

# SMPC for Discrete-time Singular Systems with Time-varying Delay

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**Abstract**—By introducing sliding mode predictive control (SMPC) techniques, a controller with passivity of a class of discrete-time uncertain singular systems is proposed in this paper. A new switching surface function is designed by taking the singular matrix into account, thus the resulting sliding mode dynamics is a full-order uncertain singular time-varying delay system. Due to feedback correction and receding horizon optimization, the influence of uncertainty can be compensated in time, strong robustness to matched or unmatched uncertainties is possessed. In addition, chattering of sliding mode control can be eliminated by predictive control method. Simulation result is given to illustrate the validity of the proposed approach.

**Keywords**—Singular systems; Discrete-time systems; Passivity; Robust stability; Sliding mode predictive control (SMPC)

## I. INTRODUCTION

Singular systems, also referred to as descriptor systems, generalized state-space systems or differential-algebraic systems, provide convenient and natural representations in the description of economic systems, power systems and circuits systems. So a great number of problems about singular systems were studied (see [1]-[4]). Time delays are frequently the main causes of instability and poor performance of systems, and are encountered in a variety of engineering systems such as chemical processes, nuclear reactors, biological systems and so on.

Sliding mode control (SMC) is a popular control approach for systems containing uncertainties or unknown disturbances, as the controllers can be designed to compensate for the uncertainties or disturbances. For a broad class of systems, this kind of control is particularly appealing due to its ability to deal with non-linearities, time-variance, as well as uncertainties and disturbances, in a direct manner in the face of modeling imprecisions (see [5]-[7]). The first step in SMC is to define a sliding surface. The second step is to design the control law in such a way that any state outside the sliding surface is driven to reach the surface in finite time and stay here.

However, it is well known that chattering is a flaw for SMC, to reduce chattering usually results in decreasing robustness of the closed-loop systems. So a SMPC strategy is proposed to overcome this problem in this paper. Model predictive control (MPC) is a control technique which permits to cope with a constrained system providing an optimal control strategy. The concepts of prediction and receding horizon that generalized prediction control provides, can improve the performance in a reaching mode, and this is one of the goals that the SMPC strategy achieve (see [8]-[15]). Another one is the capability of controlling processes with large time delays and high

controllability ratio. Moreover, the problem of the implementation of a SMC when the state is not accessible can be solved with the predictive strategy.

There are many works based on SMPC control techniques. For example, [8] applied SMPC in a solar air conditioning plant, SMPC for nonlinear systems based on lazy learning is investigated in [9], robust model predictive control with integral sliding mode in continuous-time sampled-data nonlinear systems is studied in [10], [11] investigated model-based predictive networked control systems. But they are continuous case. Due to the widespread using of digital

Implementations, discrete-time systems are very common in real plants compared with continuous-time ones. Besides, stable continuous-time systems may become unstable after being discretized. Therefore, it is necessary to design controllers for discrete-time systems directly. [12] investigated a SMPC algorithm for a class of discrete-time  $n$ -joint rigid robotic manipulator systems. Stability analysis for a triangular discrete-time nonlinear system with SMPC approach studied in [13].

In this work, we will address the problems of passivity-based SMPC for a class of uncertain discrete-time singular time-varying delay systems. We will pay particular attention to the singular matrix  $E$  in the design of a new switching surface function, which leads to a full-order uncertain singular time-delay system for describing the sliding mode dynamics. Then construct a newly Lyapunov function to derive a delay-dependent sufficient condition in the form of linear matrix inequality (LMI), which guarantees that the sliding mode dynamics is robustly passive. A numerical example illustrates the validity of the proposed approach.

Notation: Throughout this paper,  $R^n$  denotes the  $n$ -dimensional Euclidean space,  $R^{m \times n}$  is the set of all  $m \times n$  real matrices;  $P > 0$  ( $P \geq 0$ ) denotes  $P$  as a positive definite (positive semi-definite) symmetric matrix.  $\|x\|$  refers to the Euclidean norm of the vector  $x$ ;  $I$  denotes the identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by  $*$ .

## II. PROBLEM FORMULATION

Consider the following time-varying delay discrete-time singular system:

$$\begin{cases} Ex(k+1) = Ax(k) + A_d x(k-d(k)) + B_\omega \omega(k) \\ \quad + Bu(k), \\ z(k) = Cx(k) + C_d x(k-d(k)) + D_\omega \omega(k), \\ x(k) = \phi(k), k \in [-\bar{d}, 0,] \end{cases} \quad (1)$$

where  $x(k) \in R^n$  is the state,  $u(k) \in R^m$  is the control input,  $z(k) \in R^p$  is the control output,  $\omega(k) \in R^q$  is the disturbances,  $d(k)$  represents the time-varying delay and satisfied  $0 < \underline{d} \leq d(k) \leq \bar{d}$ ,  $\phi(t)$  is the initial function;  $E \in R^{n \times n}$  may be singular and it is assume that  $\text{rank}(E) = r < n$ ,  $A, A_d, B, B_\omega, C, C_d$  and  $D_\omega$  are known matrices with appropriate dimensions.

We recall the following definitions for the discrete-time singular time-delay system.

$$\begin{cases} Ex(k+1) = Ax(k) + A_d x(k-d(k)), \\ x(k) = \phi(k), t \in [-\bar{d}, 0,] \end{cases} \quad (2)$$

Definition 1. The system (1) is said to be :

- (i) regular if  $\det(zE - A)$  is not identically zero.
- (ii) causal if it is regular and  $\deg(\det(zE - A)) = \text{rank}(E)$ .
- (iii) stable if it is regular and  $\rho(zE - A) \subset D(0,1)$ , where  $D(0,1)$  expresses the interior of the identity circle which center is the origin,

$$\rho(zE - A) = \{z \mid \det(zE - A) = 0\}.$$

Definition 2. The uncertain singular time-delay system (1) with  $u(k) = 0$  is said to be robustly passive if there exists a scalar  $\gamma > 0$  such that

$$2 \sum_{s=0}^{k-1} \omega^T(k) z(k) \geq -\gamma \sum_{s=0}^{k-1} \omega^T(k) \omega(k) \quad (3)$$

Lemma 3. (Integral Inequality) For any matrices  $R > 0$ ,  $Y_1, Y_2, W$  and a time-varying delay  $d(k)$ , then

$$-\sum_{s=k-d(k)}^{k-1} \xi^T(s) E^T R E \xi(s) \leq \eta^T(k) \begin{bmatrix} E^T Y_1 + Y_1^T E & E^T Y_2 + Y_1^T E & E^T W \\ * & -E^T Y_2 + Y_2^T E & -E^T W \\ * & * & 0 \end{bmatrix} \eta(k)$$

$$+ d(k) \eta^T(k) \begin{bmatrix} Y_1^T \\ Y_2^T \\ W^T \end{bmatrix} R^{-1} [Y_1 \ Y_2 \ W] \eta(k), \quad (4)$$

where  $\xi(s) = x(s+1) - x(s)$  and  $\eta(k) = [\xi^T(k) \ \xi^T(k-d(k)) \ \omega^T(k)]$ .

### III. MAIN RULES

In this section, the problems of regularity, causally, stability and passivity for singular system (1) are investigated. Furthermore, a controller is given by SMPC techniques which make the closed-loop system is robust stability and passivity.

#### A. Passivity Analysis

The following theorem shows that the system (1) ( $u(k) = 0$ ) is passivity can be guaranteed if there exist some matrices satisfying a certain LMI.

Theorem 1. Given a scalar  $\gamma > 0$ , the discrete-time singular system (1) is regular, causal, stable and robustly passive, if there exist positive matrices  $P, Q$  and  $Z$ , matrices  $R, S, S_d, S_\omega, Y_1$  and  $Y_2$  satisfying

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \bar{d} Y_1^T & \Psi_{15} & A^T P \\ * & \Psi_{22} & \Psi_{23} & \bar{d} Y_2^T & \bar{d} A_d^T Z & A_d^T P \\ * & * & \Psi_{33} & \bar{d} W^T & \bar{d} B_\omega^T Z & B_\omega^T P \\ * & * & * & -\bar{d} Z & 0 & 0 \\ * & * & * & * & -\bar{d} Z & 0 \\ * & * & * & * & * & -P \end{bmatrix} < 0, \quad (5)$$

where  $d_{12} = \bar{d} - \underline{d}$

$$\Psi_{11} = A^T R S^T + S R^T A - E^T P E + Y_1^T E + E^T Y_1 + (d_{12} + 1) Q,$$

$$\Psi_{12} = A^T R S_d^T + S R^T A_d + E^T Y_2 - Y_1^T E,$$

$$\Psi_{13} = S R^T B_\omega + A^T R S_\omega^T - C^T,$$

$$\Psi_{15} = d_{12} (\bar{A} - E)^T Z,$$

$$\Psi_{22} = A_d^T R S_d^T + S_d R^T A_d - Y_2^T E - E^T Y_2 - Q,$$

$$\Psi_{23} = S_d R^T B_\omega + A_d^T R S_\omega^T - C_d^T,$$

$$\Psi_{33} = -\gamma I + B_\omega^T R S_\omega^T + S_\omega^T R^T B_\omega - D_\omega - D_\omega^T,$$

where  $R$  is any full column matrix that satisfied  $E^T R = 0$ .

Proof: Choose a Lyapunov function candidate as

$$V(k) = \sum_{i=1}^4 V_i(k)$$

where

$$V_1(k) = x^T(k)E^TPEx(k),$$

$$V_2(k) = \sum_{s=k-d(k)}^{k-1} x^T(s)Qx(s),$$

$$V_3(k) = \sum_{\theta=-\bar{d}+2}^{-d+1} \sum_{s=k-1+\theta}^{k-1} x^T(s)Qx(s),$$

$$V_4(k) = \sum_{\theta=-\bar{d}+1}^0 \sum_{s=k-1+\theta}^{k-1} z^T(s)E^TZEz(s).$$

By Lemma 3, we have

$$\Delta V_1(k) = x^T(k+1)E^TPEx(k+1) - x^T(k)E^TPEx(k),$$

$$\Delta V_2(k) \leq x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k))$$

$$+ \sum_{s=k+1-\bar{d}}^{k-d} x^T(s)Qx(s),$$

$$\Delta V_3(k) = (\bar{d}-d)x^T(k)Qx(k) - \sum_{s=k-\bar{d}+1}^{k-d} x^T(s)Qx(s),$$

$$\Delta V_4(k) = \bar{d}z^T(k)E^TZEz(k) - \sum_{s=k-\bar{d}}^{k-d} z^T(s)E^TZEz(s),$$

Then

$$\Delta V(k) \leq \eta^T(k) \begin{bmatrix} A^T \\ A_d^T \\ B_\omega^T \end{bmatrix} P \begin{bmatrix} A & A_d & B_\omega \end{bmatrix}$$

$$+ \begin{bmatrix} (A-E)^T \\ A_d^T \\ B_\omega^T \end{bmatrix} \bar{d}R \begin{bmatrix} (A-E) & A_d & B_\omega \end{bmatrix}$$

$$+ \begin{bmatrix} -E^TPE+Q & 0 & 0 \\ * & -Q & 0 \\ * & * & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} E^TY_1+Y_1^TE & -Y_1^TE+E^TY_2 & E^TW \\ * & -E^TY_2-Y_2^TE & -E^TW \\ * & * & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} Y_1^T \\ Y_2^T \\ W^T \end{bmatrix} \bar{d}Z^{-1} \begin{bmatrix} Y_1 & Y_2 & W \end{bmatrix} \eta(k), \tag{6}$$

where

$$\eta(k) = [x(k) \ x^T(k-d(k)) \ \omega^T(k)]^T,$$

$$z(s) = x(s+1) - x(s).$$

Notice that  $E^TR = 0$ , we have

$$2x^T(k+1)E^TR(S^T x(k) + S_d^T x(k-d(k)) + S_\omega^T \omega(k)) = 0. \tag{7}$$

Therefore,

$$\Delta V(k) - 2\omega^T(k)z(k) - \gamma\omega^T(k)\omega(k) \leq \xi^T(k)\Omega\xi(k), \tag{8}$$

where

$$\Omega = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \hat{\Psi}_{13} & \bar{d}Y_1^T & \Psi_{15} & A^TP \\ * & \Psi_{22} & \hat{\Psi}_{23} & \bar{d}Y_2^T & \bar{d}A_d^TZ & A_d^TP \\ * & * & \hat{\Psi}_{33} & \bar{d}W^T & \bar{d}B_\omega^TZ & B_\omega^TP \\ * & * & * & -\bar{d}Z & 0 & 0 \\ * & * & * & * & -\bar{d}Z & 0 \\ * & * & * & * & * & -P \end{bmatrix} \tag{9}$$

$$\hat{\Psi}_{13} = \Psi_{13} + C,$$

$$\hat{\Psi}_{23} = \Psi_{23} + C_d,$$

$$\hat{\Psi}_{33} = B_\omega^TR S_\omega^T + S_\omega^T R^T B_\omega.$$

Sum both sides of (8) from 0 to  $K$  gives rise to

$$V(K) - V(0) + 2 \sum_0^{k-1} \omega^T(k)z(k) - \gamma \sum_0^{k-1} \omega^T(k)\omega(k) \leq 0 \tag{10}$$

Under the zero initial condition, we have  $V(0) = 0$  and  $V(K) \geq 0$ , so (10) guarantees (3).

Next we prove the singular system (1) is regular and causal. Since  $\text{rank}(E) = r < n$ , there must exist two invertible matrices  $G, H \in R^{n \times n}$  such that

$$GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

Then  $R = G^T \begin{bmatrix} 0 \\ \Phi \end{bmatrix}$ ,  $GAH = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ ,

$$G^{-T}PG^{-T} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, G^{-T}Y_1H = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix},$$

$$H^T S = \begin{bmatrix} S_{11} \\ S_{12} \end{bmatrix}.$$

Since  $\Psi_{11} < 0$  and  $(\bar{d} - \underline{d} + 1)Q > 0$ , we get

$$\Theta := A^T R S^T + S R^T A - E^T P E + Y_1^T E + E^T Y_1 < 0.$$

Pre- and post-multiplying  $\Theta < 0$  by  $H^T$  and  $H$ , respectively. Then

$$A_{22}^T \Phi S_{21}^T + S_{21} \Phi^T A_{22} < 0, \tag{11}$$

i.e.,  $A_{22}$  is nonsingular. Otherwise, supposing  $A_{22}$  is singular, there must be a non-zero vector  $\zeta \in R^{n-r}$  which ensure that  $A_{22}\zeta = 0$ . And then we can conclude that :

$$\zeta^T (A_{22}^T \Phi S_{21}^T + S_{21} \Phi^T A_{22}) \zeta = 0,$$

and this contradicts (11). So  $A_{22}$  is nonsingular. Then the singular system (1) is regular and causal.

Finally, we consider the stability of nominal case of system (1) with  $\omega(k) = 0$ . Following the similar procedure as used above, we obtain

$$\Delta V(k) \leq \bar{\eta}^T(k) \Psi \bar{\eta}(k), \tag{12}$$

where  $\bar{\eta}(k) = [x^T(k) \ x^T(k-d(k))]^T$  and

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \bar{d}Y_1^T & \Psi_{15} & \bar{A}^T P \\ * & \Psi_{22} & \bar{d}Y_2^T & \bar{d}A_d^T Z & \bar{A}_d^T P \\ * & * & -\bar{d}Z & 0 & 0 \\ * & * & * & -\bar{d}Z & 0 \\ * & * & * & * & -P \end{bmatrix}.$$

Noting that (12) implies  $\Psi < 0$ , then

$$\Delta V(k) \leq -\lambda_{\min}(-\Psi) \|\bar{\eta}(k)\|^2 < 0.$$

Thus, the stability of the nominal system (1) is proved. The following, a SMPC controller will be designed.

**B. Sliding Mode Function**

Defining sliding mode function as

$$s(k) = GEx(k) - \sum_{s=0}^{k-1} G(A-E)x(s), \tag{13}$$

where  $A$  and  $B$  are system matrices defined in (1), the matrix  $G$  is to be chosen such that  $GB$  is nonsingular.

Remark 1. Notice that the discrete-time sliding surface function designed in (13) is different from that previous literatures, since the singular matrix  $E$  is considered in this paper. This enables to avoid some difficulties caused by  $E$  in deriving the sliding mode dynamics subsequently.

**C. Design of Sliding Mode Prediction Model**

In order to implement an MPC, a model of the plant is used to predict the future plant outputs. This prediction is based on past and current values of the input and the output of the plant.

The sliding mode prediction model is constructed

$$s_m(k+1) = GAx(k) + GA_d x(k-d(k)) + GBu(k) - \sum_{s=0}^k G(A-E)x(s) + \xi s(k), \tag{14}$$

where  $\xi$  is a designable parameter, which satisfies  $0 < \xi < 1$ .

**D. Design of Control Law**

In practice, because of time-variance, non-linearity or disturbances, SMPC will inevitably exist errors, therefore, the model output will not the same as real output. A desirable way to solve such problem is feedback correction

$$\tilde{s}_m(k+1) = s_m(k+1) + \sigma(s(k) - s_m(k)), \tag{15}$$

where  $\sigma$  is a weight coefficient, i.e., weighted feedback correction. The effect of feedback correction will reduce with the decreasing of  $\sigma$ . When suitable  $\sigma$  is chosen, feedback correction can compensate the model and improve control performance. From the viewpoint of practice, the appropriate range of  $\sigma$  is  $0 < \sigma < 1$ .

Let  $\bar{s}(k) = s(k) - s_m(k)$  for the sake of clarification. Then (15) reduces to

$$\begin{aligned} \tilde{s}_m(k+1) &= s_m(k+1) + \sigma \bar{s}(k) \\ &= GAx(k) + GA_d x(k-d(k)) + GBu(k) \\ &\quad - \sum_{s=0}^k G(A-E)x(s) + \xi s(k) + \sigma \bar{s}(k) \\ &= h(k) + GBu(k), \end{aligned} \tag{16}$$

where

$$h(k) = GAx(k) + GA_d x(k-d(k)) - \sum_{s=0}^k G(A-E)x(s) + \xi s(k) + \sigma \bar{s}(k).$$

Now, performance index is given as

$$J = (\tilde{s}_m(k+1) - s_r)^2 + \lambda u^2(k), \quad (17)$$

where  $s_r$  is a sliding mode reference value.

Since the control objective is to keep states on the sliding surface, the desired sliding mode reference value should be  $s_r = 0$ .

Therefore, the performance index (17) can be reduced to

$$J = \tilde{s}_m^2(k+1) + \lambda u^2(k), \quad (18)$$

where  $\lambda$  is a weight coefficient, which adjusts optimized index  $J$  and the control signal.

According to equation (16), performance index (18) can be rewritten as

$$J = (h(k) + GBu(k))^2 + \lambda u^2(k). \quad (19)$$

The solution of minimizing (19) gives control signal  $u(k)$ . By setting the partial derivative of  $J$  to zero, i.e.,  $\partial J / \partial u = 0$ , solving the resulting equation, the optimal solution of  $u(k)$  is

$$u(k) = -\frac{h(k)GB}{\lambda I + (GB)^2}. \quad (20)$$

Now, the control input  $u(k)$  for the closed-loop system (1) is obtained.

### E Robust Stability Analysis

From performance index (19), one can see that  $u(k)$  affects  $J$  less with the decreasing of  $\lambda$ . For the sake of simplifying analysis, according to optimal control theory (see [13]), it is sound to suppose  $\lambda = 0$ , i.e., the case when control signal is not optimal. Therefore, control law (20) reduces to

$$u(k) = -h(k)(GB)^{-1}. \quad (21)$$

Consider the singular system (1) and sliding mode function (13), it follows

$$\begin{aligned} s(k+1) &= GEx(k+1) - \sum_{s=0}^k G(A-E)x(s) \\ &= GAx(k) + GA_d x(k-d(k)) - h(k) \\ &\quad + GB_\omega \omega(k) - \sum_{s=0}^{k-1} G(A-E)x(s) \\ &= GB_\omega \omega(k) - \xi s(k) - \sigma \bar{s}(k). \end{aligned} \quad (22)$$

According to (14),

$$s_m(k) = GAx(k-1) + GA_d x(k-d(k)-1)$$

$$\begin{aligned} &+ GBu(k-1) - \sum_{s=0}^{k-1} G(A-E)x(s) \\ &+ \xi s(k-1) \\ &= GEx(k) - GB_\omega \omega(k-1) \\ &\quad - \sum_{s=0}^{k-1} G(A-E)x(s) + \xi s(k-1) \end{aligned} \quad (23)$$

Accordingly,

$$\begin{aligned} \bar{s}(k) &= s(k) - s_m(k) \\ &= GB_\omega \omega(k-1) - \xi s(k-1). \end{aligned} \quad (24)$$

Therefore, (22) turns to

$$\begin{aligned} s(k+1) &= -\xi s(k) - \sigma \bar{s}(k) + GB_\omega \omega(k) \\ &= -\xi s(k) - \sigma [GB_\omega \omega(k-1) \\ &\quad - \xi s(k-1)] + GB_\omega \omega(k) \\ &= \xi [-s(k) + \sigma s(k-1)] \\ &\quad + GB_\omega [\omega(k) - \sigma \omega(k-1)] \\ &= \xi M + GB_\omega N, \end{aligned} \quad (25)$$

where  $M = -s(k) + \sigma s(k-1)$ ,  $N = \omega(k) - \sigma \omega(k-1)$ .

Theorem 2. If the change rate of disturbances is bounded, i.e., the following inequality holds

$$|\omega(k) - \sigma \omega(k-1)| \leq \frac{\mu}{|GB_\omega|}, \quad (26)$$

where  $\mu$  is a positive constant, then the closed-loop system (1) which is constructed by (13) and (20) is regular, causal, robustly stable and passive.

Proof: Consider the characteristic polynomial of  $M$ ,

$$-1 + \sigma z^{-1} = 0. \quad (27)$$

Obviously, the root of equation (27) is  $z = \sigma$ . Because  $0 < \sigma < 1$ ,  $M$  is stable.

While  $0 < \xi < 1$ ,  $\xi M$  is stable. Namely,  $\forall \varepsilon > 0, \exists k_0$  such that  $|M| < \varepsilon / \xi$  when  $k > k_0$ .

According to (27),  $|N| \leq \mu$ , then

$$\begin{aligned} |s(k+1)| &= |\xi M + GB_\omega N| \\ &\leq |\xi M| + |GB_\omega| |N| \leq \varepsilon + \mu. \end{aligned}$$

Consequently, the practical sliding mode motion of the closed-loop system will converge to a  $\varepsilon$  vicinity of sliding surface and stay on it subsequently. In addition, because the stability of sliding surface has been guaranteed by (13), the closed-loop system (1) is robustly stable with the control (20),

the regularity, causality and passivity can be get from Theorem 1 directly.

Remark 2. It is well known that chattering is a flaw for SMC, to reduce chattering usually at the cost of decreasing robustness of the closed-loop systems. In SMPC, because of feedback correction and receding horizon optimization, the influence of uncertainties may be discovered in time, the control signal can be adjusted immediately to prevent system states cross sliding surface, hence chattering will be avoid. Thus the combination of SMC and predictive control can do great help to controller design.

IV. SIMULATION RESULTS

In this section, an example is given to illustrate the advantage of proposed method.

Example 1. Consider the uncertain singular time-varying delay system (1) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -0.3 & -0.2 & 0.3 \\ 0.5 & 0.3 & 0.5 \\ 0.9 & 0.4 & 0.9 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0.1 & 0 & 0 \\ 0.8 & 0 & 0.3 \\ 0.5 & 0.2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix},$$

$$B_\omega = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix}, C = [0.1 \quad 0.2 \quad 0.1],$$

$$C_d = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}, D_\omega = \begin{bmatrix} 0.2 \\ -0.2 \\ 0.4 \end{bmatrix}.$$

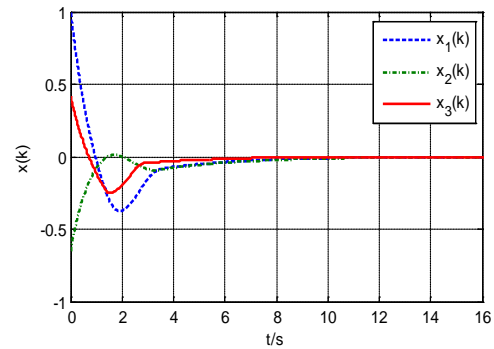
Set  $G = B^T X$  with  $X = I$  for computational simplicity, the initial state of system is  $[1, 0.6 \ 50.4]^T$ ,  $\gamma = 0.02, \sigma = 0.95$ . From (21), we get the controller. The simulation results of the closed-loop system (1) with the proposed method in this paper are illustrated in Figs. 1.

It is well known that chattering is a flaw for SMC. In SMPC, because of feedback correction and receding horizon optimization, the influence of uncertainties may be discovered in time, the control signal can be adjusted immediately to prevent system states cross sliding surface, hence chattering phenomena is eliminated with SMPC method and the closed-loop system (1) is stable.

V. CONCLUSION

This paper has investigated the problems of robust passivity analysis and SMPC of uncertain singular time-varying delay systems. The major theoretical findings are as follows. First, the delay-dependent sufficient condition in the form of LMI has been established so as to ensure that the sliding mode dynamics is stable and robustly passive. Then the desired control law is constructed by using special sliding mode prediction model, feedback correction and receding horizon

optimization. The usefulness of the proposed method has been verified by the numerical results.



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