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Research Article

Bilevel stochastic transportation problem with exponentially distributed demand

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Abstract

In this paper, we consider a bilevel stochastic transportation problem (BSTP) which is a two level hierarchical program to determine optimal transportation plan for a single product assuming that customers' demands for the product are stochastic, in particular, exponentially distributed random variables. In our model, we suppose that the leader and the follower operate two separate groups of plants in a decentralized firm. The leader, who moves first, determines quantities shipped to customers, and then, the follower decides his own quantities rationally. There are holding and shortage costs at the customer zones. The leader's objective is to minimize the sum of corresponding total transportation costs and the total expected holding cost. Holding costs can be negative which implies that the leader can sell excess quantities at some prices. Similarly, the follower's objective is to minimize the sum of the corresponding total transportation costs and the total expected shortage cost. Our proposed model is transformed into a single level nonlinear programming by using its Karush-Kuhn-Tucker (KKT) conditions, and then, it is applied with a branch and bound algorithm to obtain noncooperative solutions. A small numerical example is also given to illustrate our model.

Keywords: Bilevel programming, stochastic programming, stochastic transportation problem, exponentially distributed demand

1. Introduction

In practical optimization problems involving randomness, it is difficult to estimate modeling parameters accurately. Stochastic programming deals that kind of optimization problems under uncertainty in which the parameters are considered as random or stochastic variables to take into account the presence of uncertainty. In order to capture the impact of uncertainty, the original stochastic programming problem is usually transformed into a nonlinear deterministic equivalent problem by using probabilistic programming or two stage stochastic programming with recourse. Then standard solution techniques for nonlinear programming problems can be applied (Werner 2005).

To estimate unpredictable or uncertain problem parameters, each source of randomness is necessarily represented by a probability distribution. It can be assumed that certain random variables are exponentially distributed to simplify and to make the model mathematically tractable. The exponential distribution is a special case of both gamma and Weibull distributions. These distributions are relatively easy to work with and often allow good approximation to the actual distribution when data are highly variable. The exponential distribution is usually used to represent interarrival times of customers to a system (time between two independent events) that occur at a constant rate, and the time to the failure of a piece of equipment. (Law & Kelton 1991; Ross 2003). However, Holmberg and Tuy (1999), Daneva et al. (2010) used that distribution for customer demands in their test problems with randomly generated parameters.

Problems related to supply chain management can also have random or uncertain data in all stages, such as production and distribution from suppliers to customers. To assume otherwise is not realistic in most cases such as changing demand quantities, interarrival times between demands and prices. The purpose of the supply chain management is to coordinate suppliers, manufacturers, warehouses, retailers, transporters and customers simultaneously.

Transportation is an important part of the supply chain management. The conventional transportation problem, which is based on a network structure consisting of a finite numbers of sources and destinations, is encountered in most stages of a supply chain. Products are to be transported in such a way that the total transportation cost is minimum.

Bilevel programming (BP) is a nested hierarchical system where two decision makers act in a cooperative or noncooperative manner to optimize their individual objective functions. BP has been applied in fields which involve hierarchical relationship between two classes of decision makers, such as transportation networks, management, economic planning, engineering, chemistry, environmental sciences, optimal control, etc.

In this paper, we consider a bilevel structured transportation planning type of problem involving

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demand uncertainty. In literature, Patriksson & Wynter (1999) introduced stochastic extension of mathematical programs with equilibrium constraints, which can be regarded as a hierarchical decision making problem under uncertainty. Several methods have been suggested to solve stochastic mathematical programs with equilibrium constraints (SMPEC) such as smoothing implicit programming approach, smoothing penalty method, regularization method and sample average approximation (Lin et al. 2009).

Ryu et al. (2004) addressed a bilevel decision making problem under uncertainty in which the first level decision maker manages distributions, and the second level decision maker is responsible for production in a supply chain. They presented a solution method based on parametric programming. Roghanian et al. (2007) discussed the same problem, but they tackled uncertainty by using chance constrained programming, where constraints may not be satisfied at certain levels of probability. Werner (2005) studied bilevel stochastic programming problems as an extension of stochastic programming problems where part of the uncertainty is attributed to the behavior of another decision maker. Author applied this bilevel stochastic programming formulation to telecommunication sector. Kato et al. (2006) considered a two level manufacturing planning problem which involves random variables in some parameters to deal with hierarchical decision making problems under uncertainty. Authors applied an interactive fuzzy programming method to their probability maximization model. Katagiri et al. (2007) considered a hierarchical decision problem with two noncooperative decision makers by constructing two level expectation optimization and two level variance minimization models. Kalashnikov et al. (2010) presented a bilevel multi-stage stochastic optimization model development to balance fuel volumes over a distribution network of natural gas supply chain. In their model, the natural gas shipping company is considered as the leader, and the pipeline operating company is considered as the follower. Akdemir & Tiryaki (2011) proposed a bilevel stochastic transportation model for discrete customer demand cases.

This paper is arranged as follows; in the following section, we first present some preliminary essential concepts. In section 3, a description of BSTP and its formulation are given, after that, the KKT conditions are provided. In section 4, a simple numerical example and its computational results are given to illustrate application of the problem. Finally, in section 5, conclusions are drawn regarding the model.

2. Preliminaries

2.1. Stochastic Transportation Problem

When customers' demands for a certain product are not known with certainty, the single period problem of determining the optimal quantities of a product to be shipped from supply points to demand points is a stochastic transportation problem (STP).

2.1.1. Nomenclature

Parameters

S_i capacity of plant i ($i=1,\ldots,m$)

d_j stochastic demand of customer j ($j=1,2,\ldots,n$)

c_{ij} unit cost of transportation from plant i to customer j

p_j unit shortage cost (penalty rate for each unit of unfulfilled demand) at customer zone j

h_j unit holding cost (penalty rate for each unit in excess of quantity demanded) at customer zone j

Decision variables

x_{ij} quantity shipped from plant i to customer j

y_j total quantity shipped to customer j

Functions

$\phi_j(t)$ probability density function of customer demand j

$\Phi_j(t)$ cumulative distribution function of customer demand j

The amounts of demands are uncertain and the set of possible values is uncountable (divisible product), but they are described by stochastic variables with known distribution functions.

The resulting optimization model is a linearly constrained nonlinear convex problem (Williams 1963; Daneva et al. 2010).

STP is formulated as:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} h_j \int_{0}^{y_j} (y_j - t)\phi_j(t)dt + \sum_{j=1}^{n} p_j \int_{y_j}^{\infty} (t - y_j)\phi_j(t)dt$$

$$\begin{aligned} \text{subject to } & \sum_{j=1}^{n} x_{ij} \le S_i, \quad i=1,2,\ldots,m \\ & \sum_{i=1}^{m} x_{ij} = y_j, \quad j=1,2,\ldots,n \\ & x_{ij} \ge 0, \quad \forall i,j \end{aligned}$$

where the objective function of the problem is derived as:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n}\left((h_j + p_j)\int_{0}^{y_j} \Phi_j(t)dt - p_j y_j \right)$$

by omitting the constant term

$$\sum_{j=1}^{n} p_j E(d_j).$$

The STP has been treated in many papers and can be solved efficiently (Holmberg & Tuy 1999), for example with the Frank-Wolfe method (Frank & Wolfe 1956), by cross decomposition (Holmberg 1992), by separable programming (Holmberg 1984), by the forest iteration method (Qi 1985) and by mean value cross decomposition (Holmberg & Jörnsten 1984).

2.2. Bilevel Programming

In BP, the set of decision variables is partitioned between two vectors $\mathbf{x_1}$ and $\mathbf{x_2}$. The first level decision maker (leader) controls over the vector $\mathbf{x_1}$, and the second level decision maker (follower) controls over the vector $\mathbf{x_2}$. It is assumed that the leader decides his decision vector first, and then, the follower reacts to the leader's decision by taking into account leader's strategy.

The BP problem can be formulated as:

$$\begin{aligned}&\min_{\mathbf{x_1}} F(\mathbf{x_1},\mathbf{x_2})\\ &\text{subject to } G(\mathbf{x_1},\mathbf{x_2})\le 0\\ &\qquad \text{where } \mathbf{x_2} \text{ solves}\\ &\qquad \begin{cases}\min_{\mathbf{x_2}} f(\mathbf{x_1},\mathbf{x_2})\\ \text{subject to } g(\mathbf{x_1},\mathbf{x_2})\le 0\end{cases}\end{aligned}$$

where upper-level variables $\mathbf{x_1}\in\mathbb{R}^p$, lower-level variables $\mathbf{x_2}\in\mathbb{R}^r$, upper-level objective function $F:\mathbb{R}^p\times\mathbb{R}^r\to\mathbb{R}$, lower-level objective function $f:\mathbb{R}^p\times\mathbb{R}^r\to\mathbb{R}$, upper-level constraints $G:\mathbb{R}^p\times\mathbb{R}^r\to\mathbb{R}^{s'}$ and lower-level constraints $g:\mathbb{R}^p\times\mathbb{R}^r\to\mathbb{R}^s$.

Several methods have been proposed to solve BP problems. A survey of existing methods is given in Colson et al. (2005b). Using the branch and bound algorithm (Bard & Moore 1990) in our proposed model, we dealt with a bilevel structure for noncooperative case. So, rather than working with hierarchical form, we converted it into a standard mathematical program. This can be achieved by replacing the second level problem with its KKT optimality conditions. This operation reduces the original problem to a single level program involving nonlinear complementary constraints (Colson et al. 2005a).

The equivalent single level program of the BP problem follows as:

$$\begin{aligned}&\min_{\mathbf{x_1},\mathbf{x_2},\boldsymbol{\mu}} F(\mathbf{x_1},\mathbf{x_2})\\ &\text{s.t. } G(\mathbf{x_1},\mathbf{x_2})\le 0\\ &\quad g(\mathbf{x_1},\mathbf{x_2})\le 0\\ &\quad \nabla_{\mathbf{x_2}} f(\mathbf{x_1},\mathbf{x_2})+\mu^T\nabla_{\mathbf{x_2}} g(\mathbf{x_1},\mathbf{x_2})=0\\ &\quad \mu_i g_i(\mathbf{x_1},\mathbf{x_2})=0,\quad i=1,\ldots,s\\ &\quad \mu_i\ge 0,\qquad\qquad i=1,\ldots,s\end{aligned}$$

where $\boldsymbol{\mu}\in\mathbb{R}^s$ is the vector of Lagrange multipliers.

The KKT conditions are necessary optimality conditions for the second level problem. The KKT conditions are also sufficient, if the second level problem is a convex optimization problem in variables $\mathbf{x_2}\in\mathbb{R}^r$ for fixed parameters $\bar{\mathbf{x}}_\mathbf{1}\in\mathbb{R}^p$ (Dempe 2003). Hence, any local minimum will be global minimum for the second level. However, equivalent single level programming problem is difficult to solve due to nonlinear complementary constraints:

$$\mu_i g_i(\mathbf{x_1},\mathbf{x_2})=0,\quad i=1,\ldots,s\,.$$

In branch and bound algorithm, the complementary constraints are removed to construct the relaxed program. Supposing that the solution of the relaxed program does not satisfy some complementary constraints:

$$\mu_i g_i(\mathbf{x_1},\mathbf{x_2})=0\,,$$

branching is performed by separating two subproblems one with $\mu_i=0$ as an additional constraint, and the other with the constraint $g_i(\mathbf{x_1},\mathbf{x_2})=0$, selecting i for which $\mu_i g_i(\mathbf{x_1},\mathbf{x_2})$ is the largest. Branching is repeated until all complementary constraints are satisfied or an infeasible solution is obtained. Resulting feasible solutions are labeled as candidate solutions (Bard & Moore 1990; Colson et al. 2005a).

3. Problem Description

3.1. Assumptions

(i) The set of plants is partitioned into two sets L_1 and L_2. L_1 is the set of Level 1 plants which are operated by the leader. Similarly, L_2 is the set of Level 2 plants which are operated by the follower (Sonia et al., 2008).

(ii) The leader controls variables $x_{ij}, i\in L_1$ and the follower controls variables $x_{ij}, i\in L_2$ for $j=1,2,\ldots,n$, where

$$\mathbf{x_1}=\left(x_{ij}\right)_{i\in L_1} \text{ and } \mathbf{x_2}=\left(x_{ij}\right)_{i\in L_2}.$$

(iii) Customer demand amounts are stochastic variables with known continuous distribution functions. In this paper, it is assumed that demands are exponentially distributed random variables with mean $1/\lambda_j$, i.e., the probability density functions are chosen to be the form:

$$\phi_j(t)=\lambda_j\exp(-\lambda_j t)\ \text{ for } j=1,2,\ldots,n\,.$$

(iv) The follower determines his quantities shipped to customers after the leader does. Each decision maker has to decide before demands are realized. There are holding and shortage costs at customer zones. Holding cost can be interpreted as a penalty when the actual customer demand is lower than the total quantity shipped to the customer. In our problem, if the firm supplies more than demanded, then it is assumed that a holding cost is occurred for the leader. So, the objective of the first level is to minimize its own total transportation costs plus total expected penalty cost of oversupply. But, holding costs can be negative which implies that the leader can sell excess quantities at some prices.

Shortage cost can be interpreted as a penalty when the actual customer demand is higher than the total quantity shipped to the customer. In our problem, if some demands are not met by the firm, then it is assumed that the follower has to do make-up shipments with a unit shortage penalty. So, the objective of the second level is to minimize its own total transportation costs plus total expected penalty cost of the undersupply. The shortage costs belonging to the follower is assumed to be assuring because the lower level problem is convex. So, the KKT conditions are the necessary and sufficient optimality conditions.

Briefly, the follower decides $x_{ij}, i\in L_2$ after the leader decides $x_{ij}, i\in L_1$ for customer j. Then demand d_j is realized. For customer j, if

$$d_j\le\sum_{i=1}^m x_{ij}$$

then there is a holding cost

$$h_j(\sum_{i=1}^{m} x_{ij} - d_j)$$

for the leader, and if

$$d_j > \sum_{i=1}^{m} x_{ij}$$

then there is a shortage cost

$$p_j(d_j - \sum_{i=1}^{m} x_{ij})$$

for the follower.

3.2. Formulation of the Problem

$$(\text{P1}) \min_{\mathbf{x_1}} \sum_{j=1}^{n} \sum_{i \in L_1} c_{ij} x_{ij} + \sum_{j=1}^{n} \left(h_j \int_0^{y_j} (y_j - t)\phi_j(t) dt \right)$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \le S_i, \quad i \in L_1$$

$$x_{ij} \ge 0, \qquad i \in L_1; j = 1,2,\dots,n$$

where $\mathbf{x_2}$ solves

$$\min_{\mathbf{x_2}} \sum_{j=1}^{n} \sum_{i \in L_2} c_{ij} x_{ij} + \sum_{j=1}^{n} \left(p_j \int_{y_j}^{\infty} (t - y_j)\phi_j(t) dt \right)$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \le S_i, \quad i \in L_2$$

$$\sum_{i=1}^{m} x_{ij} = y_j, \quad j = 1,2,\dots,n$$

$$x_{ij} \ge 0 \qquad i \in L_2; j = 1,2,\dots,n$$

3.3. The KKT Conditions for BSTP

The objective functions are obtained by calculating the integrals in (P1):

$$\min_{\mathbf{x_1}} F(\mathbf{x_1}, \mathbf{x_2}) = \sum_{j=1}^{n} \sum_{i \in L_1} c_{ij} x_{ij} + \sum_{j=1}^{n} \left(h_j y_j + \frac{h_j}{\lambda_j} \exp(-\lambda_j y_j) \right)$$

and

$$\min_{\mathbf{x_2}} f(\mathbf{x_1}, \mathbf{x_2}) = \sum_{j=1}^{n} \sum_{i \in L_2} c_{ij} x_{ij} + \sum_{j=1}^{n} \frac{p_j}{\lambda_j} \exp(-\lambda_j y_j) .$$

objective function of the second level problem f is a convex function in $\mathbf{x_2}$ with partial derivatives:

$$\frac{\partial f}{\partial x_{ij}} = c_{ij} - p_j \exp(-\lambda_j \sum_{k=1}^{m} x_{kj})$$

$$\frac{\partial^2 f}{\partial x_{ab} \partial x_{ij}} = \begin{cases} p_j \lambda_j \exp(-\lambda_j \sum_{k=1}^{m} x_{kj}), & \text{if } j = b \\ 0, & \text{if } j \ne b \end{cases}, \quad a, i \in L_2$$

because the Hessian of f is a positive semidefinite matrix. The KKT conditions are the necessary and sufficient conditions for the second level programming problem due to convexity. The Lagrange function of the second level problem is:

$$L(\mathbf{x_1}, \mathbf{x_2}, \mathbf{u}, \mathbf{v}) = \sum_{j=1}^{n} \sum_{i \in L_2} c_{ij} x_{ij} + \sum_{j=1}^{n} \frac{p_j}{\lambda_j} \exp(-\lambda_j \sum_{k=1}^{m} x_{kj})$$
$$+ \sum_{i \in L_2} u_i (\sum_{j=1}^{n} x_{ij} - S_i) - \sum_{j=1}^{n} \sum_{i \in L_2} v_{ij} x_{ij}$$

where the variables $u_i, v_{ij}, i \in L_2, j = 1,2,\dots,n$ are Lagrange multipliers. The KKT conditions for the second level problem become:

$$u_i, v_{ij} \ge 0, \quad i \in L_2, j = 1,2,\dots,n$$

$$\frac{\partial L}{\partial x_{ij}} = c_{ij} - p_j \exp(-\lambda_j \sum_{k=1}^{m} x_{kj}) + u_i - v_{ij} = 0,$$
$$i \in L_2, j = 1,2,\dots,n$$

$$u_i(\sum_{j=1}^{n} x_{ij} - S_i) = 0, \ i \in L_2$$

$$v_{ij} x_{ij} = 0, \quad i \in L_2, j = 1,2,\dots,n$$

$$\sum_{j=1}^{n} x_{ij} \le S_i, \quad i \in L_2$$

$$x_{ij} \ge 0, \quad i \in L_2, j = 1,2,\dots,n$$

Variables v_{ij} can be eliminated; so, the equivalent single level program (P2) is derived as:

$$(\text{P2}) \min_{\mathbf{x_1}, \mathbf{x_2}} F(\mathbf{x_1}, \mathbf{x_2}) = \sum_{j=1}^{n} \sum_{i \in L_1} c_{ij} x_{ij}$$
$$+ \sum_{j=1}^{n} \left(h_j \sum_{i=1}^{m} x_{ij} + \frac{h_j}{\lambda_j} \exp(-\lambda_j \sum_{i=1}^{m} x_{ij}) \right)$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \le S_i, \quad \forall i$$

$$c_{ij} - p_j \exp(-\lambda_j \sum_{k=1}^{m} x_{kj}) + u_i \ge 0, \quad i \in L_2, \forall j$$

$$x_{ij}(c_{ij} - p_j \exp(-\lambda_j \sum_{k=1}^{m} x_{kj}) + u_i) = 0, \ i \in L_2, \forall j$$

$$u_i(\sum_{j=1}^{n} x_{ij} - S_i) = 0, \quad i \in L_2$$

$$u_i \ge 0, \quad i \in L_2$$

$$x_{ij} \ge 0 \quad \forall i, j$$

4. Numerical Example

A petroleum company has three refineries and four customer zones. Production capacities of those refineries are 150, 200, and 100 units of petrol, respectively. The customer demand varies from zone to zone, the demand of customers are exponentially distributed with

$$\lambda = \begin{pmatrix} 0.012 & 0.007 & 0.008 & 0.006 \end{pmatrix}^T .$$

The parameters λ_j are chosen from the interval $[0.005, 0.02]$ which yields expected demands in the interval $[50, 200]$. It is assumed that the third refinery is operated by the leader and other refineries are operated by the follower.

Transportation, holding and shortage costs are given as follows:

$$c = \begin{pmatrix} 8 & 2 & 5 & 4 \\ 2 & 4 & 6 & 7 \\ 6 & 5 & 3 & 4 \end{pmatrix}, \ h = \begin{pmatrix} -16 & -18 & 5 & 6 \end{pmatrix}^T,$$

$$p = \begin{pmatrix} 60 & 28 & 20 & 30 \end{pmatrix}^T .$$

By using the KKT conditions, reformulation of bilevel problem becomes:

$$\begin{aligned}
\min_{\mathbf{x_1},\mathbf{x_2}} F(\mathbf{x_1},\mathbf{x_2}) = & -16x_{11} - 18x_{12} + 5x_{13} + 6x_{14} \\
& -16x_{21} - 18x_{22} + 5x_{23} + 6x_{24} \\
& -10x_{31} - 13x_{32} + 8x_{33} + 10x_{34} \\
& -1333.3333\exp(-0.012(x_{11} + x_{21} + x_{31})) \\
& -2571.4286\exp(-0.007(x_{12} + x_{22} + x_{32})) \\
& +625\exp(-0.008(x_{13} + x_{23} + x_{33})) \\
& +1000\exp(-0.006(x_{14} + x_{24} + x_{34}))
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } & x_{11} + x_{12} + x_{13} + x_{14} \le 150 \\
& x_{21} + x_{22} + x_{23} + x_{24} \le 200 \\
& x_{31} + x_{32} + x_{33} + x_{34} \le 250 \\
& 8 - 60\exp(-0.012(x_{11} + x_{21} + x_{31})) + u_1 \ge 0 \\
& 2 - 28\exp(-0.007(x_{12} + x_{22} + x_{32})) + u_1 \ge 0 \\
& 5 - 20\exp(-0.008(x_{13} + x_{23} + x_{33})) + u_1 \ge 0 \\
& 4 - 30\exp(-0.006(x_{14} + x_{24} + x_{34})) + u_1 \ge 0 \\
& 2 - 60\exp(-0.012(x_{11} + x_{21} + x_{31})) + u_2 \ge 0 \\
& 4 - 28\exp(-0.007(x_{12} + x_{22} + x_{32})) + u_2 \ge 0 \\
& 6 - 20\exp(-0.008(x_{13} + x_{23} + x_{33})) + u_2 \ge 0 \\
& 7 - 30\exp(-0.006(x_{14} + x_{24} + x_{34})) + u_2 \ge 0 \\
& x_{11}(8 - 60\exp(-0.012(x_{11} + x_{21} + x_{31})) + u_1) = 0 \\
& x_{12}(2 - 28\exp(-0.007(x_{12} + x_{22} + x_{32})) + u_1) = 0 \\
& x_{13}(5 - 20\exp(-0.008(x_{13} + x_{23} + x_{33})) + u_1) = 0 \\
& x_{14}(4 - 30\exp(-0.006(x_{14} + x_{24} + x_{34})) + u_1) = 0 \\
& x_{21}(2 - 60\exp(-0.012(x_{11} + x_{21} + x_{31})) + u_2) = 0 \\
& x_{22}(4 - 28\exp(-0.007(x_{12} + x_{22} + x_{32})) + u_2) = 0 \\
& x_{23}(6 - 20\exp(-0.008(x_{13} + x_{23} + x_{33})) + u_2) = 0 \\
& x_{24}(7 - 30\exp(-0.006(x_{14} + x_{24} + x_{34})) + u_2) = 0 \\
& u_1(x_{11} + x_{12} + x_{13} + x_{14} - 150) = 0 \\
& u_2(x_{21} + x_{22} + x_{23} + x_{24} - 200) = 0 \\
& u_1, u_2 \ge 0 \\
& x_{ij} \ge 0,\ i = 1,2,3,\ j = 1,2,3,4
\end{aligned}$$

Branch and bound algorithm is applied to the problem and the resulting solution of the problem is given in Table 1.

Table 1. Noncooperative solution

$$\mathbf{x_1}^* = \left(x_{3j}^*\right) = \begin{pmatrix} 0 & 0 & 46.3298 & 53.6702 \end{pmatrix},$$

$$\mathbf{x_2}^* = \begin{pmatrix} x_{1j}^* \\ x_{2j}^* \end{pmatrix} = \begin{pmatrix} 0 & 74.3195 & 0 & 75.6805 \\ 150.9470 & 49.0530 & 0 & 0 \end{pmatrix},$$

$$u_1^* = 9.8059, u_2^* = 7.8059, F(\mathbf{x_1}^*, \mathbf{x_2}^*) = -3684.926.$$

5. Conclusion

In this paper, we propose BSTP and its KKT conditions. BSTP is a bilevel version of STP. The leader (follower) tries to optimize his total transportation costs plus total holding (shortage) costs assuming that the customer demands are exponentially distributed random variables. We also assume that any leftover quantity can be sold at a unit price which is represented by a negative holding cost. Convexity of the follower's objective function allows us to reformulate the bilevel problem as a single level nonlinear programming problem by using the KKT conditions. Equivalent single level problem involves complementary constraints which are tackled by Branch and Bound algorithm.

In our example, there are two refineries in the second level, one refinery in the first level and four customers. The number of constraints is 35 and the 59 subproblems are solved to obtain noncoopertaive solution. However, resulting solutions are only local since the first level objective function is not convex. Global optimization techniques can be applied to improve the solution algorithm, especially for large sized problems.

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