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AN INVESTIGATION OF THE CONCEPT OF VARIABLE IN TURKISH ELEMENTARY MATHEMATICS TEACHERS' GUIDEBOOKS

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Abstract
This research was aimed to investigate the presentation of the concept of variable in the algebra learning domain of Turkish elementary mathematics teachers' guide books. Furthermore, it is particularly focused on whether explicit guidance on the teaching of the concept of variable for teachers is in these books. For this purpose, mathematics teachers' guide books for sixth, seventh, and eighth grade level were examined by using document analysis as the method of research. Results indicated that variables were situated in the books with the meanings of pattern generalizer, unknown, placeholder, parameter, varying quantity and arbitrary symbols. Furthermore, although pattern generalizer was being touched in all grade levels, parameter and arbitrary symbols were only given in eight grade levels. On the other hand, explicit guidance for teachers was not found in the books. There was limited information about students' misconceptions and different definitions of variable in the teachers' guide books of all grade levels.

Key words: The concept of variable, elementary mathematics teachers' guidebooks, algebra

INTRODUCTION

Algebra is defined as a world of difference (Stacey & Kendal, 2004; Usiskin, 1988). To be more precise, algebra is conceived of different approaches as "*a way of expressing generality and pattern*", "*a study of symbol manipulation and equation solving*", "*a study of functions and their transformations*", "a *way to solve problems*", and "*modeling*" (Berdnaz, Kieran & Lee, 1996). The concept of variable is core element in all approaches due to its multiple meanings. Based on the importance of variables, some researchers aimed to explore the uses of variables in school algebra (Philipp, 1992; Schoenfeld & Arcavi, 1988; Usiskin, 1988; Wagner, 1999). They identified the uses of variable in school algebra as *labels*, *objects*, *unknowns*, *varying quantities*, *constants*, *parameters*, *generalized numbers*, *placeholders*, *arguments*, and *abstract symbols*.

Despite the heavy usage of variables in school mathematics and other school subjects, studies indicate that students of all grade levels have problems working with variables (Booth 1984, 1986; Davis, 1984; Kieran, 1988, 1992; Kuchemann, 1978, 1981; Philipp, 1992, Schoenfeld & Arcavi, 1988; Ursini & Trigueros, 1997; Usiskin, 1988; Wagner, 1999). The most important results of these studies are summarized to review the literature. In general, students rarely interpret algebraic letters standing for variables because they get used to studying numbers (Kuchemann, 1978, 1981; Macgregor & Stacey, 1997; Sleeman, 1984). After being familiar with letters as variables, they are more likely to conceive and use them as *unknown* than *varying quantity* and *generalized numbers* (Kieran, 1992; Kuchemann, 1978). In addition to these difficulties, results of some studies revealed that students who are 11-12 years old commonly interpret letters as standing for object or words (Booth, 1986; 1988; Kuchemann, 1981; Macgregor & Stacey, 1997). The reason may be teachers' explanations like "*A = l* x *w*" as "area equals length times width". Another important result is that when students accept that letters are representing for numbers, they tend to associate letters with their positions in the alphabet. For instance, they suppose the bigger one is c among a, b and c (Watson, 1990). The reason may be the numbering schema in algebra textbooks. Furthermore, a misbelief that letters always stands for numbers leads missing the point of numbers in general like $a + b = b + a$(Booth, 1986). Separately, several researchers have documented students' difficulties while they are solving word problems involving two variables as *varying quantities* (Macgregor & Stacey, 1993; White & Mitchelmore, 1996; Wollman, 1983). They concluded that misuses of letters occur due to the incorrect translation among word sentences and algebraic equations.

There are also several studies researching about Turkish elementary school students' difficulties about variables in algebra (Akgün & Özdemir, 2006; Dede & Argün, 2003; Soylu, 2008). The results show strong similarities with the results of international studies. In this regard, it is concluded that students are generally confused due to multiple meanings of variables in mathematics context according to the results of national and international studies. However, almost all studies conducted to define the types of variables and to identify students' difficulties and misconceptions about variables, especially in Turkey. To know what researchers conclude about students' difficulties and misconceptions is important, but it is necessary to find the reasons of them and to put solutions instead of replication studies. In this sense, well-defined teacher training programs and curricula may be designed to eliminate them. However, related literature indicates that there is less emphasis on the importance of curriculum and textbooks in teaching and learning of the concept of variable although they are powerful indicators to present what students learn and what they do learn (NCTM, 2000).

While many studies have investigated students' as well as teachers' understanding and misconceptions of variables (Clement, Narode, & Resnick, 1981; Küchemann, 1978; Philipp, 1992; White & Mitchelmore, 1996), very few studies have examined the extent to which this concept is introduced or developed in school mathematics curriculum, and the trend of its development over time in mathematics textbooks. Specifically, a few studies were found in the research literature that examined, how the concept of variables is introduced and developed in middle-grades mathematics curricula (Nie, Cai & Moyer, 2009; Dogbey, 2010) whereas textbooks as major curriculum materials that many students and teachers use, and their prospects in impacting students' opportunities to learn and acquire facility with the use of vital concepts such as variables in school mathematics (Robitaille & Travers, 1992). From this standpoint, the aim of this study was to investigate the presentation of the concept of variable in the Turkish elementary mathematics textbooks. Additionally, it is particularly focused on whether explicit guidance related to the teaching of the concept of variable for teachers is in textbooks. Specifically, following research questions are proposed in this article:

1. How the concept of variable is introduced in the textbooks?
2. What kind of guidance is provided to teachers who use Turkish elementary mathematics textbooks for algebra tasks related the concept of variable?

The findings of this research might make a contribution to the organization of curriculum and teachers' guidebooks in terms of the presentation of the concept of variable. Another important point is the effect of materials on the teachers' instructional approaches and their pedagogical strategies which they use while teaching mathematical concepts. Indeed, the implemented curriculum often closely mirrors the content and pedagogical approach presented in textbooks (Reys, Reys, Lapan, Holliday,& Wasman, 2003). Consequently, the results of this study could make a great contribution to design of future mathematics education programs, to enhance classroom instruction and to improve students' learning.

METHOD

This study is based on documental analysis and the use of content analysis. In this study, three documents published by the Turkish Ministry of Education were examined to identify variables in algebra sub learning domain of sixth, seventh, and eighth grade mathematics teachers' guidebooks which published in 2012 by the Minister of Education in Turkey. These mathematics teachers' guidebooks are used in all public schools in Turkey because the textbooks are given free to the teachers. In addition, the mathematics textbooks can be classified according to five main content areas such as number, geometry, measurement, algebra, probability and statistics (NCTM, 2000). Additionally, teachers' guidebooks involve all information in the student books. As a result, this study focused on the analysis of algebra content presented in the sixth, seventh and eighth grade mathematics teachers' guidebooks.

Analysis Plan

Analysis plan is prepared by using a framework based on how algebraic ideas are introduced and developed in various curricula (Cai, 2004; Cai et al., 2005). The framework concentrates on the goal specification, content coverage and process coverage of the curricula being analyzed. In this paper, the focus of analysis is to convey how the concept of variable is treated in the elementary school mathematics teachers' guidebooks. For this purpose, the three teachers' guidebooks were analyzed using an instrument consisting on a framework for

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content presentation including analysis of tasks and another framework for the *guidance for teachers* presented in Table 2.

Table 2. Analytical Framework for the Study

Frameworks	Block to examine	Examples of task to examine
Framework 1	Content Presentation	- Different uses of variable across grade levels and types of activities and tasks - Definitions of the concept of variable
Framework 2	Guidance for Teachers	- Explicit notes on the different uses of variable - Alerts and suggestions on the students misconceptions/ difficulties

The first framework takes into consideration the structure, organization and content of the concept of variable in the textbook. The second framework is specifically related to the support for teachers and includes three points: whether there are explicit notes on the different uses of variable, alerts on the students' misconceptions/difficulties and suggestions to these misconceptions/difficulties.

FINDINGS

Different Uses of Variable across Grade Levels

In the curriculum, the content area of algebra is divided into three sub learning domain for all grade levels: (1) patterns and relationships, (2) algebraic expressions, and (3) equations. General structure of curriculum regarding uses of variable is demonstrated in Table 3.

Table 3. *General Structure of Curriculum regarding the Concept of Variable*

Variable type	Sub learning Domain	Remaining Time (Hours)		
		Grade 6	Grade 7	Grade 8
Placeholder or unknown	Algebraic Expressions/Equations	7	10	-
Pattern generalizer	Patterns and Relationships	7	9	3
Varying quantity		-	6	6
Parameter	Algebraic Equations	-	-	8
Arbitrary symbols		-	-	9

Table 3 summarizes the place of variable in algebra content according to grade level, sub learning domain, and remaining time for every use of variable. All these indicate that variables are situated in the curriculum with the meanings of "*pattern generalizer*", "*unknown*", "*placeholder*", "*parameter*", "*varying quantity*", and "*arbitrary symbols*". Additionally, allocated time (hours) indicates the heavy use of variable as a placeholder and unknown in the textbooks. According to table, although patterns are being touched in all grade levels parameter and arbitrary symbols are only given in eight grade textbooks. Furthermore, the time remained for use of variable as pattern generalizer was more than the time remained for other uses of variable.

Presentation of the Concept of Variable in the Teachers' Guidebooks

Variable as pattern generalizer*:* Patterns were added as a new subject in the curriculum from grade 1 to grade 8 after the revision of curriculum in 2005. Definition of variable as a pattern generalizer is stated in the sixth grade textbook at below:

> *Definition*: *n is a sign, symbol or notation which show the order and place of numbers in a pattern. This letter is emphasized as a variable* (MEB, 2012a, p. 89*)*.

Beside explicit definition of variable as pattern generalizer**,** in activities, students are provided ample opportunities to make generalizations through number pattern activities. For instance, while there is an expectation of identifying relationship in a pattern, including objects and geometric shapes in the content area of geometry for K-2, the expectancy in grade 3-5 is based on the relationship among numbers with patterns. Afterwards, patterns are broadened throughout the grade 6-8 by giving specific examples as Fibonacci numbers

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and Pascal triangle. From this point, it is concluded that there is coherence in the grade levels of textbooks for the usage of variable as a pattern generalizer.


Aşağıda modellenen sayı örüntüsünü inceleyiniz.
a) Verilen örüntüyü bir adım daha devam ettirerek tabloyu tamamlayınız.
b) Verilen örüntünün temsilci sayısını "n" ile göstererek kuralını yazınız. n^2

Örüntü					
Adım	1.	2.	3.	4.	5.
Birim Dikdörtgen Sayısı	1	4	9	16	25

8, 16, 24, 32, ... şeklinde verilen örüntünün temsilci sayısını "k" ile göstererek kuralını bulunuz. 8 n

12, 13, 14, 15, ... şeklinde verilen örüntünün temsilci sayısını "b" ile göstererek kuralını bulunuz. 11 + n

Figure 1: An example for the concept of variable as pattern generalizer

At the same time, teacher guidebooks are to give opportunity to establish connection with numbers and other content domain such as geometry in variable as pattern generalizer activities. Furthermore, activities were prepared based on multiple representations as in Figure 1 which taken from page 89 of sixth grade mathematics teacher guidebook (MEB, 2012a).

Variable as unknown or placeholder: Another type of variable as *unknown* and *placeholder* was seen in the sub-learning domain of algebraic expressions and equations with the definition of;

> *Definition: Letters used in algebraic expression represent numbers and called as variable or unknown* (MEB, 2012a, p.256)

The definition of variable as letters that stands for numbers, has been criticized as the source of many difficulties faced by beginner algebra students (Booth 1988; Mac Gregor, 1986). They defined "fruit salad algebra" to describe an approach to algebra in which students choose variables as objects or labels rather than as numbers (Mac Gregor, 1986; Oliver, 1984). To help prevent this confusion, more useful and powerful models such as algebra tiles have been suggested to give concrete meaning to symbolic manipulation. Herein, elementary mathematics curriculum has many examples including algebraic tiles about the *unknown and placeholder use of variable* in grade 6-7.

On the other hand, students begin to learn algebra concepts in the sixth grade with the algebraic expressions with the real life examples which require transformation from verbal to symbolic language such as "a^2 represents area of a square". It is a good starting point to learn the use of variable as *unknown* or *placeholder*, but related objectives are not suitable with cognitive skills. For instance, the objective of "*students will be able to write suitable algebraic expressions for specific conditions*" in the sixth grade level. In this example, the objective cannot reflect high levels of cognition according to Bloom's (1956) Taxonomy and Marzano and Kendall's (2008) new taxonomy. The focal point in the activity is associated with making a transition from verbal statements to algebraic expressions by developing an understanding about utilization of letters as variables, specifically *unknown*, but objective is not compatible with the goal of activity.

Variable as a varying quantity and parameter*:* Variable as *varying quantities* within a functional relationship which Smith (2003) describes as "representational thinking that focuses on the relationship between two (or more) varying quantities" is seen in the sub learning domain of algebraic expressions and equations at seventh and eighth grade level. In seventh grade teacher guidebook stated some explanations given below:

> *Explanation*: *Teachers emphasize that linear equations include terms having a fixed number and two variables as ax + by + c = 0. Parameter of a and b never be equal to zero at the same time* (MEB, 2012b, p.154)

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These explanations indicate that x and y are explicitly stated as variables for displaying varying quantities. Furthermore, the use of variable as a varying quantity begins with the objective of "*students will be able to explain linear equations*" in the seventh grade. Then, multiple representations such as Cartesian coordinate system and graphs with real life context are used to explain linear relationship between quantities in a linear equation. It is strength of the textbooks because use of multiple representations will help students to understand the relationship between two varying quantities in the meaningful and contextualized situations rather than seeing variables in equations simply as objects to manipulate. However, the use same letters such as x, y in all examples lead to students misconceptions and difficulty in terms of symbolization. For instance, students may suppose that only $y = ax + b$ represents a functional relationship. It is suggested that there should be different letters representing variables in the textbooks.

On the other hand, there is no explicit emphasis on the use of variable as *parameter*. Real life examples and models were used for slope in algebraic equations of the eighth grade mathematics teacher guidebook (p.196) like in Figure 2 to demonstrate the concept of variable as a parameter (MEB, 2012c).

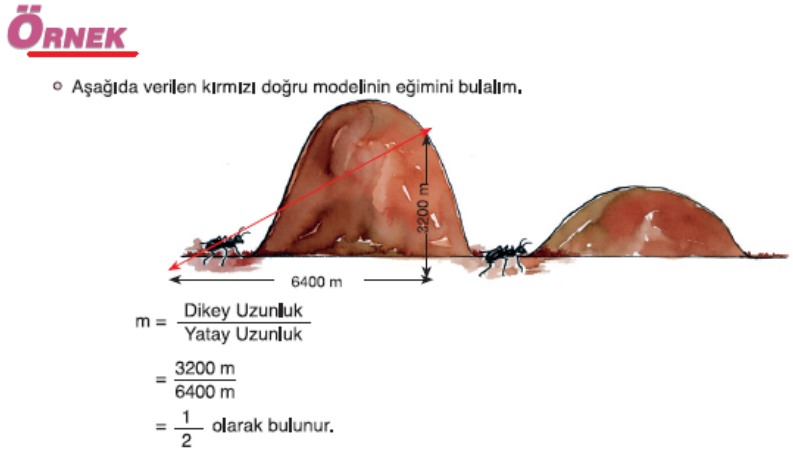

Figure 2: An example for the concept of variable as a parameter

In the eighth grade mathematics teacher guidebook, slope is defined as the ratio of vertical length to horizontal length. For this reason, students do not perceive slope as a type of variable as well as teachers do. In addition, there are many similar worked-out examples focused more on computational procedures than the meaning of concept.

Variable as arbitrary symbols: In this conception, variables are used as arbitrary symbols without number referent. For instance, students asked to factor expression such as x^2 +3x+2 is more interested in their manipulation ability rather than their ability to solve for the variable. Textbook highlighted the point of difference between variable in an equation and in a factorization by giving some explanation given below:

> *Explanation: All values given to variable make true a factorization, but only some reel number(s) satisfy equations* (MEB, 2012c, p. 92)

Furthermore, there is less emphasis on the use of *variable as abstract symbols* in the factorization subject eighth grade level by using algebraic tiles as in Figure 2 which taken from page 93 of 8th grade mathematics teacher guidebook(MEB, 2012c, p. 93).

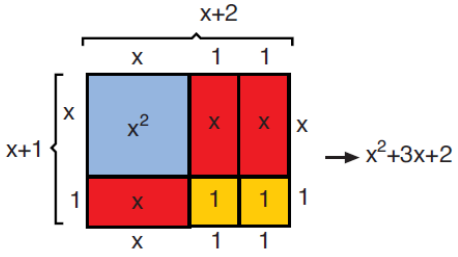

Figure 3: An example for the concept of variable as arbitrary symbols

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This finding supports Usiskin's (1988) claim that the least used category of variables in school mathematics is that of abstract symbol. This might be due to the fact that many of the concepts that employ variables as an *abstract symbol* are not of major focus in the middle grades mathematics curriculum.

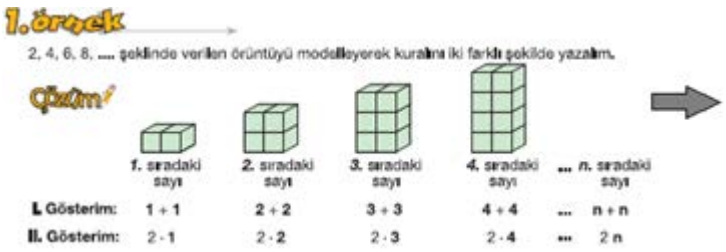
Guidance for Teachers in Mathematics Textbooks

For this part, it was examined the teacher's edition textbooks in detail, which contains the student activities and sidebar notes to teachers regarding how to implement those activities. It was developed *foci of guidance* (FOG) based on research work (especially Ball & Cohen, 1996; Davis & Krajcik, 2005) that suggests possible ways in which K-12 curriculum materials can promote teacher learning.

> **FOG 1**: *Explanation and explicit notes on how students engage in tasks about different uses of variables* (drawing on the work of Ball & Cohen, 1996; Davis & Krajcik, 2005). This form of guidance is important since the concept of variable leads problems for students because of its multiple meanings. Thus, teachers need to know mathematical and pedagogical reasons in these tasks.
>
> **FOG 2**: *Alerts on how to manage students' approaches to a task related to concept of variable* (drawing on the work of Ball & Cohen, 1996; Davis & Krajcik, 2005). Curriculum materials have importance to help teachers in terms of giving information about students' correct and incorrect solutions, misconceptions and giving suggestions how to handle these misconceptions. By so doing, curriculum materials can reinforce teachers' pedagogical content knowledge in the domain of algebra.

In this respect, it was examined the teachers' edition of selected mathematics textbooks with particular attention to FOG 1 and FOG 2.

***Definitions and explanations**:* Although the variable is formally defined in the sixth grade textbook, teachers are not reminded that variables can represent many numbers simultaneously, that they have no place value, and that representations of variables can be selected arbitrarily. Furthermore, an important detail in a prominent is that there is no explicit distinction between definitions of unknown and variable in the textbooks. Besides, teachers were expected to make the emphasis on the difference between unknown and variable in sixth and seventh grade textbooks. However, there is no enough information about how they emphasize this distinction for their students. On the other hand, the explanations and suggestions about how teachers make emphasis on letters as "*pattern generalizer*" to the students are not clear and adequate. Specific examples and related explanations are given in Figure 4 from the page 89 of teacher guidebook for the sixth grade level.



Explanation: n is called as general number of a pattern. It is emphasized that this letter is a variable.

Figure 4. An example for the concept of variable as pattern generalizer

Because of such imperfect explanations given and a lot of similar types of questions related patterns, students may find the constant and reach the general rule without receiving the role of variable as a generalizer in reality. That is, they may not construct well-developed notion for the general nature of the pattern due to the implementation of a procedure to develop a correct symbolic expression. It is suggested that some additional explanations may be added in curriculum and teachers' books. As a result, teachers will be able to use expressions to generalize, rather than focusing on answers and the steps to get the answer. Moreover, it can be concluded that there were limited instances where support was provided to enhance teachers' content knowledge on variable ideas in the teachers' textbooks.

***Teachers' attention about students' common misconceptions/ difficulties of variables and suggestions**:* In the teacher edition textbooks, teacher were not informed explicitly about misconceptions that the majority of students may carry about variables although these misconceptions are defined clearly in the findings of

previous research. For example, there is a suggestion to teachers in the page 67 of teacher guidebook for seventh grade below:

Suggestion: Models which do not lead to misconceptions are used (MEB, 2012b, p.67).

This suggestion was given for the operations of algebraic expressions. However, it is not clear about which model teacher use and how they utilize them. Related literature indicates that teachers have difficulty to understand the complex nature of the concept of variable (Boz, 2007; White & Mitchelmore, 1996) and they rarely define students 'misconceptions on the use of variables (Asquith, Stephens, Knuth, & Alibali, 2007). Nevertheless, guidance for teacher is weak in all grade level textbooks.

RESULTS AND DISCUSSION

Results of this research indicated that variables are situated in the books with the meanings of pattern generalizer, unknown, placeholder, parameter, varying quantity and arbitrary symbols. Furthermore, although pattern generalizer was being touched in all grade levels, parameter and arbitrary symbols are only given in eight grade levels. The findings of this study also reflect some possible relationships between students' understanding of variable and its presentation in the curriculum. For instance, there is relatively large emphasis on variables as *placeholders* in the teacher guidebooks. Besides, related studies indicated that the majority of students, as well as some teachers consider variables as representing a single or unique number like a placeholder (Kieran, 1992; Mohr, 2008).

On the other hand, explicit guidance for teachers was not found in the books. There was limited information about students' misconceptions and different definitions of variable in the teachers' guide books of all grade levels. Because there are not enough reminders which assist teachers to become *alert to multiple meanings* of variables in various contexts, teachers may teach most of the time the concept of variable as a place holder or constant which lead misconceptions stated literature (Booth, 1986; 1988; Küchemann, 1981; Mac Gregor & Stacey, 1997). Curriculum developers may add some explicit information to remedy this problem. Furthermore, the big deficiency of curriculum is absence of well-organized design which reflects the use of variable as *generalized arithmetic* by using properties of numbers. According to literature, children are capable of making generalizations about numbers and operations (Carpenter & Franke, 2001; Fujii & Stephens, 2001; Kaput & Blanton, 2005; NCTM, 2000). However, students are not given the opportunity to construct connections between arithmetic and algebra (Kieran & Chalouh, 1992). At this point, the commutative, associative, and distributive properties of numbers may be utilized to develop the meaning of *variable as generalized arithmetic* in students' mind by constructing rich set of connections on their prior knowledge. It allows promoting *"algebra readiness"* for solving algebraic equations (Bottoms, 2003; Welder, 2007). In this sense, it is needed that a well-designed curriculum which aim to strengthen students' arithmetic problem-solving methods and to provide opportunities for the growth of algebraic thinking. When looking from this perspective, it is expected that the findings from this study will inform curriculum developers and evaluators of middle grades mathematics curriculum in their future organizations to develop classroom materials on variables for students to use as well as for teachers to implement in their classrooms. As a result, curriculum developers might consider changes to the treatment of variable ideas in future editions of middle grades mathematics textbooks to increase students' opportunities in order to relate with the other uses of variables.

As stated before, although there are a lot of studies expressing students' difficulties and misconceptions about variables in both national and international studies, little attention has been given to answer these questions: How can we eliminate these difficulties and misconceptions? How do we develop proposed instructions on the concept of variable to enhance students' algebraic thinking and reasoning processes in terms of variables? How do we detect effects of curriculum on teaching and learning of variables in classroom environment? In order to answer these important questions, in the light of a comprehensive literature review and curriculum evaluation on variables in algebra strand, it is proposed some suggestions which will be able to shape future research on the concept of variable in learning and teaching of school algebra in Turkey. In this sense, when considering the weakness of the curriculum about variables, some recommendations are made for curriculum developers. Firstly, in the curriculum, there is limited and inadequate information for teachers on the definitions of

variables: guidance on how to enact opportunities for students to work with variables in the classroom, cautions to students' misconceptions and difficulties with variables, suggestions for these difficulties. In this regard, curriculum developers may familiarize themselves with the research findings on students' difficulties and misconceptions of fundamental topics in school mathematics. Then, they might consider designing activities that will explicitly and carefully introduce students to the various uses of variables. If teachers and students are equipped with the right materials on the meaning and uses of variable, this will automatically lead to improved mathematics instruction and students' understanding of it. Secondly, although there is a heavy emphasis on problem solving as a critical aspect of understanding and as an integral feature of all subject areas in the curriculum, as stated before, there are similar types of problem situation throughout the sub learning domains. Nevertheless, repetition of similar types of problems on variables and restricted explanations about different solution strategies in the curriculum may limit students' creativity and construction process of concepts. As remedy, curriculum developer may add problems involving different solution strategies. To conclude, it is not forgotten that the concept of variable gives meaning to algebra and determines approach to teach it. From this perspective, by taking into consideration the results of the research being conducted these suggestions will be useful to educate well-trained teachers in terms of algebra learning and teaching and to design well-equipped curricula which meet the needs of society in Turkey.

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