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Ranking Generalized Trapezoidal Fuzzy Numbers with Euclidean Distance by the Incentre of Centroids

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Abstract

This paper proposes a method on the incentre of Centroids and uses of Euclidean distance to ranking generalized fuzzy numbers. In this method, splitting the generalized trapezoidal fuzzy numbers into three plane figures and then calculating the centroids of each plane figure followed by the incentre of the centroids and then finding the Euclidean distance. For the validation the results of the proposed approach are compared with different existing approaches.

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1 Introduction

Centroid concept in ranking fuzzy number only started in 1980 by Yager [?]. Yager [?] was the first researcher who contributed the centroid concept in the ranking method and used the horizontal coordinate x as the ranking index. Murakami et al. [9] developed both the horizontal x and vertical y coordinates of the centroid point as the ranking index. Cheng [5] argued in certain cases, the value of x can also be an aid index and y becomes the important index especially when the values of x are equal or the left and right spread are the same for all fuzzy numbers. Therefore, to overcome the problems of choosing either x or y as the important index, Cheng (1998) proposed a distance index which is based on the calculation of using both values of x and y. In 2002, Chu and Tsao [6] found that the distance method and Coefficient of Variance (CV) index proposed by Cheng also contain shortcomings. Both methods are inconsistent to rank fuzzy numbers and their images. Hence, to overcome the problems of incorrect ranking order, Chen and Chen [2] derived a new method

on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations. In 2006, Wang et al. [19] found that the method provided by Cheng (1998) was shown to be incorrect and to have led to some misapplications. Therefore, based on the concept of the importance of the degree of x, Wang and Lee [20] presented a revise method which they claimed can improve Chu and Tsaos (2002) area method and Thorani [18] proposed Ordering Generalized Trapezoidal Fuzzy Numbers. Also, F. Azman and L.Abdullah [7] proposed a Review on Ranking Fuzzy Numbers Using The Centroid Point Method and S. Rezvani ([11]-[16]) evaluated the system of Fuzzy Numbers. Moreover, Rezvani [16] proposed a New Method for Ranking in Areas of two Generalized Trapezoidal Fuzzy Numbers.
This paper proposes a method on the incentre of Centroids and uses of Euclidean distance to ranking generalized fuzzy numbers. In this method, splitting the generalized trapezoidal fuzzy numbers into three plane figures and then calculating the centroids of each plane figure followed by the incentre of these centroids and then finding the Euclidean distance. For the validation the results of the proposed approach are compared with different existing approaches.

2 Preliminaries

Definition 2.1 Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A(x) = 0, -\infty < x \leq a$,

(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,

(iv) $\mu_A(x) = w, b \leq x \leq c$,

(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,

(vi) $\mu_A(x) = 0, d \leq x < \infty$

Where $0 < w \leq 1$ and $a, b, c,$ and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a, b, c, d; w) \ . \tag{1}$$

A $A = (a, b, c, d; w)$ is a fuzzy set of the real line R whose membership function $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} w\frac{x-a}{b-a} & if \quad a \leq x \leq b \\ w & if \quad b \leq x \leq c \\ w\frac{d-x}{d-c} & if \quad c \leq x \leq d \\ 0 & \quad Otherwise \end{cases} \tag{2}$$

Definition 2.2 The membership function of the real fuzzy number A is given by

$$f_A(x) = \begin{cases} f_A^L & a \leq x \leq b \\ w & b \leq x \leq c \\ f_A^R & c \leq x \leq d \\ 0 & Otherwise \end{cases} \tag{3}$$

where $0 < w \leq 1$ is a constant, a, b, c, d are real numbers and $f_A^L : [a, b] \rightarrow [0, w]$, $f_A^R : [c, d] \rightarrow [0, w]$ are two strictly monotonic and continuous functions from R to the closed interval $[0, w]$. Their inverse functions $g_A^L : [0, w] \rightarrow [a, b]$ and $g_A^R : [0, w] \rightarrow [c, d]$ are also continuous and strictly monotonic. Hence g_A^L and g_A^R are integrable on $[0, w]$.

3 Proposed Method

The Centroid of a trapezoid is considered as the balancing point of the trapezoid (Fig.1). Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC), and a triangle (CQD), respectively. Let the Centroids of the three plane figures be G_1 , G_2 and G_3 respectively. The Incenter of these Centroids G_1 , G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each Centroid point are balancing points of each individual plane figure, and the Incentre of these Centroid points is a much more balancing point for a generalized trapezoidal fuzzy number . Therefore, this point would be a better reference point than the Centroid point of the trapezoid. Consider a generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ (Fig. 1) The Centroids of the three plane figures are:

$$G_1 = (\frac{a+2b}{3}, \frac{w}{3}),\ G_2 = (\frac{b+c}{2}, \frac{w}{2})\ and\ G_3 = (\frac{2c+d}{3}, \frac{w}{3}). \tag{4}$$

Equation of the line G_1G_3 is $y = \frac{w}{3}$ and G_2 does not lie on the line G_1G_3. Therefore G_1 , G_2 and G_3 are non-collinear and they form a triangle. We define the Incentre $I_A(\overline{x_0}, \overline{y_0})$ of the triangle with vertices G_1 , G_2 and G_3 of

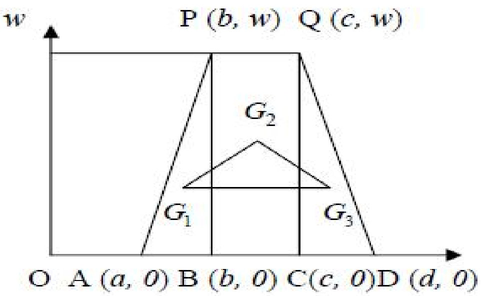

Figure 1: Trapezoidal fuzzy number

the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ as

$$I(\overline{x_0}, \overline{y_0}) = (\frac{\alpha(\frac{a+2b}{3}) + \beta(\frac{b+c}{2}) + \gamma(\frac{2c+d}{3})}{\alpha + \beta + \gamma}, \frac{\alpha(\frac{w}{3}) + \beta(\frac{w}{2}) + \gamma(\frac{w}{3})}{\alpha + \beta + \gamma}) \tag{5}$$

where

$$\alpha = \frac{\sqrt{(c - 3b + 2d)^2 + w^2}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3}, \gamma = \frac{\sqrt{(3c - 2a - b)^2 + w^2}}{6} \tag{6}$$

Ranking function of the trapezoidal fuzzy number $A = (a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as

$$R(A) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2} \tag{7}$$

which is the Euclidean distance from the incentre of the centroids.
In sum, the rank of two fuzzy numbers A and B based on the incentre of the centroids is given as follows steps:
Let $A_1 = (a_1, b_1, \alpha_1, \beta_1; w_1)$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2; w_2)$ be two generalized trapezoidal fuzzy numbers, then

∗ steps 1

Find α, β, γ

∗ steps 2

Find $I(\overline{x_0}, \overline{y_0})$

∗ steps 3

Find $R(A) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$ and using follows for ranking fuzzy numbers

(i) if $R(A_1) > R(A_2)$, then $A_1 > A_2$,

(ii) if $R(A_1) < R(A_2)$, then $A_1 < A_2$,

(iii) if $R(A_1) \sim R(A_2)$, then $A_1 \sim A_2$.

4 Results

Example 4.1 Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy number, then

$*$ steps 1

$$\alpha_A = \frac{\sqrt{(0.6-3\times0.4+2\times0.8)^2+(0.35)^2}}{6} = 0.18, \ \ \alpha_B = \frac{\sqrt{(0.3-3\times0.2+2\times0.4)^2+(0.7)^2}}{6} = 0.143$$

$$\beta_A = \frac{\sqrt{(2\times0.6+0.8-0.2-2\times0.4)^2}}{3} = 0.33, \ \ \beta_B = \frac{\sqrt{(2\times0.3+0.4-0.1-2\times0.2)^2}}{3} = 0.17$$

$$\gamma_A = \frac{\sqrt{(3\times0.6-2\times0.2-0.4)^2+(0.35)^2}}{6} = 0.18, \ \ \gamma_B = \frac{\sqrt{(3\times0.3-2\times0.1-0.2)^2+(0.7)^2}}{6} = 0.143$$

$*$ steps 2

$$I_A(\overline{x_0}, \overline{y_0}) =$$

$$\left(\frac{0.18(\frac{0.2+2\times0.4}{3})+0.33(\frac{0.4+0.6}{2})+0.18(\frac{2\times0.6+0.8}{3})}{0.18+0.33+0.18}, \frac{0.18(\frac{0.35}{3})+0.33(\frac{0.35}{2})+0.18(\frac{0.35}{3})}{0.18+0.33+0.18}\right) = (0.5, 0.15),$$

$$I_B(\overline{x_0}, \overline{y_0}) =$$

$$\left(\frac{0.143(\frac{0.1+2\times0.2}{3})+0.17(\frac{0.2+0.3}{2})+0.143(\frac{2\times0.3+0.4}{3})}{0.143+0.17+0.143}, \frac{0.143(\frac{0.7}{3})+0.17(\frac{0.7}{2})+0.143(\frac{0.7}{3})}{0.143+0.17+0.143}\right) = (0.25, 0.28),$$

$*$ steps 3

$$R(A) = \sqrt{(0.5)^2 + (0.15)^2} = 0.52, \ R(B) = \sqrt{(0.25)^2 + (0.28)^2} = 0.37,$$

So $R(A) > R(B)$, then $A > B$.

Example 4.2 Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

∗ steps 1

$$\alpha_A = \frac{\sqrt{(0.4-3\times0.2+2\times0.5)^2+1}}{6} = 0.21,\ \ \alpha_B = \frac{\sqrt{(0.3-3\times0.3+2\times0.5)^2+1}}{6} = 0.18$$

$$\beta_A = \frac{\sqrt{(2\times0.4+0.5-0.1-2\times0.2)^2}}{3} = 0.27,\ \ \beta_B = \frac{\sqrt{(2\times0.3+0.5-0.1-2\times0.3)^2}}{3} = 0.13$$

$$\gamma_A = \frac{\sqrt{(3\times0.4-2\times0.1-0.2)^2+1}}{6} = 0.21,\ \ \gamma_B = \frac{\sqrt{(3\times0.3-2\times0.1-0.3)^2+1}}{6} = 0.18$$

∗ steps 2

$$I_A(\overline{x_0}, \overline{y_0}) =$$

$$\left(\frac{0.21(\frac{0.1+2\times0.2}{3})+0.27(\frac{0.2+0.4}{2})+0.21(\frac{2\times0.4+0.5}{3})}{0.21+0.27+0.21}, \frac{0.21(\frac{1}{3})+0.27(\frac{1}{2})+0.21(\frac{1}{3})}{0.21+0.27+0.21}\right) = (0.3, 0.39),$$

$$I_B(\overline{x_0}, \overline{y_0}) =$$

$$\left(\frac{0.18(\frac{0.1+2\times0.3}{3})+0.13(\frac{0.3+0.3}{2})+0.18(\frac{2\times0.3+0.5}{3})}{0.18+0.13+0.18}, \frac{0.18(\frac{1}{3})+0.13(\frac{1}{2})+0.18(\frac{1}{3})}{0.18+0.13+0.18}\right) = (0.3, 0.38),$$

∗ steps 3

$$R(A) = \sqrt{(0.3)^2 + (0.39)^2} = 0.49,\ R(B) = \sqrt{(0.3)^2 + (0.38)^2} = 0.48,$$

Then $R(A) > R(B) \Rightarrow A > B$.

Example 4.3 Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (1, 1, 1, 1; 1)$ be two generalized trapezoidal fuzzy number, then

∗ steps 1

$$\alpha_A = \frac{\sqrt{(0.4-3\times0.2+2\times0.5)^2+1}}{6} = 0.21,\ \ \alpha_B = \frac{\sqrt{(1-3\times1+2\times1)^2+1}}{6} = 0.17$$

$$\beta_A = \frac{\sqrt{(2\times0.4+0.5-0.1-2\times0.2)^2}}{3} = 0.27,\ \ \beta_B = \frac{\sqrt{(2\times1+1-1-2\times1)^2}}{3} = 0$$

$$\gamma_A = \frac{\sqrt{(3\times0.4-2\times0.1-0.2)^2+1}}{6} = 0.21,\ \ \gamma_B = \frac{\sqrt{(3\times1-2\times1-1)^2+1}}{6} = 0.17$$

∗ steps 2

$$I_A(\overline{x_0}, \overline{y_0}) =$$

$$\left(\frac{0.21(\frac{0.1+2\times0.2}{3})+0.27(\frac{0.2+0.4}{2})+0.21(\frac{2\times0.4+0.5}{3})}{0.21+0.27+0.21}, \frac{0.21(\frac{1}{3})+0.27(\frac{1}{2})+0.21(\frac{1}{3})}{0.21+0.27+0.21}\right) = (0.3, 0.39),$$

$I_B(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.17(\frac{1+2\times1}{3})+0(\frac{1+1}{2})+0.17(\frac{2\times1+1}{3})}{0.17+0+0.17}, \frac{0.17(\frac{1}{3})+0(\frac{1}{2})+0.17(\frac{1}{3})}{0.17+0+0.17}) = (1, 0.35),$

$*$ steps 3

$R(A) = \sqrt{(0.3)^2 + (0.39)^2} = 0.49,\ R(B) = \sqrt{(1)^2 + (0.35)^2} = 1.06,$

Then $R(A) < R(B) \Rightarrow A < B$.

Example 4.4 Let $A = (-0.5, -0.3, -0.3, -0.1; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ steps 1

$$\alpha_A = \frac{\sqrt{(-0.3-3\times(-0.3)+2\times(-0.1))^2+1}}{6} = 0.18,\ \ \alpha_B = \frac{\sqrt{(0.3-3\times0.3+2\times0.5)^2+1}}{6} = 0.18$$

$$\beta_A = \frac{\sqrt{(2\times(-0.3)-0.1+0.5-2\times(-0.3))^2}}{3} = 0.13,\ \ \beta_B = \frac{\sqrt{(2\times0.3+0.5-0.1-2\times0.3)^2}}{3} = 0.13$$

$$\gamma_A = \frac{\sqrt{(3\times(-0.3)-2\times(-0.5)+0.3)^2+1}}{6} = 0.19,\ \ \gamma_B = \frac{\sqrt{(3\times0.3-2\times0.1-0.3)^2+1}}{6} = 0.18$$

$*$ steps 2

$I_A(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.18(\frac{-0.5+2\times(-0.3)}{3})+0.13(\frac{-0.3-0.3}{2})+0.19(\frac{2\times(-0.3)-0.1}{3})}{0.18+0.13+0.19}, \frac{0.18(\frac{1}{3})+0.13(\frac{1}{2})+0.19(\frac{1}{3})}{0.18+0.13+0.19}) = (-0.298, 0.37),$

$I_B(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.18(\frac{0.1+2\times0.3}{3})+0.13(\frac{0.3+0.3}{2})+0.18(\frac{2\times0.3+0.5}{3})}{0.18+0.13+0.18}, \frac{0.18(\frac{1}{3})+0.13(\frac{1}{2})+0.18(\frac{1}{3})}{0.18+0.13+0.18}) = (0.3, 0.38),$

$*$ steps 3

$R(A) = \sqrt{(-0.298)^2 + (0.37)^2} = 0.47,\ R(B) = \sqrt{(0.3)^2 + (0.38)^2} = 0.48,$

Then $R(A) < R(B) \Rightarrow A < B$.

Example 4.5 Let $A = (0.3, 0.5, 0.5, 1; 1)$ and $B = (0.1, 0.6, 0.6, 0.8; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ steps 1

$\alpha_A = \frac{\sqrt{(0.5-3\times0.5+2\times1)^2+1}}{6} = 0.23, \;\; \alpha_B = \frac{\sqrt{(0.6-3\times0.6+2\times0.8)^2+1}}{6} = 0.18$

$\beta_A = \frac{\sqrt{(2\times0.5+1-0.3-2\times0.5)^2}}{3} = 0.23, \;\; \beta_B = \frac{\sqrt{(2\times0.6+0.8-0.1-2\times0.6)^2}}{3} = 0.23$

$\gamma_A = \frac{\sqrt{(3\times0.5-2\times0.3-0.5)^2+1}}{6} = 0.18, \;\; \gamma_B = \frac{\sqrt{(3\times0.6-2\times0.1-0.6)^2+1}}{6} = 0.23$

$*$ steps 2

$I_A(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.23(\frac{0.3+2\times0.5}{3})+0.23(\frac{0.5+0.5}{2})+0.18(\frac{2\times0.5+1}{3})}{0.23+0.23+0.18}, \frac{0.23(\frac{1}{3})+0.23(\frac{1}{2})+0.18(\frac{1}{3})}{0.23+0.23+0.18}) = (0.52, 0.4),$

$I_B(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.18(\frac{0.1+2\times0.6}{3})+0.23(\frac{0.6+0.6}{2})+0.23(\frac{2\times0.6+0.8}{3})}{0.18+0.23+0.23}, \frac{0.18(\frac{1}{3})+0.23(\frac{1}{2})+0.23(\frac{1}{3})}{0.18+0.23+0.23}) = (0.58, 0.4),$

$*$ steps 3

$R(A) = \sqrt{(0.52)^2 + (0.4)^2} = 0.66, \; R(B) = \sqrt{(0.58)^2 + (0.4)^2} = 0.7,$

Then $R(A) < R(B) \Rightarrow A < B$.

Example 4.6 Let $A = (0, 0.4, 0.6, 0.8; 1)$ and $B = (0.2, 0.5, 0.5, 0.9; 1)$ and $C = (0.1, 0.6, 0.7, 0.8; 1)$ be three generalized trapezoidal fuzzy number, then

$*$ steps 1

$\alpha_A = \frac{\sqrt{(0.6-3\times0.4+2\times0.8)^2+1}}{6} = 0.23, \;\; \alpha_B = \frac{\sqrt{(0.5-3\times0.5+2\times0.9)^2+1}}{6} = 0.21,$

$\alpha_C = \frac{\sqrt{(0.7-3\times0.6+2\times0.8)^2+1}}{6} = 0.19$

$\beta_A = \frac{\sqrt{(2\times0.6+0.8-0-2\times0.4)^2}}{3} = 0.4, \;\; \beta_B = \frac{\sqrt{(2\times0.5+0.9-0.2-2\times0.5)^2}}{3} = 0.23,$

$\beta_C = \frac{\sqrt{(2\times0.7+0.8-0.1-2\times0.6)^2}}{3} = 0.3$

$\gamma_A = \frac{\sqrt{(3\times0.6-2\times0-0.4)^2+1}}{6} = 0.29, \;\; \gamma_B = \frac{\sqrt{(3\times0.5-2\times0.2-0.5)^2+1}}{6} = 0.19,$

$\gamma_C = \frac{\sqrt{(3\times0.7-2\times0.1-0.6)^2+1}}{6} = 0.27$

$*$ steps 2

$I_A(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.23(\frac{0+2\times0.4}{3})+0.4(\frac{0.4+0.6}{2})+0.29(\frac{2\times0.6+0.8}{3})}{0.23+0.4+0.29}, \frac{0.23(\frac{1}{3})+0.4(\frac{1}{2})+0.29(\frac{1}{3})}{0.23+0.23+0.18}) = (0.49, 0.41),$

$I_B(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.21(\frac{0.2+2\times0.5}{3})+0.23(\frac{0.5+0.5}{2})+0.19(\frac{2\times0.5+0.9}{3})}{0.21+0.23+0.19}, \frac{0.21(\frac{1}{3})+0.23(\frac{1}{2})+0.19(\frac{1}{3})}{0.18+0.23+0.23}) = (0.51, 0.39),$

$I_C(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.19(\frac{0.1+2\times0.6}{3})+0.3(\frac{0.6+0.7}{2})+0.27(\frac{2\times0.7+0.8}{3})}{0.19+0.3+0.27}, \frac{0.19(\frac{1}{3})+0.3(\frac{1}{2})+0.27(\frac{1}{3})}{0.19+0.3+0.27}) = (0.62, 0.4),$

$*$ steps 3

$R(A) = \sqrt{(0.49)^2 + (0.41)^2} = 0.639, \quad R(B) = \sqrt{(0.51)^2 + (0.39)^2} = 0.642,$

$R(C) = \sqrt{(0.62)^2 + (0.4)^2} = 0.74$

Then $R(A) < R(B) < R(C) \Rightarrow A < B < C$.

Example 4.7 Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (-2, 0, 0, 2; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ steps 1

$\alpha_A = \frac{\sqrt{(0.4-3\times0.2+2\times0.5)^2+1}}{6} = 0.21, \quad \alpha_B = \frac{\sqrt{(0-3\times0+2\times0)^2+1}}{6} = 0.69$

$\beta_A = \frac{\sqrt{(2\times0.4+0.5-0.1-2\times0.2)^2}}{3} = 0.27, \quad \beta_B = \frac{\sqrt{(2\times0+2+2-2\times0)^2}}{3} = 1.33$

$\gamma_A = \frac{\sqrt{(3\times0.4-2\times0.1-0.2)^2+1}}{6} = 0.21, \quad \gamma_B = \frac{\sqrt{(3\times1-2\times1-1)^2+1}}{6} = 0.69$

$*$ steps 2

$I_A(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.21(\frac{0.1+2\times0.2}{3})+0.27(\frac{0.2+0.4}{2})+0.21(\frac{2\times0.4+0.5}{3})}{0.21+0.27+0.21}, \frac{0.21(\frac{1}{3})+0.27(\frac{1}{2})+0.21(\frac{1}{3})}{0.21+0.27+0.21}) = (0.3, 0.39),$

$I_B(\overline{x_0}, \overline{y_0}) =$

$(\frac{0.69(\frac{-2+2\times0}{3})+1.33(\frac{0+0}{2})+0.69(\frac{2\times0+2}{3})}{0.69+1.33+0.69}, \frac{0.69(\frac{1}{3})+1.33(\frac{1}{2})+0.69(\frac{1}{3})}{0.69+1.33+0.69}) = (0.08, 0.41),$

$*$ steps 3

$R(A) = \sqrt{(0.3)^2 + (0.39)^2} = 0.49,\ R(B) = \sqrt{(0.08)^2 + (0.41)^2} = 0.42,$

Then $R(A) > R(B) \Rightarrow A > B$.

For the validation, in Table 1, the results of the approach are compared with different existing approaches.

Table (1): A comparison of the ranking results for different approaches

Approaches	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
Cheng[5]	$A < B$	$A \sim B$	Error	$A \sim B$	$A > B$	$A < B < C$	Error
Chu[6]	$A < B$	$A \sim B$	Error	$A < B$	$A > B$	$A < B < C$	Error
Chen[3]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < C < B$	$A > B$
Abbasbandy[1]	Error	$A \sim B$	$A < B$	$A \sim B$	$A < B$	$A < B < C$	$A > B$
Chen[4]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Kumar[8]	$A > B$	$A \sim B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Singh[10]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Thorani[17]	$A < B$	$A > B$	$A < B$	$A < B$	$A < B$	$A < B < C$	$A > B$
Rezvani[15]	$A \sim B$	$A > B$	$A > B$	$A \sim B$	$A \sim B$	$A > C > B$	$A < B$
Proposed approach	$A > B$	$A > B$	$A < B$	$A < B$	$A < B$	$A < B < C$	$A > B$

5 Conclusions

In this method, splitting the generalized trapezoidal fuzzy numbers into three plane figures and then calculating the centroids of each plane figure followed by the incentre of the centroids and then finding the Euclidean distance. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

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