EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 10, No. 2, 2017, 167-198
ISSN 1307-5543 – www.ejpam.com
Published by New York Business Global

Measuring set latent variables that explain attitude toward statistic through exploratory factor analysis with principal components extraction and confirmatory analysis

Arturo, GARCÍA-SANTILLÁN

UCC Business School, Universidad Cristóbal Colón, MÉXICO

Abstract. The aim of this paper is to show a path to measure a set latent variable through exploratory factorial analysis and confirmatory analysis. It starts with the theoretical mathematical procedure and then, with a database, it shows the re-specified model of study. This procedure has been used to explain anxiety towards mathematics. Many students often come to these subjects with negative attitude and usually with high levels of anxiety, which affects performance when they face classes, exercises or tests. Due to the importance of this subject, this behavior is formally analyzed in several studies, with the use of these statistical techniques previously mentioned

2010 Mathematics Subject Classifications: 62H15, 62H25 JEL: C26, C38, C51

Key Words and Phrases: Factor analysis, confirmatory analysis, structural equations, factorial weights, communalities, variance

1. Introduction

Preliminary notes and notation

In exploratory factor analysis (EFA), we seek to identify the measures of the model, i.e., the number of factors and its indicators, because the theory establishes that some variables are indicators of some factors, as we can see in any model of study. After this, the model structure is specified in order to be validated by confirmatory analysis in a later phase, i.e., we seek to validate the model obtained in the exploratory phase, and subsequently confirmed by another statistic technique, which could be with the use of structural equations.

In the social science field, frequently we require measure some scales that its utilized in order to obtain a set data to measure some aspect about perception. For example, in the research about attitude, anxiety, beliefs and perception toward mathematics, all of this, on high school students and college students as well. Hence, with this type of scales

Email address: agarcias@ucc.mx, arturogarciasantillan@yahoo.com.mx

we may build a latent variables structure which underlying in the phenomena previously mentioned.

In this idea and in order to align on the one hand, the set of latent variables that explain this kind of phenomena, and secondly the use of statistical techniques, then is discussed the phenomena of attitude and anxiety towards math. The set of latent variables that have tried to explain anxiety towards mathematics was initiated with the work of mathematics teachers at the beginning of 1950. In 1957 Dreger and Aiken [1] introduced the term "math anxiety" as a variable which allows describing the difficulties and attitude of students with mathematics. They defined as "the presence of a syndrome of emotional reactions to the arithmetic and math." Although, since then it became difficult explain this concept, it was begun explanation of this phenomenon only with the opinions of the authors, without the use of statistical techniques to assess anxiety towards mathematics [36].

Later, in a second term the studies focused on measuring attitudes toward mathematics through surveys which included several variables, which necessarily requires the use of multivariate statistical techniques [7]. In a third period was continued with the study of mathematics and from this, the integrated scales associated with aspects that explain the attitude factors, anxiety, beliefs and perception towards mathematics, were developed. In this regard Dreger and Aiken [1] designed the first instrument, the numerical anxiety scale in 1957. In 1972 Richardson and Suinn [33] developed a scale called the Mathematics Anxiety Rating Scale (MARS). Subsequently was developed the Fennema-Sherman Mathematics Attitudes Scales [30]. Afterwards other scales were designed: the Mathematics Anxiety Scale [28] and Math Anxiety Questionnaire [24].

Furthermore, some authors developed an abbreviated version of the scale MARS, for example, Suinn and Winston [35] investigated the previous studies that attempted to shorten the original MARS, e.g., [19, 18, 26, 3] and generated 30 items from Alexander and Cobb (1984), Alexander and Martray, and Rounds and Hendel (1980). The 30 collected items were subjected to a principal components analysis with oblique rotation, and two factors that emerged accounted for 70.3% of the total variability in the MARS items. Mathematics Test Anxiety accounted for 59.2% of the variance, whereas Numerical Anxiety accounted for 11.1% of the variance.

Extensive research has been done on the scale MARS and its psychometric properties, e.g., [21, 33, 18, 32, 13]. However, the second and more important study about this scale is, the shortcoming of the instrument, due to the proposed underlying construct of the scala MARS, is unidimensional [33, 27]. Nonetheless, others studies have revealed that there may be more than one underlying construct in mathematics anxiety, e.g., Alexander & Cobb; Alexander & Martray; Brush; Ferguson,; mentioned by [15], also [26, 21, 18, 2]. Ling [23] investigated the validity of mathematics anxiety as a multidimensional construct and found six factors (i.e., Personal Effectiveness; Assertiveness; Math Anxiety; Outgoingness; Success; and Dogmatism) that accounted for 76% of the total variance. Also, Bessant cited by [15] revealed that 43% of the variance in the MARS scores was explained by six factors: General Evaluation Anxiety, Everyday Numerical Anxiety, Passive Observation Anxiety, Performance Anxiety, Mathematics Test Anxiety, and Problem-Solving Anxiety.

Kazelskis [22] investigated the factor structure of the three most widely used mathematics anxiety scales: the RMARS [3], the Mathematics Anxiety Questionnaire (MAQ) [24] and the Mathematics Anxiety Scale (MAS) designed by [30]. When an exploratory factor analysis, with a principal component analysis and oblique rotation, was applied, the results revealed six dimensions of mathematics anxiety, which accounted for approximately 61% of the total variance. These six dimensions were: Mathematics Test Anxiety, Numerical Anxiety, Mathematics Course Anxiety, Worry, Positive Affect toward Mathematics, and Negative Affect toward Mathematics. Kazelskis [22] also pointed out that because Numerical Anxiety appears to be distinct from the other dimensions . . . it could be argued that anxiety as a result of the manipulation of numbers is the sine qua non of mathematics anxiety.

In other study, Bowd and Brady cited by [15] conducted principal components analysis followed by Varimax rotation on the results of 357 senior undergraduates in education and found three factors that accounted for 73% of the variability in the RMARS scores. The three factors were named Mathematics Test Anxiety (11 items), Mathematics Course Anxiety (8 items), and Numerical Task Anxiety (4 items).

Initial concurrent validity of the instrument was tested by comparing it with the Fennema-Sherman Attitude Scale [30], and negative relationships were found, which meant that students who had more favorable attitudes toward mathematics experienced less mathematics anxiety [3]. In addition, Moore, Alexander, Redfield, and Martray [3] found high to moderate correlations between the RMARS and the MAS [30], the State-Trait Anxiety Inventory [9], and the Test Anxiety Inventory [31].

Moore, Alexander and Martray [3] also found that the RMARS discriminated between students who took geometry or algebra in high school and students who did not. Students who took an algebra course (F = 18.07, p ¡ .001) and a geometry course (F = 25.60, p ¡ .001) in high school experienced significantly less mathematics anxiety compared with students who did not take these courses, as measured with the RMARS. Moore et al, [3] also revealed that the RMARS scores were significantly correlated with the American College Testing mathematics scores and mathematics course grades. Moderate-to-high-reliability evidence was found for the total and subscales of the RMARS. Initial internal consistency reliability coefficients of the RMARS subscales were .96 for the Mathematics Test Anxiety, 0.86 for the Numerical Task Anxiety, and 0.84 for the Math Course Anxiety.

The arguments presented in previous paragraphs about the constructs that measure attitudes, anxiety, and perception towards mathematics, are structures of latent variables that have been measured through exploratory factor analysis (EFA) and in some cases with confirmatory analysis (CA). With the use of these techniques primarily it seeks reduce the number of factors. With the factors that present the greatest possible load factor, can extract greater variance components present and allow explain the study object. Afterwards it is possible confirm the resulting model with structural equations in the confirmatory analysis. Therefore, following steps are utilized to perform this procedure.

1.1. Statistic for EFA

Following the work of Garca-Santilln, Venegas-Martnez and Escalera-Chvez [34], firstly we carry out the test of Sphericity with KMO, and goodness of fit index X2 with significance α=0.01, all this, in order to validate the pertinence of using this technique. Also, we obtain the communalities and factorial weights, in order to identify the explanatory power of the model, its mean, component matrix and communalities to obtain eigenvalue and its percentage of total variance.

Once the first statistics to validate the relevance of using the multivariate technique of factor analysis are obtained, we follow the method proposed by Carrasco Arroyo (s/f) and replicated in several studies by [16, 34, 15, 14] in order to measure the set of random variables observed; X_1 X_2. . . X_{297} which are defined in the population that share m (m<p) common causes to find m+p new variables, that we call common factors (Z_1, Z_2 ... Z_m).

Also, we should consider the unique factors ($\epsilon\ 1\ \epsilon\ 2\ \ \epsilon\ p$), in order to determine their contribution to the original variables (X_1 X_2 ..X_{p-1} X_p). Hence, we may define a model from the following equations:

$$\left|\begin{array}{l}
X_1 = a_{11}Z_1 + a_{12}Z_2 + ...a_{1m}Z_m + b_1\xi_1 \\
X_2 = a_{21}Z_1 + a_{12}Z_2 + ...a_{2m}Z_m + b_2\xi_2 \\
X_3 = a_{31}Z_1 + a_{32}Z_2 + ...a_{3m}Z_m + b_3\xi_3 \\
X_4 = a_{41}Z_1 + a_{42}Z_2 + ...a_{4m}Z_m + b_4\xi_4 \\
X_5 = a_{51}Z_1 + a_{52}Z_2 + ...a_{5m}Z_m + b_5\xi_5 \\
X_6 = a_{61}Z_1 + a_{62}Z_2 + ...a_{6m}Z_m + b_6\xi_6 \\
X_7 = a_{71}Z_1 + a_{72}Z_2 + ...a_{7m}Z_m + b_7\xi_7 \\
X_8 = a_{81}Z_1 + a_{82}Z_2 + ...a_{8m}Z_m + b_8\xi_8 \\
X_9 = a_{91}Z_1 + a_{92}Z_2 + ...a_{9m}Z_m + b_9\xi_9 \\
X_{10} = a_{101}Z_1 + a_{102}Z_2 + ...a_{10m}Z_m + b_{10}\xi_{10} \\
\text{...} \\
X_p = a_{p1}Z_1 + a_{p2}Z_2 + ...a_{pm}Z_m + b_p\xi_p
\end{array}\right| \quad (1)$$

Where:

Z_1, Z_2, Z_m are common factors

$\epsilon\ 1\ \epsilon\ 2\ \ \epsilon\ p$ are unique factors

Therefore, $\epsilon\ 1\ \epsilon\ 2\ \ \epsilon\ p$ have an influence on the totality of variables X_i (i=1 p) $\varepsilon\ _i$ influence in Xi(i=1..p)

Thus, the model equations can be expressed in matrix form according to the following:

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \\ \ldots \\ X_p \end{vmatrix} = \begin{vmatrix} a_{11}a_{12}a_{13}a_{14}\ldots\ldots\ldots\ldots a_{1m} \\ a_{21}a_{22}a_{23}a_{24}\ldots\ldots\ldots\ldots a_{2m} \\ a_{31}a_{32}a_{33}a_{34}\ldots\ldots\ldots\ldots a_{3m} \\ a_{41}a_{42}a_{43}a_{44}\ldots\ldots\ldots\ldots a_{4m} \\ a_{51}a_{52}a_{53}a_{54}\ldots\ldots\ldots\ldots a_{5m} \\ a_{61}a_{62}a_{63}a_{64}\ldots\ldots\ldots\ldots a_{6m} \\ a_{71}a_{72}a_{73}a_{74}\ldots\ldots\ldots\ldots a_{7m} \\ a_{81}a_{82}a_{83}a_{84}\ldots\ldots\ldots\ldots a_{8m} \\ a_{91}a_{92}a_{93}a_{94}\ldots\ldots\ldots\ldots a_{9m} \\ a_{101}a_{102}a_{103}a_{104}\ldots\ldots\ldots a_{10m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{p1}a_{p2}a_{p3}a_{p4}\ldots\ldots\ldots\ldots a_{pm} \end{vmatrix} \cdot \begin{vmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \\ Z_8 \\ Z_9 \\ Z_{10} \\ \ldots \\ Z_p \end{vmatrix} + \begin{vmatrix} b_1\xi_1 \\ b_2\xi_2 \\ b_3\xi_3 \\ b_4\xi_4 \\ b_5\xi_5 \\ b_6\xi_6 \\ b_7\xi_7 \\ b_8\xi_8 \\ b_9\xi_9 \\ b_{10}\xi_{10} \\ \ldots\ldots \\ b_p\xi_p \end{vmatrix} \tag{2}$$

The resulting model may be condensed as follows:

$$X = AZ + b_p\xi_p \tag{3}$$

Where:

We assume that $m < p$ because they explain the variables through a small number of new random variables and all the factors $(m + p)$are correlated variables, i.e., the variability explained by a variable factor, has no relation with the other factors.

We know that each observed variable of model is a result of lineal combination of each common factor with different weights(a_{ia}). Those weights are called saturations, but one of part of $x_i$is not explained for common factors. As we know, all intuitive problems can be inconsistent when obtaining solutions and therefore, we require the approach of hypothesis; hence, in the factor model we used the following assumptions:

$\mathrm{H}_{1:}$ The factors are typified random variables, and intercorrelated, as follow:

$$\mathrm{E}\left[\mathrm{Z_i}\right] = 0\mathrm{E}\left[\xi_\mathrm{i}\right] = 0\mathrm{E}\left[\mathrm{Z_iZ_i}\right] = 1$$

$$\mathrm{E}\left[\xi_\mathrm{i}\xi_\mathrm{i}\right] = 1\mathrm{E}\left[\mathrm{Z_iZ_{i'}}\right] = 0\mathrm{E}\left[\xi_\mathrm{i}\xi_{\mathrm{i'}}\right] = 0$$

$$\mathrm{E}\left[\mathrm{Z_i}\xi_\mathrm{i}\right] = 0$$

Also, we should consider an important point, that factors have a primary goal to study and simplify the correlations between variables, measures, through the correlation matrix. Then, we should understand that:

$\mathrm{H}_{2:}$The original variables could be typified by transforming these variables of type

$$x_i = \frac{x_i - \bar{x}}{\sigma_x} \tag{4}$$

Therefore, and considering the variance property, we have:

$$var(x_i) = \ a_{i1}^2 var(z_1) + a_{i2}^2 var(z_2) + \ldots\ldots\ldots\ldots a_{im}^2 var(z_m) + b_i^2 var(\xi_i) \tag{5}$$

Resulting:

$$1 = a_{i1}^2 + a_{i2}^2 + a_{i3}^2 + \ldots + \ldots\ldots a_{im}^2 + b_i^2 \forall_i = 1 \ldots\ldots p \tag{6}$$

After, we calculate: Saturations, communalities and uniqueness as follow:

a).- We denominate *saturations* of the variable x_i in the factor z_a at coefficient (a_{ia})

Therefore, in order to show the relationship between the variables and the common factors, it is necessary to determine the coefficient of **A** (assuming the hypotheses $\mathbf{H}_1$ y $\mathbf{H}_2$), where **V** is the matrix of eigenvectors and Λ matrix eigenvalues; thus we obtained:

$$A = \begin{bmatrix} a_{11}\, a_{12}\, a_{13}\, a_{14} \ldots\ldots\ldots\ldots a_{1m} \\ a_{21}\, a_{22}\, a_{23}\, a_{24} \ldots\ldots\ldots\ldots a_{2m} \\ a_{31}\, a_{32}\, a_{33}\, a_{34} \ldots\ldots\ldots\ldots a_{3m} \\ a_{41}\, a_{42}\, a_{43}\, a_{44} \ldots\ldots\ldots\ldots a_{4m} \\ a_{51}\, a_{52}\, a_{53}\, a_{54} \ldots\ldots\ldots\ldots a_{5m} \\ a_{61}\, a_{62}\, a_{63}\, a_{64} \ldots\ldots\ldots\ldots a_{6m} \\ a_{71}\, a_{72}\, a_{73}\, a_{74} \ldots\ldots\ldots\ldots a_{7m} \\ a_{81}\, a_{82}\, a_{83}\, a_{84} \ldots\ldots\ldots\ldots a_{8m} \\ a_{91}\, a_{92}\, a_{93}\, a_{94} \ldots\ldots\ldots\ldots a_{9m} \\ a_{101}\, a_{102}\, a_{103}\, a_{104} \ldots\ldots\ldots a_{10m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{p1}\, a_{p2}\, a_{p3}\, a_{p4} \ldots\ldots\ldots\ldots a_{pm} \end{bmatrix} \tag{7}$$

$$\mathrm{R} = \mathrm{V}\Lambda V' = \mathrm{V}\Lambda^{1/2}\Lambda^{1/2}\mathrm{V}' = \mathrm{AA}' \tag{8}$$

$$A = V\Lambda^{1/2}$$

The above suggests that (a_{ia}) coincides with the correlation coefficient between the variables and factors. In the other sense, for the case of non-standardized variables, A is obtained from the covariance matrix S, hence the correlation between x_i and z_a is the ratio:

$$corr(i, a) = \frac{a_{ia}}{\sigma_a} = \frac{a_{ia}}{\sqrt{\lambda_a}} \tag{9}$$

Thus, the variance of the factor (a_{ia}) is the result of the sum of the squares saturations of a_i from column A (formula 7):

$$\lambda_a = \sum_{i=1}^{p} a_{ia}^2 \tag{10}$$

Considering that:

$$\mathrm{A'A} = (\mathrm{V}\Lambda^{1/2})'(\mathrm{V}\Lambda^{1/2}) = \Lambda^{1/2}\mathrm{V'V}\Lambda^{1/2} = \Lambda^{1/2}\mathrm{I}\Lambda^{1/2} = \Lambda \tag{11}$$

b).- We denominate *communalities* to the next theorem:

$$h_i^2 = \sum_{a=1}^{m} a_{ia}^2 \tag{12}$$

The communalities show a percentage of variance of each variable (i) and are explained by m factors.

Thus, every coefficient h_i^2 is called variable specificity. Therefore the matrix model $X = AZ + \xi, \xi$ (unique factors matrix), Z (common factors matrix) will be lower while greater is the variation explained for every m(common factor). If we work with typified variables and considering the variance property, we have:

$$1 = a_{i1}^2 + a_{i2}^2 + ... + a_{ia}^2 + b_2^2 \qquad (13)$$
$$1 = h_i^2 + b_1^2$$

Remember that the variance of any variable, is the result of adding their communalities and the uniqueness $\mathrm{b_i^2}$, thus, in the number of factors obtained, there is a part of the variability of the original variables unexplained and will correspond to a residue (unique factor).

Subsequently, based on the correlation matrix between the variables i and i,we now obtain:

$$\mathrm{corr}(\mathrm{x_i x_{i'}}) = \frac{\mathrm{cov}(\mathrm{x_i x_{i'}})}{\sigma_\mathrm{i}\sigma_\mathrm{i'}} \qquad (14)$$

Also, we know

$$\mathrm{x_i} = \sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{ia} z_a + b_i \varepsilon_i}, \mathrm{x_{i'}} = \sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{i'\,a} z_a + b_{i'} \varepsilon_{i'}} \qquad (15)$$

From the hypothesis which we started, now we have:

$$\mathrm{corr}(\mathrm{x_i x_{i'}}) = \mathrm{cov}(\mathrm{x_i x_{i'}}) = \sigma_{\mathrm{ii'}} = \mathrm{E}\left[\left(\sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{ia} z_a + b_i \varepsilon_i}\right)\left(\sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{i'a} z_a + b_{i'} \varepsilon_{i'}}\right)\right] \qquad (16)$$

Developing the product:

$$= \mathrm{E}\left[\sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{ia} a_{i'a} z_a z_a} + \sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{ia} b_{i'} z_a \varepsilon_{i'}} + \sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{b_i a_{i'i} \varepsilon_i z_a} + \sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{b_i b_{i'} \varepsilon_i \varepsilon_{i'}}\right] \qquad (17)$$

From the linearity of hope and considering that the factors are uncorrelated (hypotheses of starting), now we have:

$$\begin{array}{l} \mathrm{cov}(\mathrm{x_i x_{i'}}) = \sigma_{\mathrm{ii'}} = \sum_{\mathrm{a=1}}^{\mathrm{m}} \mathrm{a_{ia} a_{i'a}} = \mathrm{corr}(\mathrm{x_i x_{i'}}) \\ \forall \mathrm{i, i'} \rightarrow 1................................\mathrm{p} \end{array} \qquad (18)$$

The variance of variable $i-^{esim}$ is given for:

$$\begin{array}{l} \mathrm{var}(\mathrm{x_i}) = \sigma_\mathrm{i}^2 = \mathrm{E}\left[\mathrm{x_i x_i}\right] = 1 = \mathrm{E}\left[\sum_{\mathrm{a=1}}^{\mathrm{m}} \left(\mathrm{a_{ia} z_a + b_i \varepsilon_i}\right)^2\right] = \\ = \mathrm{E}\left[\sum_{\mathrm{a=1}}^{\mathrm{m}} \left(\mathrm{a_{ia}^2 z_a^2 + b_i^2 \varepsilon_i^2 + 2 a_{ia} b_i z_a \varepsilon_a}\right)\right] \end{array} \qquad (19)$$

If we take again the starting hypothesis, we can prove the follow expression:

$$\sigma_i^2 = 1 = \sum_{a=1}^{m} a_{ia}^2 + b_i^2 = h_i^2 + b_i^2 \tag{20}$$

In this way, we can test how the variance is divided into two parts: communality and uniqueness, which is the residual variance not explained by the model

Therefore, we can say that the matrix form is: $R = AA' + \xi$where $R* = R - \xi^2$.

R^* is a reproduced correlation matrix, obtained from the matrix R

$$R^* = \begin{vmatrix} h_1^2 r_{12} r_{13} r_{14} \ldots\ldots r_{1p} \\ r_{21} h_2^2 r_{23} r_{24} \ldots\ldots r_{2p} \\ r_{31} r_{32} h_3^2 r_{34} \ldots\ldots r_{3p} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ r_{p1} r_{p2} r_{p3} r_{p4} \ldots h_p^2 \end{vmatrix} \tag{21}$$

The fundamental identity is equivalent to the following expression:R^*AA'.Therefore the sample correlation matrix is a matrix estimator AA'. Meanwhile, a_{ia} saturation coefficients of variables in the factors, should verify this condition, which certainly, is not enough to determine them. When the product is estimatedAA', we diagonalizable the reduced correlation matrix, whereas a solution of the equation would be: $R - \xi^2 = R^* = AA^{'}$ is the matrix$A$, whose columns are the standardized eigenvectors ofR^*. From this reduced matrix, through a diagonal, as a mathematical instrument, we obtain through vectors and eigenvalues, the factor axes.

How to demonstrate if factor analysis is pertinent?

To evaluate the appropriateness of the factor model, it is necessary to design the sample correlation matrix R, from the data obtained. Also, beforehand we should perform hypothesis tests in order to determine the relevance of the factor model, i.e., whether it is appropriate to analyze the data with this model.

A contrast to be performed is the Bartlett Test of Sphericity. It seeks to determine whether there is a relationship or not among the original variables. The correlation matrix R show the relationship between each pair of variables (rij) and its diagonal will be composed for 1(ones).

Hence, if there is no relationship between the variables h, then, all correlation coefficients between each pair of variable would be zero. Therefore, the population correlation matrix coincides with the identity matrix and determinant will be equal to 1.

$$H_o: |R| = 1$$
$$H_1: |R| \neq 1$$

If the data are a random sample from a multivariate normal distribution, then, under the null hypothesis, the determinant of the matrix is 1 and is shown as follows:

$$-\left[n - 1 - \frac{(2p+5)}{6}\right] \ln |R| \tag{22}$$

Under the null hypothesis, this statistic is asymptotically distributed through aX^2distribution with $p\,(p-1)\,/2$ degrees freedom. So, in case of accepting the null hypothesis it would not be advisable to perform factor analysis.

Another index is the contrast of Kaiser-Meyer-Olkin (KMO), whose purpose is to compare the correlation coefficients and partial correlation coefficients. This measure is called sampling adequacy (KMO) and can be calculated for the whole or for each variable (MSA)

$$\mathrm{KMO} = \frac{\sum_{\mathrm{j}\neq\mathrm{i}}\sum_{\mathrm{i}\neq\mathrm{j}}\mathrm{r}_{\mathrm{ij}}^2}{\sum_{\mathrm{j}\neq\mathrm{i}}\sum_{\mathrm{i}\neq\mathrm{j}}\mathrm{r}_{\mathrm{ij}}^2+\sum_{\mathrm{j}\neq\mathrm{i}}\sum_{\mathrm{i}\neq\mathrm{j}}\mathrm{r}_{\mathrm{ij(p)}}^2}\mathrm{MSA} = \frac{\sum_{\mathrm{ij}}\mathrm{r}_{\mathrm{ij}}^2}{\sum_{\mathrm{ij}}\mathrm{r}_{\mathrm{ij}}^2+\sum_{\mathrm{ij}}\mathrm{r}_{\mathrm{ij(p)}}^2};\mathrm{i} = 1, \ldots.., \mathrm{p} \qquad (23)$$

Where:

$r_{ij}(p)$ Is the partial coefficient of correlation among variables X_i y $X_j$in all cases.

In the same idea, to measure data obtained as a result of applied questionnaires, we follow the procedure utilized by García-Santillán, Venegas-Martínez and Escalera-Chávez (2013); hence, we have the next data matrix:

Table 1: Data matrix.

Students	Variables
	X_1 X_2 . . . . . X_p
1	X_{11} X_{12} . . . . x_{1p}
2	X_{21} X_{22} . . . . x_{2p}
3	X_{31} X_{32} . . . . X_{3p}
4	X_{41} X_{42} . . . . X_{4p}
5	X_{51} X_{52} . . . . X_{5p}
6	X_{61} X_{62} . . . . X_{6p}
7	X_{71} X_{72} . . . . X_{7p}
8	X_{81} X_{82} . . . . X_{8p}
9	X_{91} X_{92} . . . . X_{9p}
10	X_{101} X_{102} . . . . X_{10p}
...	....
n	X_{n1} X_{n2} . . . . x_{np}

1.2. Acceptation or Rejection of null hypothesis in EFA

Preliminary notes and notation

In order to measure data obtained and carry out the hypothesis test (Hi) about variables set that integrate the construct of the phenomena under study, we started from the hypothesis: Ho $\rho = 0$ have no correlation Ha: $\rho \neq 0$ have correlation [14].

The statistic test: $\chi 2$, and the Bartlett test of Sphericity KMO (Kaiser-Meyer-Olkin), MSA (Measure sample adequacy), with significance level: $\alpha = 0.05$; p <0.05; load factor

0.70; calculated critical value $\chi2 > \chi2$ theoretical, then the decision rule is: Reject Ho if calculated $\chi2 > \chi2$ theoretical.

This is given from the following equation:

$$\begin{array}{l} X_1 = a_{11}F_1 + a_{12}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{1k}F_k + u_1 \\ X_2 = a_{21}F_1 + a_{22}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{2k}F_k + u_2 \\ X_3 = a_{31}F_1 + a_{32}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{3k}F_k + u_3 \\ X_4 = a_{41}F_1 + a_{42}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{4k}F_k + u_4 \\ X_5 = a_{51}F_1 + a_{52}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{5k}F_k + u_5 \\ X_6 = a_{61}F_1 + a_{62}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{6k}F_k + u_6 \\ X_7 = a_{71}F_1 + a_{72}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{7k}F_k + u_7 \\ X_8 = a_{81}F_1 + a_{82}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{8k}F_k + u_8 \\ X_9 = a_{91}F_1 + a_{92}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{9k}F_k + u_9 \\ X_{10} = a_{101}F_1 + a_{102}F_2 + \ldots\ldots\ldots + a_{10k}F_k + u_{10} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_p = a_{p1}F_1 + a_{p2}F_2 + \ldots\ldots\ldots\ldots\ldots + a_{pk}F_k + u_p \end{array} \tag{24}$$

Where $F_1 \ldots F_k\,(K << p)$are common factors; $u_1, \ldots\ u_p$ are specific factors and the coefficients $\{a_{ij};\ \mathrm{i} = 1,\ \ldots,\mathrm{p};\ \mathrm{j} = 1, \ldots, \mathrm{k}\ \}$ are the factorial load. It is assumed that the common factors have been standardized or normalized$E\,(F_i) = 0$, $Var\ (f_i) = 1$, the specific factors have a mean equal to zero and both factors have a correlation $Cov\,(F_i, u_j) = 0, \forall_i = 1, \ldots, k; j = 1...p.$ with the following consideration: if the factors are correlated $(Cov\ (F_i,\ Fj) = 0,\ if\ i \neq j;\ j,\ i = 1, \ldots.., k)$, we are facing a model with orthogonal factors, but if they are not correlated, it is a model with oblique factors.

Therefore, the equation can be expressed as follows:

$$\mathrm{x} = \mathrm{Af} + \mathrm{uUX} = \mathrm{FA}' + \mathrm{U} \tag{25}$$

Where:

Data matrix

$$\mathrm{x} = \begin{pmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \ldots \\ \mathrm{x}_\mathrm{p} \end{pmatrix}, \mathrm{f} = \begin{pmatrix} \mathrm{F}_1 \\ \mathrm{F}_2 \\ \ldots \\ \mathrm{F}_\mathrm{k} \end{pmatrix}, \mathrm{u} = \begin{pmatrix} \mathrm{u}_1 \\ \mathrm{u}_2 \\ \ldots \\ \mathrm{u}_\mathrm{p} \end{pmatrix}$$

Factorial load matrix *Factorial matrix*

$$\mathrm{A} = \begin{pmatrix} \mathrm{a}_{11}\mathrm{a}_{12}\ldots..\mathrm{a}_{\mathrm{ik}} \\ \mathrm{a}_{21}\mathrm{a}_{22}\ldots..\mathrm{a}_{2\mathrm{k}} \\ \ldots\ldots\ldots\ldots \\ \mathrm{a}_{\mathrm{p}1}\mathrm{a}_{\mathrm{p}2}\ldots..\mathrm{a}_{\mathrm{pk}} \end{pmatrix} \quad \mathrm{F} = \begin{pmatrix} \mathrm{f}_{11}\mathrm{f}_{12}\ldots..\mathrm{f}_{\mathrm{ik}} \\ \mathrm{f}_{21}\mathrm{f}_{22}\ldots..\mathrm{f}_{2\mathrm{k}} \\ \ldots\ldots\ldots\ldots \\ \mathrm{f}_{\mathrm{p}1}\mathrm{f}_{\mathrm{p}2}\ldots..\mathrm{f}_{\mathrm{pk}} \end{pmatrix}$$

With a variance equal to:

$$\mathrm{Var}(X_i) = \sum_{j=1}^{k} a_{ij}^2 + \Psi_i = h_i^2 + \Psi_i;\ i = 1, \ldots.., p \tag{26}$$

Where:

$$h_i^2 = \mathrm{Var}\left(\sum_{j=1}^{k} a_{ij} F_j\right) \ldots..y\ldots\ldots..\psi_i = \mathrm{Var}\,(u_i) \tag{27}$$

This equation corresponds to the communalities and the specificity of the variable X_i. Thus the variance of each variable can be divided into two parts: a) in their communalities $h_i{}^2$representing the variance explained by common factors and b) the specificity $\Psi\ _I$ that represents the specific variance of each variable. Therefore, we get:

$$\mathrm{Cov}\ (X_{i\ ,} X_l) = \mathrm{Cov}\left(\sum_{j=1}^{k} a_{ij} F_{j,} \sum_{j=1}^{k} a_{lj} F_j\right) = \sum_{j=1}^{k} a_{ij} a_{lj} \forall i \neq \ell \tag{28}$$

With the transformation of the correlation matrix's determinants, we obtain Bartlett's test of Sphericity, and it is given by the following equation:

$$d_R = -\left[n - 1 - \frac{1}{6}(2p+5)\ln|R|\right] = -\left[n - \frac{2p+11}{6}\right]\sum_{j=1}^{p} \log(\lambda_j) \tag{29}$$

$$\left[n - \frac{2p+11}{6}\right] \log \frac{\left[\frac{1}{p-m}\left(\mathrm{traz}R^* - \left(\sum_{a=1}^{m} \lambda_a\right)\right)\right]^{p-m}}{|R^*| / \prod_{a=1}^{m} \lambda_a} \tag{30}$$

After this exploratory factor analysis, where we seek to reduce the number of factors in order to set the final adjusted model, we shall now proceed to explain the path of the confirmatory model, according to the following theoretical foundations.

2. Confirmatory Analysis

With confirmatory analysis, we seek to validate the theoretical model resulting of the first phase with EFA. To do this, we utilized some measure for its evaluation.

1. Likelihood ratio Chi Square (X^2)
2. NFI Normed Fit Index (Benlert & Bonnet, 1980)
3. NNFI Non Normed Fit Index or Tucker Lewis Index (TLI)
4. CFI Compare Fit Index (Benlert, 1989), (Hair, et al. 1999).
5. GFI Goodness of Fit Index,

6. AGFI Adjusted Goodness of Fit Index

7. RMSEA Root Mean Square Error of Approximation

The above has the purpose of assessing the level at which the data support the proposed theoretical model. Therefore, first we design the construction of the resulting structural model of the previous phase exploratory. We started with the estimating step, where theoretically it is explained if the variables are related or not.

With the foregoing, statements on the set of parameters are formulated: If these are free (unknown and not restricted), not restricted (two or more parameters must take the same value, although they are restricted) or fixed (known parameters, to which a fixed value is assigned) and finally, we should define if the maximum number of relationships and statistics associated with them, are established. These should be able to structure the data according to the theory, in order to define the statistic model

The structural model shows causal relationships between latent variables, same as that will have many structural equations as latent constructs, which are explained by other exogenous variables, if they are latent or observed. The structure can be expressed as follows:

$$n = \beta n + \Gamma\xi + \zeta \tag{31}$$

Where:

$n =$ is a vector *"p x 1"* of endogenous latent variables.

$\varepsilon =$ is a vector *"q x 1"* of exogenous latent variables.

$\Gamma=$ is a matrix *"p x q"* of coefficients γ ι j that relate to exogenous latent variables with endogenous latent variables.

$\beta=$ is a matrix *"q x p"* of coefficients that relate exogenous latent variables between them

$\zeta=$ is a vector *"q x1"* of error or disturbance terms. Indicate that the endogenous variables are not predicted by the structural equations.

On the other hand, the latent variables are related with observable variables through the measurement model, which is defined by endogenous and exogenous variables through the following expression:

$$y = \Lambda_y n + \varepsilon y x = \Lambda_x \xi + \delta \tag{32}$$

Where:

$\eta=$ it is a vector *"p x 1"* of endogenous latent variables.

$\varepsilon=$ it is a vector *"q x 1"* of exogenous latent variables.

Λ x = it is a matrix "q x k" coefficients of exogenous variables.

Λ y = it is a matrix "p x m" coefficient of endogenous variables.

δ = it is a vector "q x 1" measurement errors for exogenous indicators.

ε = it is a vector "p x 1" measurement errors for endogenous indicator.

x = it is a set of observable variables of measurement model.

y = it is a set of observables variables of the model structure.

Furthermore, the estimation of model parameters is performed in order to determine which one best fit has maximum likelihood; weighted least squares and generalized least squares.

2.1. Estimation models

Estimation Maximum Likelihood (ML)

This estimating model requires them to have a normal distribution, although the multivariate normal condition does not affect the ability of the method to estimate an unbiased manner, the model parameters. Therefore the log-likelihood function is given by:

$$\log L = -\frac{1}{2}(N-1)\left\{\log\left|\sum(\theta)\right| + tr\left|S\sum(\theta)^{-1}\right|\right\} + c \tag{33}$$

In order to maximize (33), is equivalent to minimize the following function:

$$F_{ML} = \log\left|\sum(\theta)\right| - \log|S| + tr\left[S\sum(\theta)^{-1}\right] - p \tag{34}$$

Where:

L = likelihood function,

N = sample size,

S = covariance matrix,

Σ (Θ) = covariance matrix of the model and Θ is the vector of parameters

2.2. Weighted Least Squares (WLS)

If needed and considering that some ordinals, dichotomous and continuous variables, do not adjust to the criteria of normality, this method may be used. With this statistic procedure, the adjustment function may be minimized:

$$F_{WLS} = [S - \sigma(\theta)]\, W^{-1}\, [S - \sigma(\theta)] \tag{35}$$

Where:

S = is the vector of non-redundant elements in the empirical covariance matrix,

σ (Θ) = is the vector of non-redundant elements in covariance matrix of the model,

Θ = is a parameters vector (*t x 1*),

W^{-1} = is a matrix (k x k) defined positive with k =*(p+1)/2* where p is the number of observed variables, where $W^{-1} = H$ the function of the fourth order moments of observable variables.

2.3. Estimating by generalized least squares (GLS)

In this method, the data must be in a condition of multivariate normality and it is asymptotically equivalent to FML; both of them have the same criteria and may be used in the same conditions. The adjustment function is calculated from the following expression:

$$F_{GLS} = \frac{1}{2}tr\left\{\left[S - \sum(\theta)\right]S^{-1}\right\}^2 \tag{36}$$

Where:

S = covariance matrix, Σ (Θ) = covariance matrix of the model and Θ =is the vector of parameters (*t x 1*)

2.4. Identification phases

To develop the structural model, it will be necessary to estimate the unknown parameters of the specified model. After this, it is possible to contrast statistically. Thus, for the identifying model, the parameters may be identified from the elements of the covariance matrix of the observed variables. With this, the problem of identifiability of the model may be studied under conditions that ensure the unicity in the determination of the model parameters. Then, and starting from the difference between the number of variances and covariances, and the parameters to be estimated, the degree of freedom is defined, so, g should not be negative to develop the study.

The total of variables we denote with $s = p + q$, where q are exogenous variables and p are endogenous variables, and non-redundant elements $a = \frac{s(s+1)}{2}$ and total parameters to estimate in the model as *t,* is defined as $g = \frac{s(s+1)}{2} - t$. Of this way and depending on the value of$g$, so, the model is classified as:

1. Never identified (g<0) models with infinite values in its parameters
2. Possibly identified (g=0) there may be a single solution for the parameters that equals the observed covariance matrix, and finally,
3. Possibly identified (g> 0), that is, the model includes fewer parameters than variances and covariances.

2.5. Modeling with Structural Equation

Afterwards, we reach the most important phase of modeling with structural equations. This phase relates to the diagnosis of the goodness of fit. With this test it is determined if the model is correct and aligned to the purpose of the study.

Statistic X^2 is the only measure of goodness of fit associated with a test of significance and comes from adjustment function$F$, which follows a distribution X^2 with similar degrees of freedom, allowing testing the hypothesis regarding, if the model fits the observed data correctly. Furthermore, it should also have probability p of having a high value of X^2 as the model.

The X^2 Statistic it is influenced by three factors, such as: the sample must be greater than 200 in order to statistic X^2 be meaningful; otherwise if we accept models with small samples, we have a risk that it does not adjust the data. The greater the complexity of the model, the greater the likelihood that the test accepts the model. Therefore, with saturated models further adjustment will be obtained.

It is important to indicate that the statistic X^2 is highly sensitive to the violation of the assumption of multivariate normality in the observed variables. We shall remember that the methods for the estimation previously explained, some of them have certain requirements of normality. A summary of these is presented below in table 2:

Table 2: Requirements of normality in the estimations methods

Maximum Likelihood (ML)	It does not require multivariate normality, just normality unvaried
The Weighted Least Squares (WLS)	Don't need normality
Generalized Least Squares (GLS)	Multivariate normality is necessary
Source: own	

Hence, the adjusted goodness of fit index is:

$$X^2(df) = (N-1)F\left[S, \Sigma(\widehat{\theta})\right] \tag{37}$$

Where:

df = s –t degree freedom,

s =is the number of non-redundant elements in S

t =is the number of total parameters to estimate,

N = is the simple size,

S = is the empirical matrix,

Σ (Θ)= is the matrix of estimated covariances.

Depending on the estimation method is the statistic X^2 and shall be calculated as follows:

$$X^2_{ML}(df) = (N-1)\left[Tr(S\Sigma(\widehat{\theta})^{-1}) - (p+q)\ln\left|\Sigma(\widehat{\theta})\right| - \ln^{\ell}|S|\right] \tag{38}$$

$$X^2_{GLS}(df) = (N-1)\left[0,5Tr(S-\Sigma(\widehat{\theta}))S^{-1})^2\right] \tag{39}$$

$$X^2_{WLS}(df) = (N-1)\left[0,5Tr(S-\Sigma(\widehat{\theta}))^2\right] \tag{40}$$

After performing the tests of goodness of fit, adjustment measures or incremental measures are calculated; these are performed by comparative statistic $\chi\ ^2$ with a more restrictive model called "base model".

The measures are: normed fit index (NFI), non-normed fit index (NNFI) and index fit compared (IFC). All these indices of goodness of fit usually have values between 0 and 1, which is compared with the statistic $\chi\ ^2$. It is expected the result will be as near as possible to 1, which will represent a perfect fit.

Normed fit index (NFI) of [8] is the easiest way to adjust indices. Furthermore, it evaluates the statistic decrease of $\chi\ ^2$, of the research model versus the null model.

Although it is true, some authors suggest re-specifying the model when values below 0.90 are obtained. It is also true that a cut point is supported.

Its representation is:

$$NFI = X_b^2 - X^2/X_b^2 \tag{41}$$

Where:

$X\ ^b\ _2$ = it is statistic of base model

Now we have the so-called Tucker Lewis Index (TLI) or normed fit index. This index is corrected and the aim is to take into account the model complexity, hence the statistic χ ^{2} is not introduced directly. Previously it is compared to the expectation and the degrees of freedom of the null model with the model in question.

If we add parameters to the model, then the NNFI or TLI may only increase if χ ^{2} decreases a greater extent than the degrees of freedom. NNFI values are usually between 0 and 1 but are not restricted to this range. That is, if the values exceed unity, this indicates an over parameterization of the model. Therefore, in order for the index to indicate a good fit of the model, the values should be as close to 1, and the expression is:

$$NFI = \frac{\left(\frac{X_b^2}{glb} - \frac{X^2}{g}\right)}{\left(\frac{X_b^2}{glb} - 1\right)} \tag{42}$$

Subsequently, the Comparative Fit Index (CFI); [8] is calculated. With this index we will compare the discrepancy between the covariance matrix predicted by the model and the observed covariance matrix with the observed discrepancy between the covariance matrix of the null model and the observed covariance matrix. With this, may be evaluated the degree of loss which occurs in the adjustment to change the model of the ongoing investigation to the null model.

The index value varies between 0 and 1. Some authors such as [8, 6, 20] suggest that, for convenience the CFI value should be higher than 0.90 which it indicates that at least 90% of the covariance data may be reproduced by the model. This model is corrected with respect to the complexity of the model and its expression is:

$$CFI = 1 - \frac{Max\left[\left(X^2 - gl\right), 0\right]}{Max\left[\left(X^2 - gl\right), \left(X_b^2 - glb\right), 0\right]} \tag{43}$$

2.6. Measures for model choice

For the test of the goodness of global adjustment, there are other indices that do not belong to the family of indices incremental adjustments. These indices, if not being delimited are hard to interpret for an isolated model, although they can be useful to compare models that are based on the same variables and data, but with different numbers of parameters.

These indices are the AIC and CAIC. The usefulness of these indices consists of comparing models with different numbers of latent variables, being the best model that which provides the smallest value. One of these indices is the AIC (Akaike Information Criterion; Akaike, 1987) this index adjusts the statistic $\chi 2$ of the model, penalizing over parameterization.

$$AIC = X^2 - 2gl \tag{44}$$

The other index is CAIC (Consistent Akaike Information Criterion; Bozdgan, 1987) which is a consistent transformation of the previous index.

$$CAIC = X^2 - \mathit{2}gl(ln(N) + \mathit{1}) \tag{45}$$

Finally other indices are developed as from the covariance of the model:

The RMSEA (Root Mean Square Error of Approximation index penalizes the adjustment for the loss of parsimony with increasing complexity [15]. This index may be interpreted as the average approximation error by degrees of freedom. The values below 0.05 indicate good model adjustment, and below 0.08 indicate adequate model fit. The sampling distribution of the RMSEA has been deducted, allowing construct confidence intervals [5, 4]. Where it is considered that the ends of the confidences intervals should be less than 0.05 (or 0.08) for the model fit is acceptable. This statistic can be calculated from the following formula:

$$RMSEA = \sqrt{\frac{NCP}{Nxgl}} \tag{46}$$

Where:

NCP: It is called non-centrality parameter that may be calculated as CP=Max[χ 2 $-2df$, 0].

RMSEA index depends on the units of measurement; hence, frequently another statistic is taken, such as the SRMR (Standardized Root Mean Square Residual). This statistic is the result of standardizing the previous RMSEA, so we get SRMR dividing the value of RMSEA, by the standard deviation. A value indicative of a good fit will be, if it is below the value 0.05.

As a consideration to take into account we can say that we must be extremely careful in the use of these indices, because according to the indications of Hu and Bentler [5] some indices such as: SRMR, RMSEA, NNFI and IFC frequently give results, which indicates that we must reject suitable models when the sample is very small. Therefore, the suggestion on these types of studies is that we should use the largest number of indices 0, that will allow us to accept or reject the model with the best possible arguments.

Finally, it should be noted that the intention of this essay is to explain the steps to follow for the analysis of data (which are obtained in field research) for the corresponding empirical study. The multivariate technique used was exploratory factor analysis and confirmatory analysis. With this clarification, now are performed calculations using SPSS v.21 software, on the hypothetical data base, which seeks to measure the attitude toward statistics in undergraduate students.

2.7. Hypothetical assumption

In order to explain this statistical procedure, we utilized the result obtained in an empirical study published by [34] entitled "*An exploratory factorial analysis to measure attitude toward statistics: Empirical study in undergraduate students*". Where they find the answer to the research question, objective and hypothesis as follow:

2.7.1. Research question

RQ_1. What is the attitude toward statistics in undergraduate students?

2.7.2. Objectives:

O_1. Identify the factors that explain the attitude towards statistics

2.7.3. Hypotheses:

H1: Liking is the factor that most explained the student's attitude towards statistics
H2: Anxiety is the factor that most explained the student's attitude towards statistics
H3: Confidence is the factor that most explained the student's attitude towards statistics
H4: Motivation is the factor that most explained the student's attitude towards statistics
H5: Usefulness is the factor that most explained the student's attitude towards statistics

In their study, they used the statistic technique of exploratory factor analysis with principal components extraction. The purpose is to determine the number of indicators that compose each of the factors for selecting those with a load factor higher to 0.70

Furthermore, in order to obtain data for their empirical study, they used the ATS scale of [12]. This scale was applied to 298 students. This instrument indicates the existence of five factors: usefulness, anxiety, confidence, liking and motivation. Table 3 described the indicators, definitions and codes/items.

Table 3: Scale factors attitude toward statistics.

Indicators	Definition	Code/items
Liking	Refers to the liking of working with statistics.	LIK,4,9,14,19 and 24
Anxiety	Can be understood as the fear the students manifest towards statistics	ANX,2,7,12,17 and 22
Confidence	Can be interpreted as the feeling of confidence of the skill in statistics.	CNF,3,8,13,18 and 23
Motivation	What the student feels towards the studying and usefulness of statistics.	MTV,5,10,15,20,and 25
Usefulness	It is related to the,value that a student gives to statistics for his/her professional future.	USF,1,6,11,16,and 21

Source: taken from [34]

With this information and the same data base, we carry out the calculations in order to validate the pertinence to use the exploratory factorial analysis technique (EFA) as mentioned in point 1.1 and 1.2 in this work.

Firstly, we obtained the descriptive statistics and afterwards, we carry out the test of Sphericity with KMO, and goodness of fit index X^2 df with significance α =0.01, Also, we obtain the correlation matrix with its determinant, Anti-image Matrices, Component matrix and communalities, Component Matrix^a rotated and communalities, Total Variance Explained and finally sedimentation plot.

Table 4: Descriptive Statistics

	Mean	Std. Deviation	N	Variation coefficient VC=Sd/mean
Liking	13.4613	4.69826	297	0.34902
Anxiety	11.4411	4.04586	297	0.35363
Confidence	17.7138	3.46443	297	0.19558
Motivation	14.2155	3.13341	297	0.22042
Usefulness	17.2458	3.05969	297	0.17742

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Table 5: KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.630
Bartlett's Test of Sphericity	Approx. Chi-Square	361.034
	df	10
	Sig.	0.000

Source: own (created with data base of García-Santillán, Escalera-Chávez Venegas-Martínez, 2014)

Table 6: Correlation Matrix[a]

		Liking	Anxiety	Confidence	Motivation	Usefulness
Correlation	Liking	1.000				
	Anxiety	−.248	1.000			
	Confidence	.383	−.561	1.000		
	Motivation	−.126	.501	−.195	1.000	
	Usefulness	.480	−.222	.455	−.090	1.000
Sig. (Unilateral)	Liking	.000				
	Anxiety	.000	.000			
	Confidence	.000	.000	.000		
	Motivation	.015	.000	.000	.000	
	Usefulness	.000	.000	.000	.061	.000

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Table 7: Anti-image Matrices

		Liking	Anxiety	Confidence	Motivation	Usefulness
Correlation	Liking	.732				
	Anxiety	.025	.523			
	Confidence	−.096	.272	.549		
	Motivation	.025	−.299	−.080	.736	
	Usefulness	−.263	−.038	−.195	.009	.681
Sig. (Unilateral)	Liking	.734**(a)**				
	Anxiety	.040	.578**(a)**			
	Confidence	−.151	.507	.640**(a)**		
	Motivation	.034	−.482	−.125	.557**(a)**	
	Usefulness	−.373	−.064	−.319	.013	.668**)(a)**

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

At the end, all this procedure allows us to identify the explanation power of the model, i.e., with the component matrix and its communalities, we obtain the eigenvalue and its percentage of total variance.

Now we carry out a measurement of the attitude toward statistics through structural equations, all this, in order to identify if the components of model proposed by Auzmendi [12] will be able to show an alternative model.

2.8. Measurement attitude toward statistics through structural equation (AMOS)

For this purpose it set the following Question, objective and hypothesis:

Table 8: Component Matrix[a] and communalities

Variable		Component		Communalities	
		1	2	Inicial	Extraction
Liking		.654	.450	1.000	.631
Anxiety		-.756	.464	1.000	.786
Confidence		.802	.076	1.000	.650
Motivation		-.505	.690	1.000	.731
Usefulness		.665	.516	1.000	.709
	Eigenvalues	2.340	1.166		
	% variance	46.83	23.32		
	Total variance	70.15%			

Extraction Method: Principal Component Analysis. a. 2 components extracted.

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Table 8.1: Component Matrix[a] rotated and communalities

Variable		Component		Communalities	
		1	2	Inicial	Extraction
Liking		.792	-.067	1.000	.631
Anxiety		-.290	.838	1.000	.786
Confidence		.669	-.450	1.000	.650
Motivation		.047	.854	1.000	.731
Usefulness		.842	-.023	1.000	.709
	Eigenvalues	1.870	1.639		
	% variance	37.371	32.776		
	Total variance	70.145%			

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization. a
a. Rotation converged in 6 iterations.

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

2.8.1. Research question

RQ_1. *What factors can help explain the attitude toward statistics in college students?*

Table 9: Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.341	46.825	46.825	2.341	46.825	46.825
2	1.166	23.320	70.145	1.166	23.320	70.145
3	.666	13.325	83.470			
4	.514	10.273	93.743			
5	.313	6.257	100.000			

Extraction Method: Principal Component Analysis.
Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Continue table 9

	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	1.868	37.370	37.370
2	1.639	32.776	70.145

Extraction Method: Principal Component Analysis.
Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

2.8.2. Objectives:

So_1. Develop a theoretical model that integrates the factors that explain attitude toward statistics.

So_2. Evaluate the model using the elements of each factor.

So_3. Evaluate the adjusted model.

2.8.3. Hypotheses:

H1: There are factors that can help explain the attitude toward statistic in undergraduate Students

We recall that, the structural equation modeling (SEM) is a statistical technique which allows us to test the hypothesis of relationships between variables through the estimation of a number of independent variables. Furthermore, it allows performing multiple regressions in a simultaneous way. Hence, its capacity to evaluate multiple regressions of dependence favors the development in this work [11]. Therefore, again we take the data base of [34] indicated in point 2.7, and for the development of the model SEM we take as a reference the work of Escalera-Chávez, García-Santillán and Venegas-Martínez [17] for the use of all tables and figures.

For that, we start from the model developed by Auzmendi [30] which includes latent variables that represent non-observable concepts and their possible measurement through the use of SEM, due to its capacity to include latent variables. Furthermore, this model SEM represents the measurement error in the estimation process [20].

To get objectives SO2 and SO3, we should use the approach of two steps for SEM; measurement of model and structural model. The measurement model followed by an estimate of the structural model is estimated. The measurement model consists of a confirmatory factor analysis (CFA) that assesses the contribution of each variable and its indicators to measure the adequacy of the measurement model.

Several tests of Goodness of Fit (GOF) measures are used in this study; these include the likelihood ratio chi-square (X^2), the ratio of X^2 to degrees of freedom (X^2/df), the Goodness of Fit Index (GFI), the Adjusted Goodness of Fit Index (AGFI), the Root Mean Square Error of Approximation (RMSEA) and Tucker-Lewis (TLI) index [20]. The guidelines for acceptable values for these measures are discussed below.

A non-significant X^2 (p>0.05) is considered a good fit for the X^2 the GOF measure; however, this does not necessarily mean a model with significant X^2, to be a poor fit. As a result, a consideration of the ratio of X^2 to degrees of freedom (X^2 /df) is proposed to measure as an additional measure of GOF. A value smaller than 3 is recommended for the ratio (X^2 /df) for accepting the model to be a good fit [10].

The GFI has been developed to overcome the limitations of the sample size dependent X^2 measures as GOF (Joreskog, et al. 1993). A GFI value higher than 0.9, is recommended as a guideline for a good fit. An extension of the GFI is the AGFI, adjusted by the ratio of degrees of freedom for the proposed model to the degrees of freedom for the null model. An AGFI value greater than 0.9 is an indicator of good fit [29].

RMSEA measures the mean discrepancy between the population estimates from the model and the observed sample values. RMSEA < 0.1 indicates good model fit [20]. TLI, an incremental fit measure, with a value of 0.9 or more indicates a good fit (Hair, et al. 1998). Except for TLI, all the other measures are absolute GOF measures. The TLI measure compares the proposed model to the null model.

Based on the guidelines for these values, problematic items that caused unacceptable model fit were excluded to develop a more parsimonious model with a limited number of items. The steps to follow are the following:

1. The first step in CFA is the model specification.

2. The second step is an iterative model which consists of the modification of the process to develop a more parsimonious set of items to represent a construct through refinement and retesting.

3. The third step is to estimate the parameters of the specified model.

4. The overall model is evaluated by several measures of goodness of test to assess the extent to which the data support the conceptual model.

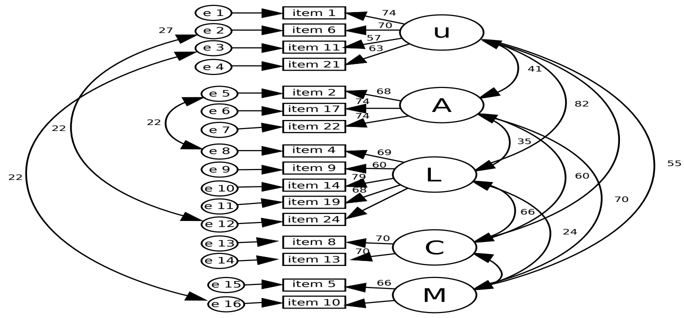

Figure 1: Sequence Diagram (source: taken from [17])

Table 10: Weighting of constructs

	Usefulness	Anxiety	Confidence	Likeness	Motivation
Variable Weighting Significance	Ítem 1 0.793	Ítem 2 0.702	Ítem 8 0.702	Ítem 4 0.711	Ítem 5 0.665
Variable Weighting Significance	Ítem 6 0.704 11.610	Ítem 17 0.732 10.403	Ítem 13 0.69 9.130	Ítem 9 0.599 9.433	Ítem 10 0.534 6.387
Variable Weighting Significance	Ítem 11 0.624 10.241	Ítem 22 0.743 10..497		Ítem 14 0.794 12.224	
Variable Weighting Significance	Ítem 21 0.627 10.301			Ítem 19 0.69 10.793	
Variable Weighting Significance				Ítem 24 0.688 10.764	

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

2.9. Conclusion

At the end of this essay, we could say that the main purpose of this work was achieved. Firstly, the theoretical path was showed. Afterwards the measurements of a set of latent variables associated with the phenomenon under study –the specific case the attitude of students towards statistics– were performed.

The theoretical path indicated in sections 1.1 and 2, promotes the understanding towards the procedure for calculations of exploratory factor analysis with extraction of rotated components, and the use of structural equation modeling (SEM) to carry out confirmatory analysis, with the purpose of validating the theoretical model proposed by

Table 11: Correlation between constructs

	Usefulness	Anxiety	Likeness	Motivation	Confidence
Usefulness	1	0.380	0.770	-0.509	0.546
Anxiety		1	0.360	-0.699	0.676
Likeness			1	-0.238	0.658
Motivation				1	-0.222
Confidence					1

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Table 12: Measures Goodness of Fit: Revised model and null

Chi-square (X^2)	236.851	Comparative Fit Index (CFI)	0.907
Degree of freedom (df)	94	Adjusted Goodness of Fit Index (AGFI)	0.874
Significance level (sig.)	0.000	Root Mean Square Error of Approximation (RMSEA)	0.072
Normed Chi-square (X^2/gl)	2.374	Tucker Lewis Index (TLI)	0.893
Goodness of Fit Index (GFI)	0.913	Normed Fit Index (NFI)	0.869

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Auzmendi [12].

Afterwards, the development of section 2.7 called hypothetical assumptions, used the databases of the work of García-Santillán et al [34] and Escalera-Chavez et al [17]. For performing the EFA software SPSS v.23 was used, and for structural equation modeling (SEM) software AMOS v.23 was used.

In the case of EFA we could observe first, the value of the Bartlett Test of Sphericity with KMO (0.648), Chi square X^2 379 674 with df 10, sig. $0.00 < p$ 0.01, the value of each variable MSA (LIK 0.628; ANX 0.602; CNF 0.731; MTV 0.610 and USF 0.649, are within acceptable values > 0.50 as well as values of the correlation matrix, gave support to validate the use of this technique, besides giving enough evidence to reject the null hypothesis in the study of García-Santillán [34].

Following this criterion, the calculation of the factor weights of each of the factors and

Table 13: Reliability and variance of constructs

Indicators	Reliability	Extracted means variance
Usefulness	0.783	0.476
Anxiety	0.769	0.526
Likeness	0.657	0.489
Motivation	0.825	0.488
Confidence	0.530	0.363

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

Table 14: Discriminant validity

	Usefulness	Anxiety	Confidence	Likeness	Motivation
Usefulness (U)	0.690	0.144	0.592	0.259	0.298
Anxiety (A)		0.726	0.129	0.488	0.456
Confidence (C)			0.602	0.056	0.432
Likeness (L)				0.700	0.049
Motivation (M)					0.700

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

extracting the proportion of variance represented by their communalities, was performed. Hence, the value of each of the eigenvalues and the percentage of the total variance explained was obtained.

As a result of this calculation two factors were obtained: one composed of three elements (usefulness, confidence and liking) and the other composed of two elements (anxiety and motivation). Eigenvalues 2.340 and 1.166 (with 46.83% of the variance and 23.32% respectively) give an explanation of the total variance of 70.15%. Also in the rotated matrix were obtained two factors: one composed of three elements (usefulness, liking and confidence) and the other composed of two elements (anxiety and motivation). Eigenvalues 1.870 and 1.639 (with 37.37% of the variance and 32.77% respectively) give an explanation of the total variance of 70.14%.

With the resulting model, now we proceed to develop confirmation of the model through structural equation modeling (SEM). In the empirical study of Escalera-Chávez et al [17], they evaluated the results to see if there are no estimates infringing. As we can see, Table 10 shows the weights of each indicators that included each construct. Furthermore, in the correlations between constructs, none have values greater than 1.0 (Table 11).

Table 15: Measures of Goodness Fit (Model 2)

Chi-square (X^2)	151.580	Comparative Fit Index (CFI)	0.885
Degree of freedom (df)	88	Adjusted Goodness of Fit Index (AGFI)	0.970
Significance level (sig.)	0.000	Root Mean Square Error of Approximation (RMSEA)	0.049
Normed Chi-square (X^2/gl)	1.722	Tucker Lewis Index (TLI)	0.949
Goodness of Fit Index (GFI)	0.940	Normed Fit Index (NFI)	0.916

Source: own (created with data base of García-Santillán, Escalera-Chávez and Venegas-Martínez, 2014)

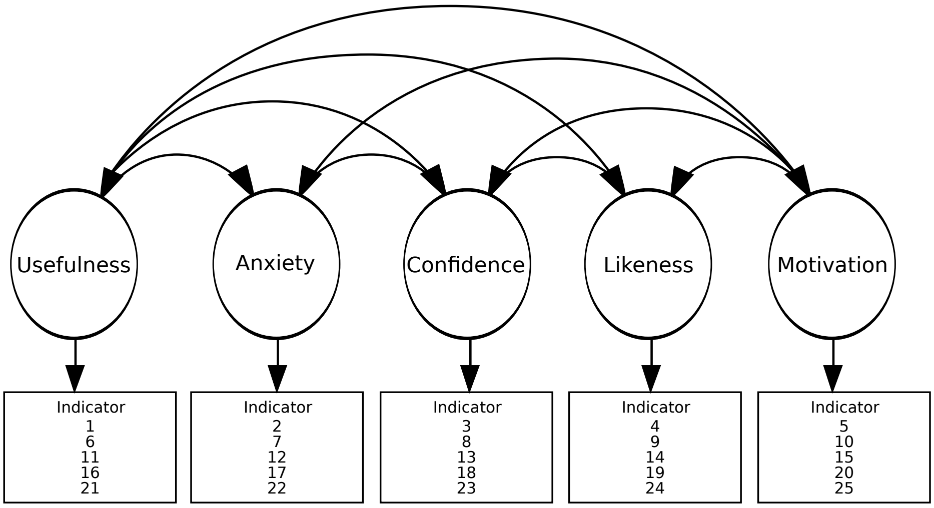

Figure 2: Sequence Diagram (source: taken from [17])

Regarding goodness of fit of the revised model and null, Table 12 provides the quality measures of absolute fit, as well as, the value of Chi square and the indices GFI, AGFI and RMSR. The values obtained were: X^2 (236.851 with *df* 94) is not significant 0.000 but, GFI 0.913 and AGFI 0.874 showed a satisfactory fit, therefore, they are satisfactory because their values tend to 1 and are > 0.5. Also, RMSEA 0.072 is according to the acceptance parameters.

The results shown in Table 13 indicate the reliability values related to the constructs, which is a range from 0.530 onwards (>), which means that not all indicators are consistent with its measure. Regarding the extracted variance, –which must be higher 0.50– we may see that the values are below 0.5 except anxiety (>0.5) which means that more than half

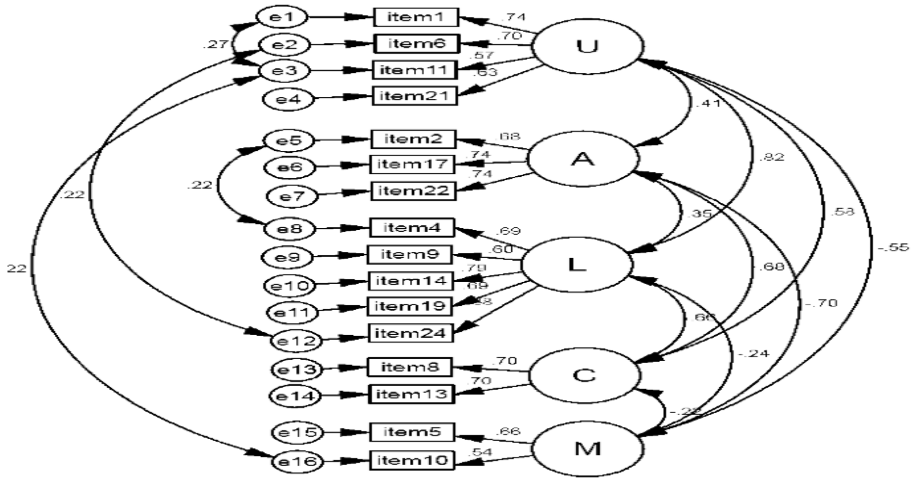

Figure 3: Model 2 Factorial structure of Auzmendi (source: taken from [17])

of the variance is not considered for the construct. But, usefulness .476), liking 0.489 and motivation 0.488 are very close to 0.5 which is the recommended value for the average variance extracted (Fornell and Larcker, 1981, cited by [25])

Regarding discriminant validity, the values shown in Table 14 reveal that all are less than 1; hence, none of the items were part of the different factors, shown in the other constructs. Thompson (2004) refers that confirmatory factor analysis type should confirm the theoretical model fit, because it is recommendable to compare the fit indices of several alternative models to select the best. Therefore, Escalera-Chávez et al [17] verified the model obtained from exploratory factor analysis, which included paths between latent variables, also, they carried out the model estimation (Figure 2).

In the work of Escalera-Chávez et al [17] and after a review of the theoretical criteria in terms of their optimal values, we observed that there are values indicating a model with a poor fit. Hence, it was necessary to make some changes in specifications in order to identify a model that best represents the data. Thus, for the re-specification of the hypothesized model, it was necessary to add estimated parameters for the model, resulting in a model 2 (figure 3).

Finally, comparing the results of model 1 and model 2, we may observe the value of Chi-square (X^2) decreased from 236.851 to 151.58 and the value of RMSEA also decreased from 0.072 to 0.049, while the goodness of fit indices GFI and AGFI improved from 0.913 to 0.940 and from 0.874 to 0.970 respectively. In the same way, the incremental fit measures (TLI and NFI) have enriched and exceeded the recommended level of 0.90. Regarding the error covariances, these suggest a redundancy between items 1 and 11, 2 and 12, and 10 and 11 due to overlap.

In summary, the main purpose of this essay is reached. Firstly, it described the theoretical path that follows the statistical procedure EFA and SEM that allows us to measure a set of latent variables and subsequent confirmation of the model. Following this, in order to perform calculations of each formula, SPSS statistic software v.21 and AMOS v.23 software were used.

Moreover, to perform calculations that allowed us to see the function for each the formulas described in sections: 1, 1.1, 2 and 2.1,the authorization was obtained to utilize the database of the works of of [17, 34], hence, it was possible to develop section 2.7 and 2.8 in this work.

As additional data, just to confirm what Garca Santilln et al [34] and Escalera-Chavez et al [17] demonstrated: the five-factor model (usefulness, motivation, likeness, confidence and anxiety) proposed by Auzmendi [12] has an impact on the student's attitude towards statistics. Moreover, Escalera-Chavez et al [17] identified that there is an alternative model which best fits the proposed model by Auzmendi (CFI = 0.907) and (CFI = 0.885).

ACKNOWLEDGEMENTS The authors thank the readers of European Journal of Pure and Applied Mathematics, for making our journal successful. Also my gratitude goes to Josefina C. Santana, Ph. D., for her support and suggestion in grammar and style in this paper. To my partners Milka, Escalera-Chvez and Francisco Venegas-Martnez who authorized the use of the database to carry out this essay, my eternal gratitude to them. Also my gratitude goes to Felipe de Jess Pozos Texon for his support in the latex format of this paper.

References

[1] R. M. Dreger & L. R. Aiken. The identification of number anxiety in a college population. *Journal of Educational Psychology*, (48):344351, 1957.

[2] E. Satake & P. P. Amato. Mathematics anxiety and achievement among Japanese elementary school students. *Educational and Psychological Measurement*, 55:10001007, 1995.

[3] D. Redfield & C. Martray B. Moore, L. Alexander. The mathematics anxiety rating scale abbreviated: A validity study. In *Paper presented at the meeting of the American Educational Research Association*, New Orleans, 1988.

[4] L. Hu & P. M. Bentler. Cutoff criteria for fit indices in covariance structure analysis: Conventional criteria versus new alternatives. Structural Equation Modeling. 1(6):1–55, 1999.

[5] L. T. Hu & P. M. Bentler. *Evaluating model fit. In R. H. Hoyle (Ed.), Structural Equation Modeling. Concepts, Issues, and Applications.* London: Sage, 1995.

[6] P.M. Bentler. Eqs structural equations program manual. *BMDP Statistic Software*, 1989.

[7] W. H. Dutton & M. P. Blum. The measurement of attitudes towards arithmetic with a Likert-type test. *Elementary School Journa*, 1(2):259263, 1968.

[8] P.M. Bentler & D.G. Bonett. Significance tests and goodness of fit in the analysis of covariance structures. *Psychological Bulletin*, 88:588–606, 1980.

[9] R. E. Lushene P. R. Vagg & G. A. Jacobs C. D. Spielberger, R. L. Gorsuch. State-Trait Anxiety Inventory for adults sampler set: Manual, test, scoring key. Mind Garden, California, 1983.

[10] W.W. Chin and P. A. Todd. On the use, usefulness and ease of structural equation modeling in MIS research: A note of caution. *MIS quarterly*, 19(2):237–247, 1995.

[11] D. W. Straub D. Gefen and M. Boudreau. Structural equation modelling and regression: Guidelines for research practice. *Communications of the Association for Information Systems (AIS)*, 7(4), 2000.

[12] E.Auzmendi. *Evaluacin de las actitudes hacia la estadstica en alumnos universitarios y factores que las determinan. PhD thesis.* PhD thesis, Universidad de Deusto, Bilbao, 1992.

[13] R. M. Suinn & R. Edwards. The measurement of mathematics anxiety. The Mathematics Anxiety Rating Scale for AdolescentsMARS-A. *Journal of Clinical Psychology*, 38:576577, 1982.

[14] A. Garca-Santilln; M. E. Escalera-Chvez and A. Crdova-Rangel. Variables to measure interaction among mathematics and computer through structural equation modeling. *Journal of Applied Mathematics and Bioinformatics*, 3(2):51–67, 2012.

[15] A.Garca-Santilln; F. Venegas-Martnez; M. E. Escalera-Chvez and A. Crdova-Rangel. Attitude towards statistics in engineering college: An empirical study in public university (UPA). *Journal of Statistic and Econometric Methods*, 1(2):43–60, 2013.

[16] A.Garca-Santilln; M. E. Escalera-Chvez and F. Venegas-Martnez. Principal components analysis and Factorial analysis to measure latent variables in a quantitative research: A mathematical theoretical approach. *Bulletin of Society for Mathematical Service and Standars,*, 2(3):3–14, 2013.

[17] M. E. Escalera-Chvez; A. Garca-Santilln and F. Venegas-Martnez. Modeling attitude toward Statistic with structural equation. *Eurasia Journal of Mathematics, Science & Technology Education*, 10(1):23–31, 2014.

[18] J. B. Rounds & D. D. Hendel. Measurement and dimensionality of mathematics anxiety. *Journal of Counseling Psychology*, 27:138149, 1980.

[19] E. Levitt & L. Hutton. A psychometric assessment of the Mathematics Anxiety Rating Scale. *International Review of Applied Psychology*, 33:233242, 1984.

[20] R. L. Tatham & W. C. Black J. F. Hair, R. E. Anderson. *Multivariate data analysis.* Prentice Hall, Spain, 1999.

[21] J. Viehe & S. Segal J. H. Resnick. Is math anxiety a local phenomenon? A study of prevalence and dimensionality. *Journal of Counseling Psychology*, 29:3947, 1982.

[22] R. Kazelskis. Some dimensions of mathematics anxiety: A factor analysis across instruments. *Educational and Psychological Measurement*, 58:623633, 1998.

[23] J. L. Ling. *A factor analytic study of mathematics anxiety. Unpublished doctoral dissertation.* PhD thesis, Virginia Polytechnic Institute and State University, 1982.

[24] A. Wigfield & J. L. Meece. Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, 80:210–216, 1988.

[25] A. Calvo De Mora and F. Criado. Anlisis de la validez del modelo europeo de excelencia para la gestin de la calidad en las Instituciones universitarias: un enfoque directivo. *Revista Europea de Direccin y Economa de la Empresa*, 14(3):41–58, 2005.

[26] B. S. Plake & C. S. Parker. The development and validation of a revised version of the Mathematics Anxiety Rating Scale. *Educational and Psychological Measurement*, 42:551557, 1982.

[27] E. Nicoletti & P. R. Spinelli R. M. Suinn, C. A. Edie. The MARS, a measure of mathematics anxiety: Psychometric data. *Journal of Clinical Psychology*, 28:373375, 1972.

[28] R. S. Sandman. The mathematics attitude inventory: Instrument and users manual. *Journal for Research in Mathematics Education*, 11:148149, 1980.

[29] A. H. Segars and V. Grover. Re-examining perceived ease of use and usefulness: A confirmatory factor analysis. *MIS quarterly*, 17(4):517–525, 1993.

[30] E. Fennema & J.A. Sherman. Fennema-Sherman Mathematics Attitude Scale: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *JAS Catalog of Selected Documents in Psychology,*, 31(6), 1976.

[31] C. D. Spielberger. Preliminary professional manual for the test anxiety inventory. Consulting Psychologists Press, Palo Alto, CA, 1980.

[32] V. W. Strawderman. *A description of mathematics anxiety using an integrative model. Unpublished doctoral dissertation.* PhD thesis, Georgia State University., 1985.

[33] F.C. Richardson & R. M. Suinn. The Mathematics Anxiety Rating Scale: Psychometric data. *Journal of Counseling Psychology*, 19:551554, 1972.

[34] A.Garca-Santilln; F. Venegas-Martnez and M. E. Escalera-Chvez. An exploratory factorial analysis to measure attitude toward statistic. Empirical study in undergraduate students. *International Journal of Research and Reviews in Applied Sciences*, 14(2):356–366, 2013.

[35] R. M. Suinn & E. H. Winston. The mathematics anxiety rating scale, a brief version: Psychometric data. Psychological Reports 92, 2003.

[36] E. F. Wood. Math anxiety and elementary teachers: What does research tell us? *For the Learning of Mathematics*, 8:8–13, 1988.