Developing a Mathematical Model Based on Multiple Traveling Salesman Problem for Determining Machine Groups in a Cell Formation Problem

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Abstract- Cellular manufacturing system is one of the modern manufacturing methods which has been recently utilized with industries because of its advantages. Cellular manufacturing is one of the applications of group technology in manufacturing systems which deals with the determining of machine cells and part families. The determining process is called cell formation problem. In this paper, the cell formation problem is formulated as a multiple travelling salesman problem with applying the dissimilarity coefficient defined as the cost of traveling between two nodes. To verify the performance of the proposed method, a number of test problems selected from the literature are solved and the obtained solutions are compared with those of previous well-known methods using the grouping efficacy measure.

Keywords- Cell Formation Problem; Multiple Traveling Salesman Problem; Dissimilarity Coefficient; Grouping Efficacy

I. INTRODUCTION

Group technology, introduced by Mitrofanov [1], is a manufacturing procedure that arranges and uses information for considering various parts and products with similar machining requirements into part families and grouping machines into machine cells. Cellular manufacturing (CM) is one of the applications of group technology concept in manufacturing systems which deals with determining machine cells and part families. In the design of CM, similar parts are grouped into families of parts and dissimilar machines into machine groups so that one or more part families can be processed within a single machine group. The major advantages of CM presented in the literature include reduction in setup time, reduction in throughput time, reduction in work-in-process inventories, reduction in material handling costs, better quality and production control, increment in flexibility, etc. [2, 3]. The process of determining part families and machine groups is called the cell formation problem (CFP). At the conceptual level, most cell formation models have ignored many manufacturing factors and only considered the machining operations of the parts. Therefore, a manufacturing system was represented by a binary machine-part incidence matrix A=[a_{ij}], which was a zero-one matrix of order P×M, where P is the number of parts and M is the number of machines. If a_{ij}=1, it means that part i needs processing on machine j, otherwise a_{ij}= 0. Many researches have been developed for solving the CFP and comprehensive summaries and taxonomies of CFP were presented by Wemmerlov and Hyer [4], Selim et al. [5], Mansouri et al. [6], Yin and Yasuda [7] and Jabalameli et al. [8]. Some recent approaches and methods considering the CFP as a binary part-machine incidence matrix are reviewed at the following.

Chen and Cheng [9] considered a neural network-based cell formation algorithm in cellular manufacturing. They used an adaptive resonance theory (ART) based neural network to the cell formation problem. The advantages of applying an ART network over the other conventional methods were fast computation and the outstanding ability to handle large scale industrial problems. Mahdavi et al. [10] proposed a graph-neural network manufacturing approach for cell formation problems. Effort was made to develop an algorithm that was more reliable than conventional methods. Their research had the ability to handle large scale industrial problems with promising results in the presence of bottleneck machines and/or exceptional parts. On wubolu and Muting [11] developed a genetic algorithm, which accounts for inter-cellular movements and the cell-load variation. Soleymanpour et al. [12] applied a transiently chaotic neural network approach (TCNN) for solving a mathematical model in design of CM. The approach adopted for the simultaneous grouping of similar machines and parts was based on minimizing the total number of exceptional elements and number of voids.

Chen and Chen [13] integrated a modified ART1 network with an effective technique, Tabu Search (TS), to solve cell formation problems. The number of exceptional elements and group efficiency are considered as the objectives for the problem under the constraints of the number of cells and cell size. Goncalves and Resende [14] presented a hybrid algorithm combining a local search and a genetic algorithm with very promising results. Albadawi et al. [15] proposed a mathematical model for forming manufacturing cells. The proposed approach involved two phases. In the first phase, machine cells were identified by applying factor analysis to the matrix of similarity coefficients. In the second phase, an integer-programming model was used to allocate parts to the identified machine cells. Mahdavi et al. [16] proposed a new mathematical model for CFP in cellular manufacturing based on cell utilization concept. The objective of the model was to minimize the number of voids in cells to achieve the higher performance of cell utilization. Yang and Yang [17] presented a modified adaptive resonance theory (ART1) neural learning algorithm, in which the vigilance parameter could be simply estimated by the data so that it was more efficient and reliable compared with previous neural network approaches. Mahdavi et al. [18] developed a mathematical model for the CFP based on cell utilization concept in CM. An efficient algorithm based on GA was designed to solve the mathematical model. Díaz et al. [19] proposed a greedy randomized adaptive search procedure (GRASP) heuristic to obtain lower bounds for the optimal solution of CFP. Their method consisted of two phases. In the first phase, an initial partition of machines into machine-cells or parts into part families was obtained, while in the second phase the assignment of parts to machine cells or machines to part-families were considered. Anvari et al. [20] developed a particle swarm optimization-based optimization algorithm for the cell formation problem. Paydar et al. [21] proposed a mathematical model to identify machine cells and part families simultaneously, so that the number of voids and exceptional elements were minimized. Arkat et al. [22] presented a multi-objective programming model with the aim of minimizing the number of exceptional elements and the number of voids, simultaneously. They also developed a bi-objectives genetic algorithm for large-scale problems.

In this paper, a mathematical model based on multiple traveling salesman problem for determining machine groups in cell formation problem is developed with applying the dissimilarity coefficient as the cost of travelling between two nodes. To verify the proposed model, a number of test problems selected from the literature are solved. The obtained solutions are compared with those of previous well-known methods using a grouping efficacy measure.

II. MULTIPLE TRAVELING SALESMAN PROBLEM

A generalization of the well-known traveling salesman problem is the multiple traveling salesman problem (mTSP). The problem can be introduced simply as the determination of a set of routes for *m* salesman who all start from and return to a single home city (depot). This problem consists of finding tours for all the salesmen such that all customers are visited exactly once and the total cost of all the tours is minimized.

In this section, we develop a mathematical model where the number of salesman is unknown and its optimal value is determined by the proposed model. Moreover, a constant cost, *f*, is considered for each salesman who is used. Beside, because of constraint capacity of salesman, *L* is defined as the maximum number of nodes which a salesman can visit. $C=c_{ij}$ is the distance matrix associated with each arc *(i, j)*. For problem formulation, tree variables are needed as:

x_{ij} 1 if arc *(i, j)* is in the optimal solution; 0, otherwise.

m the number of salesmen who is employed.

For any traveler, u_i is the number of nodes visited on the traveler's path from the origin up to node *i* (i.e., the visit number of the i^{th} node). Thus, $1 \le u_i \le L$ for all $i \ge 2$.

The following is the integer linear programming formulation for the mTSP which is derived from the mathematical model proposed by Kara and Bektas [23].

$$Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \times x_{ij} + f \times m$$

s.t.:

$$\sum_{j=2}^{n} x_{1j} = m \tag{1}$$

$$\sum_{j=2}^{n} x_{j1} = m \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \forall\ i = 2,3...,n; i \ne j \tag{3}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \forall\ j = 2,3...,n; i \ne j \tag{4}$$

$$u_i + (L-2) \times x_{1i} - x_{i1} \le L-1 \qquad \forall\ i = 2,3...,n \tag{5}$$

$$u_i + x_{1i} \ge 2 \quad \forall\ i = 2,3...,n \tag{6}$$

$$u_i - u_j + L \times x_{ij} + (L-2) \times x_{ji} \le L-1 \quad \forall\ 2 \le i \ne j \le n \tag{7}$$

$$x_{ij} \in \{0,1\} \quad \forall i, j \tag{8}$$

The objective function minimizes the total cost of the travelled tours and the total constant cost of the employed salesmen. Constraints (1) and (2) guarantee that exactly *m* salesmen depart and arrive the origin. Constraints (3) and (4) are the degree constraints. Inequalities (5) and (6) control the upper and lower bound limitations on the number of times a node can be visited. It is shown in inequality (6) that the lower bound is 1. Inequality (7) guarantees that $u_j = u_i + 1$, if $x_{ij} = 1$. This constraint prohibits forming sub-tours between the nodes.

III. EMPLOYING MTSP FOR CFP

As mentioned above, mTSP has been used in diverse applications. In this paper, we employ the presented formulation of the mTSP to determine machine cells in a CFP. We redefine the notations used for mTSP to be suited into a CFP and present them in Table I.

It is assumed that each cell starts from the virtual node. It is also assumed that each machine should be assigned to only one cell. It means that each customer (machine) is visited by only one salesman (cell). The number of arcs starting from the virtual node is equal to *m* (the number of cells) and their related costs are considered as 0. The number of arcs returning to the virtual node is also equal to *m* by related costs considered as 0. It is worth mentioning that the number of cells is an integer variable and its optimal value is determined by the proposed model.

TABLE I DEFINITION OF NOTATIONS IN mTSP AND CORRESPONDING CFP

Notations	mTSP	CFP
M	The number of salesmen	The number of cells
f	The constant cost of employing each salesman	The constant cost of forming each cell
$j=1$	The depot as start and end node	The virtual node
$j=\{2,3,...,n\}$	The set of customers	The set of machines
L	The maximum number of nodes a salesman can visit	The maximum number of machines can be allocated to each cell
$x_{ij} \in \{0,1\}$	1 if arc *(i, j)* is in the optimal solution; 0 otherwise	1 if machine *j* is assigned after machine *i*; 0 otherwise
c_{ij}	The cost associated with each arc*(i, j)*	The relation between machine *i* and *j*

TABLE II THE WELL-KNOWN DISSIMILARITY COEFFICIENTS

No.	The name of DF (d_{ij})	Formulation	Range
1	Minkowski	$\left(\sum_{k=1}^{n} \lvert a_{ki}-a_{kj}\rvert^{r}\right)^{1/r}$	Real
2	Euclidean	$\left(\sum_{k=1}^{n} \lvert a_{ki}-a_{kj}\rvert^{2}\right)^{1/2}$	Real
3	Manhattan (City Block)	$\sum_{k=1}^{n} \lvert a_{ki}-a_{kj}\rvert$	Real
4	Average Euclidean	$\left(\sum_{k=1}^{n} \lvert a_{ki}-a_{kj}\rvert^{2} / n\right)^{1/2}$	Real
5	Weighted Minkowski	$\left(\sum_{k=1}^{n} w_k \lvert a_{ki}-a_{kj}\rvert^{r}\right)^{1/r}$	Real
6	Bray-Curtis	$\sum_{k=1}^{n} \lvert a_{ki}-a_{kj}\rvert \Big/ \sum_{k=1}^{n} \lvert a_{ki}+a_{kj}\rvert$	0-1
7	Canberra Metric	$\frac{1}{n}\sum_{k=1}^{n}\left(\frac{\lvert a_{ki}-a_{kj}\rvert}{a_{ki}+a_{kj}}\right)$	0-1

The main important issue is the way of the calculation of the cost matrix between machines. Dissimilarity coefficient (DF) between machines can be used for the entries of this matrix. Similarity/dissimilarity coefficients are calculated between machines or parts. Table II shows the well-known dissimilarity coefficients between machines where:

n: is the number of parts;

a_{ki} : 1 if part k requires to be processed by machine j; 0 otherwise.

In the proposed model, the relation between two machines i and j (c_{ij}) is calculated by the dissimilarity coefficient Bray-Curtis.

IV. COMPUTATIONAL RESULTS

To verify the performance of the proposed method, ten test problems are selected from the literate. The grouping efficacy (GE) obtained by the proposed model for each problem is compared with the results reported in the literature. The sources and sizes for the test problems, the grouping efficacy (%) and the best cell size obtained for each problem are shown in Table III. Optimal solutions for the problems are obtained by the LINGO 9 software using branch-and-bound (B&B).

TABLE III COMPUTATIONAL RESULTS OF PROPOSED MODEL

No.	Problem source	No. of machine	No. of part	GE	No. of Cells
1	King and Nakornchai [24]	5	7	82.35	2
2	Waghodekar and Sahu [25], Fig. 4a	5	7	69.57	2
3	Seifoddini [26]	5	18	79.59	3
4	Kusiak and Cho [27]	6	8	76.92	3
5	Boctor [28], Fig. 1b	7	11	70.87	4
6	Seifiddini and Wolf [29]	8	12	68.30	4
7	Chandrasekharan and Rajagopalan [30]	8	20	58.72	5
8	Chandrasekharan and Rajagopalan [30]	8	20	85.25	2
9	Mosier and Taube [31]	10	10	73.33	5
10	Chan and Milner [32]	10	15	92.00	3

TABLE IV THE DISSIMILARITY COEFFICIENT FOR THE FIRST TEST PROBLEM

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	1	1	1	0.14	1
3	0	1	1	0.2	1	0.5
4	0	1	0.2	1	1	0.2
5	0	0.14	1	1	1	1
6	0	1	0.5	0.2	1	1

The most commonly-used objectives in cell formation are to minimize inter-cell movements and maximize utilization of machines [33]. The presence of exceptional elements displays inter-cell movements. Unlike, forcing exceptional elements to go to manufacturing cells reduces the utilization of machines. Therefore, a trade-off between these conflicting objectives is the chief problem of interest in the design of cell formation.

In order to decrease inter-cell movements, the number of ones out of the diagonal blocks in the machine-part incidence matrix should be minimized. Concurrently, to increase utilization of machines, the number of 'zeros' inside the diagonal blocks should be minimized. Chandrasekharan and Rajagopalan [34] proposed a grouping efficiency measure clarified as a collective measure of desired parameters. This measure simultaneously concerns the intended objectives. It has been widely used in the literature.

Although grouping efficiency is applied as a measure of the quality of solutions, it suffers from some limitations. For example, it requires a weight factor which is determined by designer and it is dependent to the number of cells. In addition, when the matrix size increases, the effect of exceptional elements becomes smaller, and in some cases, the effect of intercellular moves is not reflected in the grouping efficiency. Hence, Kumar and Chandrasekharan [35] presented a grouping efficacy measure. The definition of this measure is given as:

$$\mu = \frac{e - e_0}{e + e_v}$$

In this equation, e is the total number of '1's in the given machine-part incidence matrix, e_v is the total number of voids, and e_0 is the total number of exceptional elements.

The results of the proposed method are compared with those of the following methods:

1. ZODIAC method [36],
2. GRAFICS method [37],
3. GATSP-Genetic algorithm [38],
4. GA-Genetic algorithm [11],
5. EA-evolutionary algorithm [14],
6. HGA- hybrid genetic algorithm [39],
7. GRASP- GRASP heuristic [19].

In order to provide a comprehensive understanding of the mTSP procedure, the dissimilarity coefficient of the first test problem is calculated as given in Table IV.

As can be seen, the matrix of dissimilarity coefficients is symmetric. The first problem with 5 machines (nodes) contains 6 nodes by considering one dummy node as a departure point of salesmen. As mentioned before, arrival and departure cost of dummy node is considered as zero. Then, the entries in the first row and first column are zero. Lingo code for the first problem is given in Appendix 1. The optimal number of cells obtained for this problem is 2 and the obtained path for each cell (salesman) is as follows:

Cell I: Machine 1→ Machine 2 → Machine 5→ Machine 1

Cell II: Machine 1→ Machine 3 → Machine 4→ Machine 6→ Machine 1

It means Machines 1 and 4 are assigned to Cell 1 and Machines 2, 3 and 5 are assigned to Cell 2.

Table V compares the solutions of 10 test problems obtained by applying 7 methods previously introduced and the proposed method. As can be seen, for seven Test Problems 2, 3, 4, 6, 7, 8 and 10 the efficacy measures obtained by the proposed model are as good as the best measures obtained by other methods. Also for three Test Problems 1, 5 and 9 the efficacy measures obtained by the proposed model are better than those obtained by the other methods. The efficacy measures for four Test Problems 4, 6, 7 and 10 obtained by the all methods are the same. Generally, we can conclude the performance of the proposed model in terms of grouping efficacy is superior in compare to the other methods. Hyphen in Table V means the related test problem has not been solved by that method.

TABLE V PERFORMANCE OF PROPOSED MODEL COMPARED TO OTHER METHODS

No.	ZODIAC	GRAFICS	GATSP	GA	EA	HGA	GRASP	Proposed method
1	73.68	73.68	-	-	73.68	73.68	73.68	**82.35**
2	56.22	60.87	68.00	62.50	62.50	**69.57**	62.50	**69.57**
3	-	-	77.36	77.36	**79.59**	**79.59**	**79.59**	**79.59**
4	-	-	**76.92**	**76.92**	**76.92**	**76.92**	**76.92**	**76.92**
5	-	-	70.37	70.37	70.37	70.37	70.37	**70.87**
6	**68.30**	**68.30**	-	-	**68.30**	**68.30**	**68.30**	**68.30**
7	**85.25**	**85.25**	**85.25**	**85.25**	**85.25**	**85.25**	**85.25**	**85.25**
8	58.33	58.13	58.33	55.91	**58.72**	**58.72**	**58.72**	**58.72**
9	70.59	70.59	70.59	72.79	70.59	70.59	70.59	**73.33**
10	**92:00**	**92:00**	**92:00**	**92:00**	**92:00**	**92.00**	**92.00**	**92:00**

V. CONCLUSIONS

We have introduced a mathematical model based on multiple traveling salesman problem for solving manufacturing cell formation problem. The objective of the proposed approach is to minimize the dissimilarity coefficient as the cost of travelling between two machines and the constant cost of installing the cells. Hence, this approach has the flexibility to allow the system designer to identify the required number of cells. This model has the ability of finding good solutions in comparing to other existing approaches. Based on ten test problems adopted from the literature, we found that the proposed model performs very promising. Achieving an exact solution for such a hard problem in a reasonable time is computationally intractable. Thus, it is necessary to apply a heuristic or meta-heuristic approach to solve the proposed model for real-sized problems. Moreover, considering other similarity/dissimilarity coefficients for the relation between two machines is suggested.

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APPENDIX 1

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MODEL:
 SETS:
machine/1..6/;
ijlink(machine,machine):c,x;
ilink(machine):u;
ENDSETS
! Objective function;
min=@sum(machine(i):@sum(machine(j):c(i,j)*x(i,j)))+f*
m;
! Constraints;
!1;
@sum(machine(j)|j#ge#2:x(1,j))=m;
!2;
@sum(machine(j)|j#ge#2:x(j,1))=m;
!3;
@for(machine(j)|j#ge#2:@sum(machine(i)|i#ne#j:x(i,j))=1);
!4;
@for(machine(i)|i#ge#2:@sum(machine(j)|i#ne#j:x(i,j))=1);
!5;
@for(machine(i)|i#ge#2:u(i)+((L-2)*x(1,i))-x(i,1)<=(L-1));
!6;
@for(machine(i)|i#ge#2:u(i)+x(1,i)>=2);
!7;
@for(machine(i)|i#ge#2:@for(machine(j)|j#ge#2        #and#
i#ne#j:u(i)-u(j)+(L*x(i,j))+((L-2)*x(j,i))<=L-1));
!8;
@for(ijlink:@bin(x));
@gin(m);
data:
L=4;
f=0.4;
c=@ole(' D:\test problems\problem1.xls', 'problem1');
enddata
End
```

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