with the exact solution, Fig. 1 to Fig. 13.

IV. EXPLANATION ON RESULTS OF NONLINEAR SCHRÖDINGER EQUATIONS

Fig.1 and 3 are the exact solution for $u(x,t)$ and Fig. 5 and 7 are the exact solution for $v(x,t)$ with fixed values of α = 0.05 and k = 0.05 for different values of time. And also Fig.2 and 4 are the numerical results for $u(x,t)$ and Fig. 6 and 8 are the numerical results for $v(x,t)$ which that have been obtained by using the Laplace iteration method with a fixed values of $\alpha = 0.05$ and $k = 0.05$, for a different values of time. In Figs. 10 and 13 comparing of the exact solution for $v(x,t)$ with a fixed values of α=0.05 and k =0.05, for $0 \le t \le 10$ and $-100 \le x \le 100$ with the numerical results of LIM has been placed. And also in Fig. 9 comparing the exact solution for $u(x,t)$ with the numerical results obtained by the Laplace iteration method for fixed values of α = 0.05 and k = 0.05, t =10 and $-1000 \le x \le 1000$ has been illustrated. Also in Fig.11 comparison between the exact solution for $u(x,t)$ and the numerical results obtained by the Laplace iteration method for fixed values of $\alpha = 0.05$ and $k = 0.05$, $t = 10$ and $-100 \le x \le 100$ has been considered. And finally in Fig.12 comparison between the exact solution for $u(x,t)$ with a fixed values of α = 0.05 and k = 0.05, for $0 \le t \le 10$ and $-100 \le x \le 100$ with the numerical results of LIM has been considered.

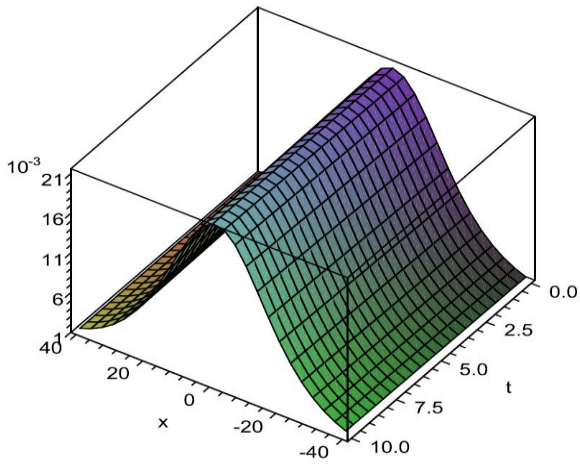

Figure 1. Exact Solution of u(x,t)

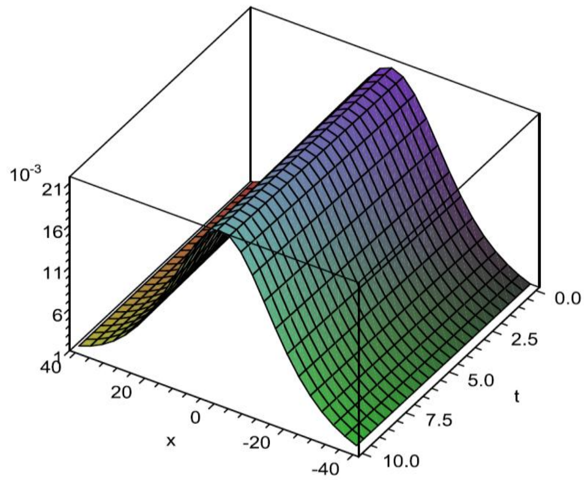

Figure 2. Approximate Solution (LIM) of u(x,t)

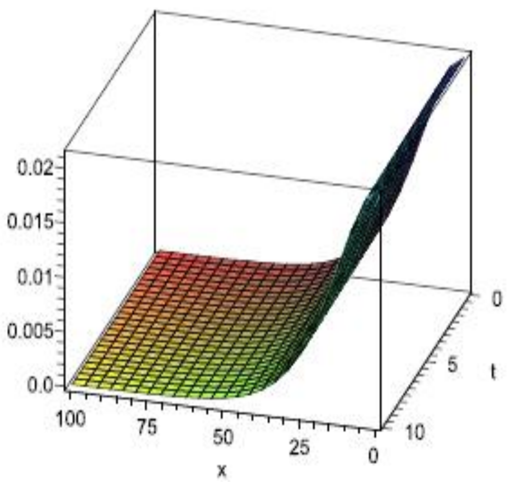

Figure 3. Exact Solution of u(x,t)

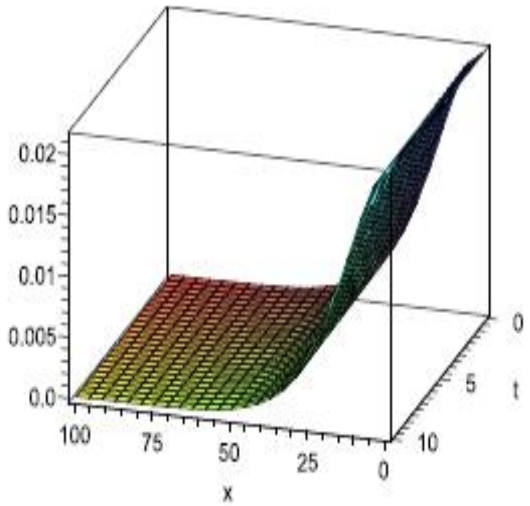

Figure 4. Approximate Solution(LIM) of u(x,t)

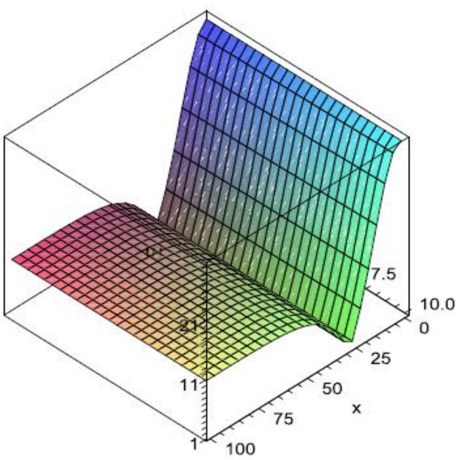

Figure 5. Exact Solution of V(x,t)

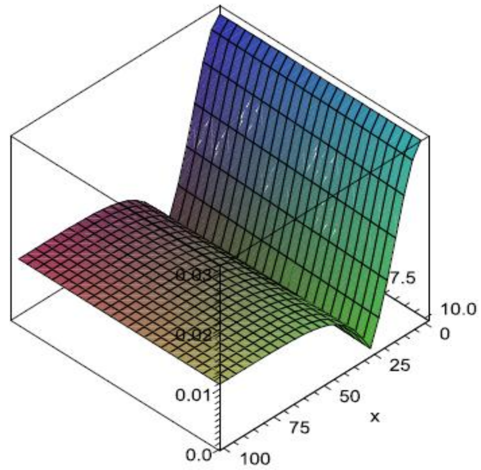

Figure 6. Approximate Solution(LIM) of V(x,t)

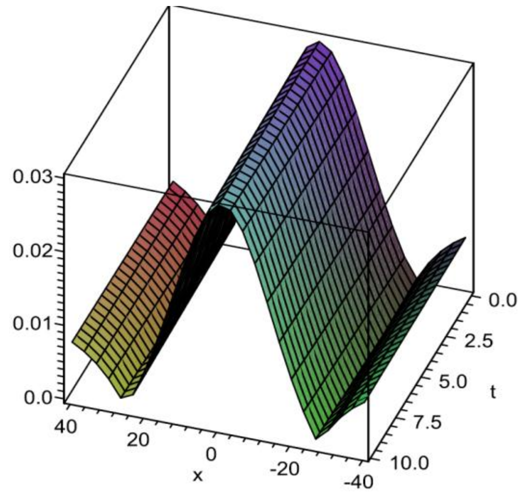

Figure 7. Exact Solution of V(x,t)

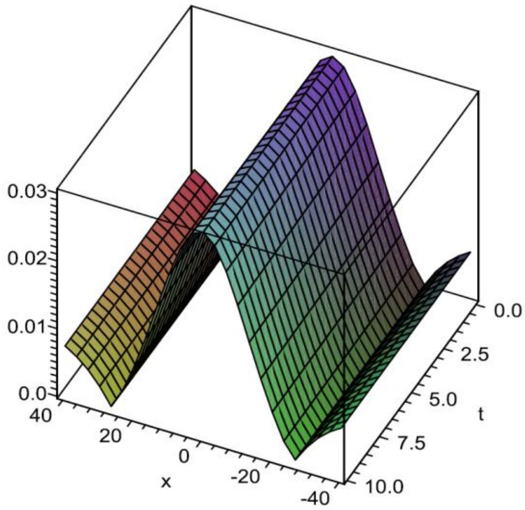

Figure 8. Approximate Solution(LIM) of V(x,t)

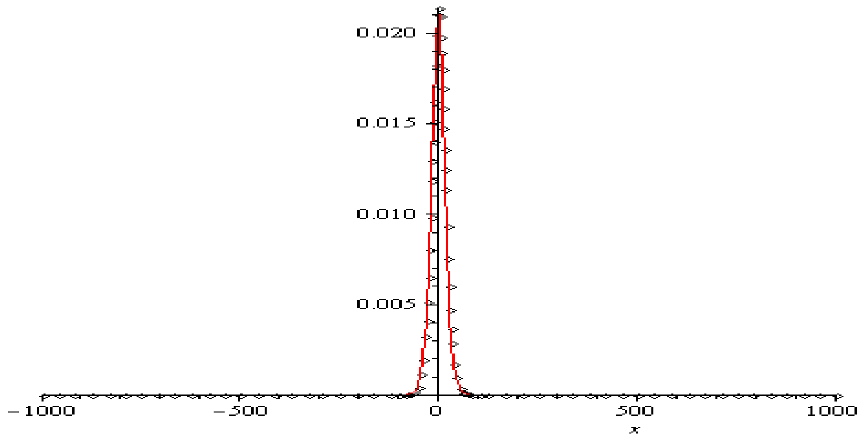

Figure 9. Comparing the exact solution (Red Line) for $u(x,t)$ with the numerical results obtained by the Laplace iteration method (Black Point) for $u(x,t)$ for fixed values of $\alpha = 0.05$ and $k = 0.05$, $t = 10$ and $-1000 \le x \le 1000$.

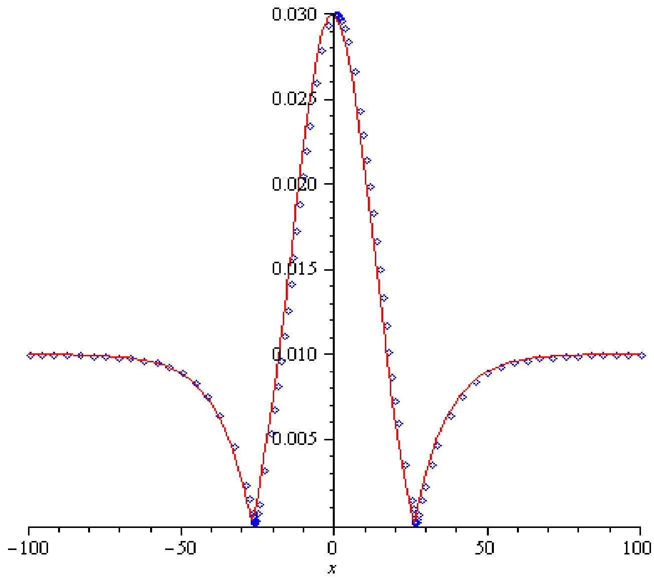

Figure 10. Comparing the exact solution (Red Line) for $v(x,t)$ with the numerical results obtained by the Laplace iteration method (Blue Point) for $v(x,t)$ for fixed values of $\alpha = 0.05$ and $k = 0.05$, $t = 5$ and $-100 \le x \le 100$.

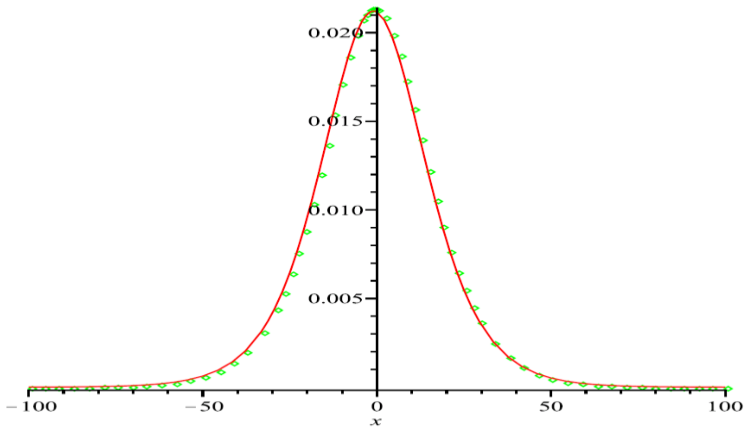

Figure 11. Comparing the exact solution (Red Line) for $u(x,t)$ with the numerical results obtained by the Laplace iteration method (Green Point) for $u(x,t)$ for fixed values of $\alpha = 0.05$ and $k = 0.05$, $t = 10$ and $-100 \le x \le 100$.

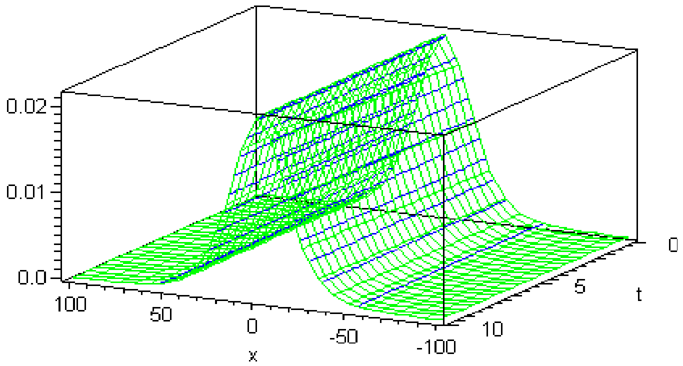

Figure 12. Comparing the exact solution for $u(x,t)$ with a fixed values of $\alpha = 0.05$ and $k = 0.05$, for $0 \le t \le 10$ and $-100 \le x \le 100$ with the numerical results (LIM) for $u(x,t)$

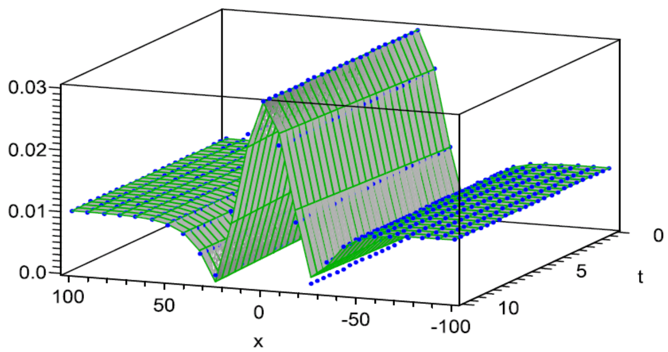

Figure 13. Comparing the exact solution for $v(x,t)$ with a fixed values of $\alpha = 0.05$ and $k = 0.05$, for $0 \le t \le 10$ and $-100 \le x \le 100$ with the numerical results (LIM) for $v(x,t)$.

V. DISCUSSION

In this letter, Laplace Iteration Method (LIM) was successfully employed to obtain the approximate analytical and exact solutions of systems of linear and nonlinear PDEs. Comparisons with the exact solutions revealed that LIM was very effective and convenient. The first main goal was to employ the powerful LIM to investigate system of partial differential equations and coupled Schrodinger-KdV equations the second one was to show the power of this method and its significant features by numerical results. The two goals were achieved.

Evidently, the method presented rapidly convergent successive approximations without any restrictive assumptions or transformation which may change the physical behavior of the problem. Laplace iteration method gave several successive approximations through by using the LIM's iteration relation. Moreover, the LIM reduced the size of calculations and also the iteration was direct and straightforward. The LIM used the initial values for selecting the 0^{th} approximation. Also, boundary conditions, given for bounded domains, could be used for the justification only.

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