Damping Evaluating of Laminated Beams by Dynamic Analysis

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Abstract- **The paper presents an analysis of the damping of laminated materials with four different stacking sequences. The impulse technique was chosen to perform modal analysis of the ease of implementation and quickness of the test. The numerical analysis is performed by the finite element method using beam element. The results obtained are compared with the experimental responses in frequency of the structure. The decrease in frequency for different rates of loading shows the loss of stiffness for all studied materials. The structural damping of the different beams is extracted from a finite element modelling and evaluated from a handling of damaged and undamaged modal energies.**

Keywords- Damping; Finite Element Method; Frequency; Composite Materials; Modal Energy; Modal Analysis

I. INTRODUCTION

Damping is an important parameter for vibration control, fatigue endurance, impact resistance, etc. although the damping of composite materials is not very high; it is significantly higher than that measured for most usual metallic materials. At the constituent level, the energy dissipation in fibre reinforced composites is induced by different mechanisms such as the fibre- matrix interface, the damping due to defects or damage, etc. At the laminate level, damping is strongly depending on the layer constituent properties as well as layers orientations, interlaminar effects and stacking sequence. Viscoelastic materials combine the capacity of an elastic type material to store energy with the capacity to dissipate energy. So, the use of an energy approach for evaluating the material or structure damping is widely considered. In this energy approach, the dissipated energy is related to the strain energy stored by introducing a damping parameter [1].

The initial works on the damping analysis of fibre composite materials were reviewed extensively in review paper by Gibson and Plunkett [2] and Gibson and Wilson [3]. A damping process has been developed initially by Adams and Bacon [4] in which the energy dissipation can be described as separable energy dissipations associated to the individual stress components. This analysis was refined in later paper of Ni and Adams [5]. The damping of orthotropic beams is considered as function of material orientation and the papers also consider cross-ply laminates and angle-ply laminates, as well as more general types of symmetric laminates.

The damping concept of Adams and Bacon was also applied by Adams and Maheri [6] to the investigation of angle-ply laminates made of unidirectional glass fibre or carbon layers. The finite element analysis has been used by Lin et al. [7] and Maheri and Adams [8] to evaluate the damping properties of free-free fibre reinforced plates. These analyses were extended to a total of five damping parameters, including the two transverse shear damping parameters. More recently the analysis of Adams and Bacon was applied by Yim [9] and Jang [10] to different types of laminates, then extended by Yim and Gillespie [11] including the transverse shear effect in the case of 0° and 90° unidirectional laminates. For thin laminate structures the transverse shear effects can be neglected and the structure behaviour can be analysed using the classical laminate theory.

The natural frequencies and mode shapes of rectangular plates are well described using the Ritz method introduced by Young [12] in the case of homogeneous plates. The Ritz method was applied by Berthelot and Safrani [13] to describe the damping properties of unidirectional plates. The analysis was extended to the damping analysis of laminates [14]. The objective of this work is to study different stacking sequences effect on damping by using a finite element analysis to evaluate the damping and the natural frequencies of the structure.

II. EXPERIMENTAL TESTS

A. Tested Materials

The experimental study was achieved in the case of glass fibre composites. The laminates were prepared by hand lay-up process from SR1500 epoxy resin with SD2505 hardener and unidirectional E-glass fibre fabrics of weight 300gm-2. Beams of 200 mm length and 20 mm width were cured at room temperature with a pressure of 30 kPa using vacuum moulding process, and then post-cured for 8h at 80 ℃ in an oven. Beams had a nominal thickness of 2 mm with a volume fraction of fibres equal to 0.40. The laminated beams with four different stacking sequences were analysed (Table 1):

TABLE I STACKING SEQUENCES OF COMPOSITE MATERIALS

Designation	Stacking sequences
U	$[(0)]_8$
C1	$[(0/90)_s]_2$
C2	$[(0/90)_2]_s$
C3	$[(0_2/90_2)]_s$

The mechanical modulus of elasticity of the unidirectional materials referred to the fibre direction was measured in static tensile (Table 2):

TABLE IIIII MECHANICAL CHARACTERISTICS OF COMPOSITE MATERIALS

Materials	Young's modulus GPa)	Max load at fracture (KN)
U	21.08	35.165
C1	14.51	20.020
C2	15.04	20.915
C3	15.06	18.428

The experimental investigation was conducted using tensile cyclic tests for different laminates studied. The applied load ratio increases with 10 % of maximum load failure for each cycle. Fig. 1 shows the results obtained for the Young's modulus reduction as a function of cycle number.

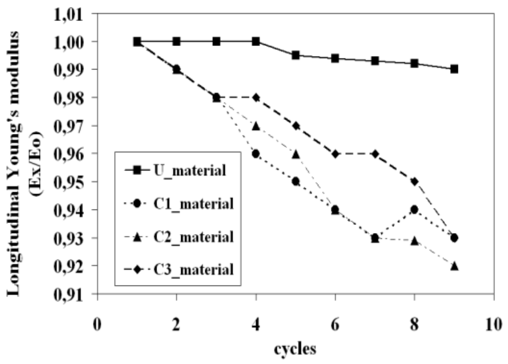


Fig. 1 Stiffness reduction of U, C1, C2 and C3 laminates as a function of cycle number

B. Experimental Equipment

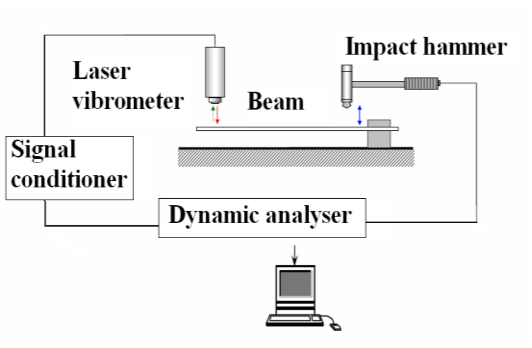


Fig. 2 Experimental equipment

The damping characteristics of the materials were obtained by subjecting beams to flexural vibrations. The equipment used is shown in Fig. 2. The test specimen is supported horizontally as a cantilever beam in a clamping block. An impulse hammer is used to induce the excitation of the flexural vibrations of the beam and the beam response is detected using a laser vibrometer. Next, the excitation and the response signals are digitalized and processed by a dynamic analyzer of signals.

This analyzer associated with a PC computer performs the acquisition of signals, controls the acquisition conditions and next performs the analysis of the signals acquired (Fourier transform, frequency response, mode shapes, etc.). In the case of laminate materials, the damping characteristics of the beams are deduced from the Fourier transform of the beam response to an impulse input by fitting this experimental response with the analytical response of the beam which was derived in [13] using the Ritz method.

C. Analysis of the Experimental Results

Figs. 3-6 report the frequency response of specimen beams obtained for three different loading rates (0 %, 50 % and 90 %). These responses show peaks, which correspond to the natural frequencies of the flexural vibrations of the beams.

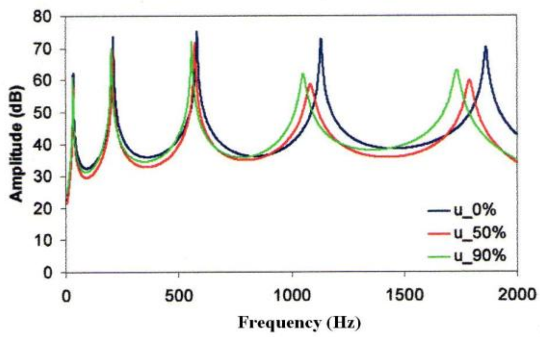


Fig. 3 Loading rates influence on the frequency responses of structure constituted of U material

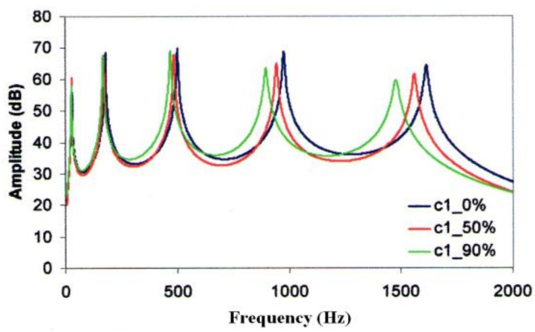


Fig. 4 Loading rates influence on the frequency responses of structure constituted of C1 material

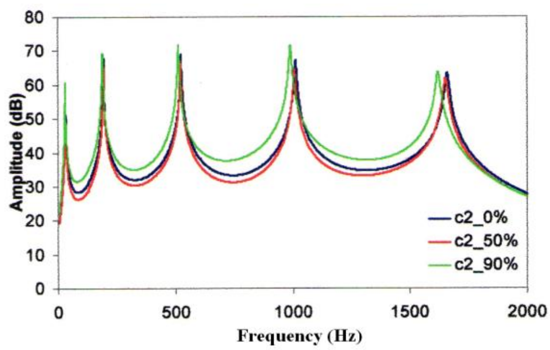


Fig. 5 Loading rates influence on the frequency responses of structure constituted of C2 material

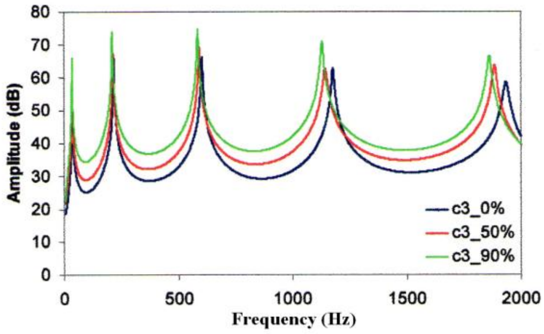


Fig. 6 Loading rates influence on the frequency responses of structure constituted of C3 material

III. FINITE ELEMENT ANALYSIS

The flexural vibrations of beams are analysed by the finite element method, using the stiffness matrix and mass matrix of beam element with two degrees of freedom per node (Fig. 7) and the number of elements using in this study is 40 elements:

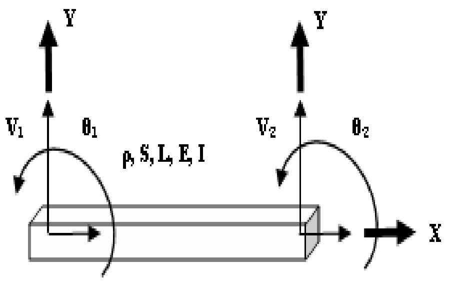


Fig. 7 Beam element with four degrees of freedom

Where:

E: the Young modulus.

I: the moment of inertia of the beam.

L: the length of the beam.

S: the section of the beam.

ρ: the density.

$$K_e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{1}$$

$$M_e = \frac{\rho S L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

The global matrix of mass and stiffness are obtained by using assembly method:

$$K_G = B^T K_{des} B \qquad M_G = B^T M_{des} B \tag{2}$$

Where:

- B is the Boolean matrix;
- Kdes and Mdes are unassembled matrix, they contain only elementary matrix of mass and stiffness.

$$K_{des} = \begin{bmatrix} [K_e^{\,1}] & & 0 \\ & \ddots & \\ 0 & & [K_e^{\,N}] \end{bmatrix}$$

$$M_{des} = \begin{bmatrix} [M_e^{\,1}] & & 0 \\ & \ddots & \\ 0 & & [M_e^{\,N}] \end{bmatrix} \tag{3}$$

IV. RESOLUTION OF THE EIGENVALUE PROBLEM

We have two cases where the structure is:

- undamaged;
- damaged.

The equation of motion (undamped and free vibration):

$$m\,\ddot{q}(t) + k\,q(t) = 0 \tag{4}$$

The equation (4) can be written in matrix form:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \tag{5}$$

With:

- q: the vector of degrees of freedom;
- for the first case [K] = $[K_D]$;
- for the second case [K] = $[K_{GD}]$.

Where $[K_{GD}]$ is the global stiffness matrix with damage, that takes into account the decrease in the rigidity of the structure when the loading rates change [8].

The general solution of Equation (5) is:

$$\{q\} = \{q_0\}\, e^{i\omega t} \tag{6}$$

By substituting the Equation (6) in Equation (5), we have:

$$[K]\{q_0\} = \omega^2 [M]\{q_0\} \tag{7}$$

Then, the determinant must be zero:

$$det([K] - \omega^2 [M]) = 0 \tag{8}$$

There are many methods to calculate the eigenvalues; the most of these methods are to write the Equation (7) as follows:

$$[H]\{X\} = \lambda\{X\} \tag{9}$$

Where [H] Is A Positive And Symmetric Matrix, It Is Clear That If We Write Directly The Equation (7) As:

$$[K]^{-1}[M]\{q_0\} = \frac{1}{\omega^2}\{q_0\} \tag{10}$$

Where $[K]^{-1}$ is the inverse of the matrix [K], the symmetry property is not always preserved. Therefore, it is necessary to write the matrix [K] using the Cholesky decomposition [7]:

$$[K] = [L][L]^T \tag{11}$$

$[L]^T$ is the transpose of the matrix [L] and [L] is a lower triangular matrix. The Equation (7) is written:

$$[L]^{-1}\,[M]\,[L]^{-T}\,[L]\,\{q_0\} = \frac{1}{\omega^2}\,[L]\,\{q_0\} \tag{12}$$

By writing Equation (12) as similar form as Equation (7):

$$[H] = [L]^{-1}\,[M]\,[L]^{-T} \tag{13}$$

$$\{X\} = [L]\,\{q_0\} \tag{14}$$

$$\lambda = \frac{1}{\omega^2} \tag{15}$$

V. RESULTS

The modal analysis of the structure for different loading rates is based on the analytical method used to solve the equation of free vibrations. The programming of this resolution method was performed under the Matlab software. The Tables 3-6 show the frequencies obtained by model and experiment for the laminates studied. The decrease in frequency for the four materials studied for different loading rates shows the loss of stiffness [15-16].

TABLE IVVVI FREQUENCIES OBTAINED BY THE MODEL AND EXPERIMENT FOR THE MATERIAL U

Experimental frequencies with Different loading rates (Hz)			Modelling frequencies with Different loading rates (Hz)		
0%	50%	90%	0%	50%	90%
34.800	34.370	33.570	35.570	35.481	35.392
210.60	208.00	203.00	207.67	207.15	206.63
581.90	571.90	558.00	525.84	524.52	523.20

TABLE VIIV FREQUENCIES OBTAINED BY THE MODEL AND EXPERIMENT FOR THE MATERIAL C1

Experimental frequencies with Different loading rates (Hz)			Modelling frequencies with Different loading rates (Hz)		
0%	50%	90%	0%	50%	90%
28.00	27.50	26.87	27.205	26.516	26.376
178.75	173.12	167.00	180.27	175.71	167.05
500.00	485.00	470.00	436.26	425.22	422.97

TABLE V FREQUENCIES OBTAINED BY THE MODEL AND EXPERIMENT FOR THE MATERIAL C2

Experimental frequencies with Different loading rates (Hz)			Modelling frequencies with Different loading rates (Hz)		
0%	50%	90%	0%	50%	90%
31.87	31.87	30.65	32.497	32.005	31.339
193.00	193.00	188.00	191.96	189.06	185.12
524.00	521.00	511.00	444.16	437.45	428.33

TABLE VVIII FREQUENCIES OBTAINED BY THE MODEL AND EXPERIMENT FOR THE MATERIAL C3

Experimental frequencies with Different loading rates (Hz)			Modelling frequencies with Different loading rates (Hz)		
0%	50%	90%	0%	50%	90%
35.6	34.40	34.00	35.076	34.546	34.008
216.80	213.00	210.60	219.29	215.98	212.61
603.00	589.60	584.00	444.46	437.74	430.92

VI. NUMERICAL EVALUATION OF DAMPING

The calculation of loss factors of modal energies for the first three modes of vibration of the structure is done by evaluating the ratio of the strain energies of beam for damaged and undamaged cases [17].

The modal strain energy of the beam for the undamaged case is given by:

$$U_n = \frac{1}{2}[\phi_n]^T\;[K_G]\,[\phi_n] \tag{16}$$

With:

- $[K_G]$: Stiffness matrix;
- $[\O_n]$: Eigenvector of displacement.

The modal strain energy for damaged case is given by:

$$U_{nD} = \frac{1}{2}[\phi_{nD}]^T\;\left[K_G{}^D\right][\phi_{nD}] \tag{17}$$

With:

- $[K_G{}^D]$: Stiffness matrix (damaged case);
- $[\O_{nD}]$: Eigenvector of displacement (damaged case).

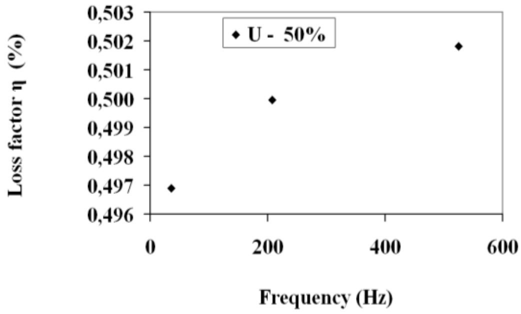
The loss factor coefficient [17] for different stages of damage (different loading rates) is given by:

$$\eta_n = \frac{\Delta U_n}{U_n} = \frac{U_n - U_{nD}}{U_n} \tag{18}$$

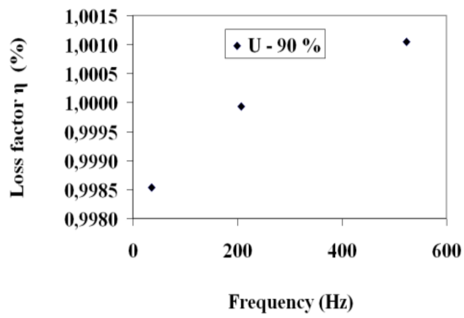
With:

- U_n: modal strain energy for undamaged case ;
- U_{nD}: modal strain energy for damaged case.

Figs. 8-12 report the results deduced for the damping by finite element analysis for the first three modes. The evaluation of laminate damping by modelling takes account of the variation of the loss factor η_n with frequency. The material damping is derived as function of laminate orientation.

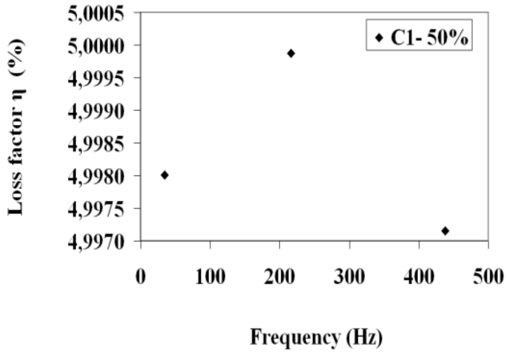


(a)

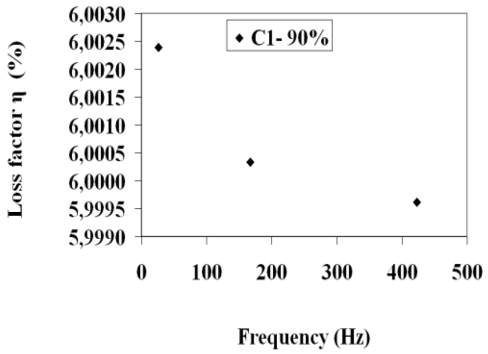


(b)

Fig. 8 Modelling results obtained for the damping as function of the frequency for U material in the case: (a) load 50 % and (b) load 90 %

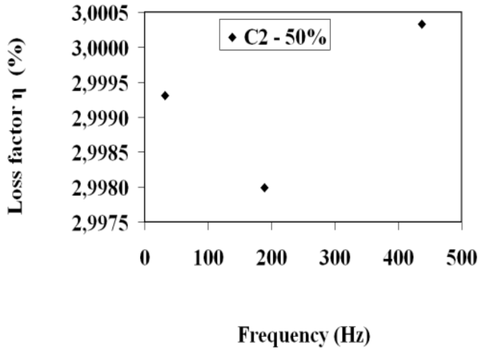


(a)

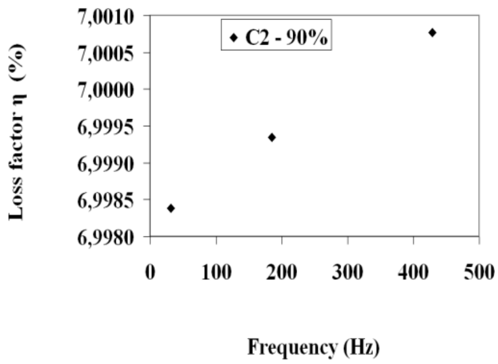


(b)

Fig. 9 Modelling results obtained for the damping as function of the frequency for C1 material in the case: (a) load 50 % and (b) load 90 %

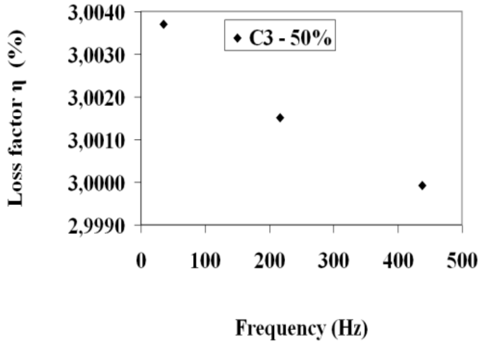


(a)

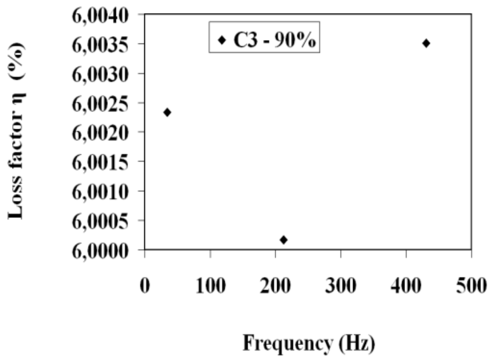


(b)

Fig. 10 Modelling results obtained for the damping as function of the frequency for C2 material in the case: (a) load 50 % and (b) load 90 %



(a)



(b)

Fig. 11 Modelling results obtained for the damping as function of the frequency for C3 material in the case: (a) load 50 % and (b) load 90 %

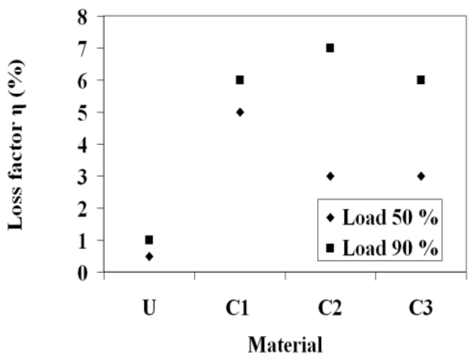


Fig. 12 Comparison between damping of different laminates studied for different loading rates (50 % - 90 %)

For 0 ° orientation of the laminate $[(0)]_8$, it is observed that damping increases when the frequency and the loading rates are increased (Fig. 8). The stacking sequence leads to a more variation of damage as function of the loading rates is increased, two cases are shown:

- When the loading rate is 50 % (Figs. 9-12): it is observed that damping is slightly higher for C1 than that laminates U, C2 and C3. The damping of the laminates U, C2 and C3 is clearly reduced (about 90 % for U , 40 % for C2 and 40 % for C3);
- When the loading rate is 90 % (Figs. 9-12): the damping behaviour is practically as function of the fibre orientation which is more important than in the case in 50 % of loading rates. The maximum rate of damping for C2

laminate is (7 %). The damping of laminate U, C1 and C3 is clearly reduced (about 85.71 % for U, 14.29 % for C1 and 14.29 % for C3).

VII. CONCLUSIONS

An evaluation of the damping of different composite materials was presented based on a finite element analysis. The analysis derived the strain energy stored in the different materials.

The damping behaviour is practically as function of the fibre orientation which is more important when the loading rate is increased.

The decrease in frequency of different loading rates shows the loss of stiffness for the four studied materials. This evolution constitutes one of the most used methods to follow the progression of fatigue damage of the composites.

The loss factors of the composite materials can be deduced by applying modelling to the flexural vibrations of free-clamped beams. The loss factors of the laminates with different stacking sequences are very higher than the laminate with 0 °orientation for different loading rates.

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