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Invited Paper

Functions and weakly $\mu\mathcal{H}$-compact spaces

Abdo Qahis[1,*], Takashi Noiri[2]

[1] *Department of Mathematics, Faculty of Science and Arts, Nagran University, Saudi Arabia.*
[2] *Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken, Japan.*

Abstract. A GTS (X, μ) is said to be weakly $\mu\mathcal{H}$-compact if for every μ-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X there exists a finite subset Δ_0 of Δ such that $X \setminus \cup\{c_\mu(V_\alpha) : \alpha \in \Delta_0\} \in \mathcal{H}$. In this paper we study the effect of functions on weakly $\mu\mathcal{H}$-compact spaces. The main result is that the $\theta(\mu, \nu)$-continuous image of a weakly $\mu\mathcal{H}$-compact space is weakly $\nu f(\mathcal{H})$-compact.

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1. Introduction

The ideas of generalized topology and hereditary classes were introduced and studied by Császár in [3] and [5], respectively. The strategy of using generalized topologies and hereditary classes to extend classical topological concepts have been used by many authors such as [5], [8], [15], and [18]. Moreover, investigations of continuity on generalized topological spaces have been recently of major interest among general topologists. They are studied by many authors, including Min [10], Al-omari and Noiri [1], Császár [3], and Jayanthi [6]. In fact, mathematicians introduced in several papers different and interesting new types of functions as well as generalized continuous functions in generalized topological spaces. The purpose of this paper is to study the effect of functions on weakly $\mu\mathcal{H}$-compact spaces. We also show that some functions preserve this property. The main result is that the image of a weakly $\mu\mathcal{H}$-compact space under a $\theta(\mu, \nu)$-continuous function is weakly $\mu\mathcal{H}$-compact.

*Corresponding author.

Email addresses: `cahis82@gmail.com` (A. Qahis), `t.noiri@nifty.com` (T. Noiri)

2. Preliminaries

Let X be a nonempty set and $p(X)$ the power set of X. A subfamily μ of $p(X)$ is called a generalized topology [3] if $\emptyset \in \mu$ and the arbitrary union of members of μ is again in μ. The pair (X, μ) is called a generalized topological space (briefly GTS). The elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A and by $i_\mu(A)$ the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A (see [3], [4]). A nonempty subcollection $\mathcal{H}$ of $p(X)$ is called a hereditary class (briefly HC) [5] if $A \subset B$, $B \in \mathcal{H}$ implies $A \in \mathcal{H}$. An HC $\mathcal{H}$ is called an ideal if $\mathcal{H}$ satisfies the additional condition: $A, B \in \mathcal{H}$ implies $A \cup B \in \mathcal{H}$ [9]. Some useful hereditary classes in X are: $p(A)$, where $A \subseteq X$ and $\mathcal{H}_f$, the HC of all finite subsets of X. A subset A of a GTS (X, μ) is said to be weakly μ-compact [17] if any cover of A by μ-open sets of X has a finite subfamily, the union of the μ-closures of whose members covers A. If $A = X$, then (X, μ) is called a weakly μ-compact space. Given a generalized topological space (X, μ) with an HC $\mathcal{H}$, for a subset A of X, the generalized local function of A with respect to $\mathcal{H}$ and μ [5] is defined as follows: $A^*(\mathcal{H}, \mu) = \{x \in X : U \cap A \notin \mathcal{H} \text{ for all } U \in \mu_x\}$, where $\mu_x = \{U : x \in U \text{ and } U \in \mu\}$. If there is no confusion, we simply write A^* instead of $A^*(\mathcal{H}, \mu)$. $\mathcal{H}$ is said to be μ-codense if $\mu \cap \mathcal{H} = \emptyset$ [5]. And for a subset A of X, $c^*_\mu(A)$ is defined by $c^*_\mu(A) = A \cup A^*$. The family $\mu^* = \{A \subset X : X \setminus A = c^*_\mu(X \setminus A)\}$ is a GT on X which is finer than μ [5]. The elements of μ^* are said to be μ^*-open and the complement of a μ^*-open set is called a μ^*-closed set. It is clear that a subset A is μ^*-closed if and only if $A^* \subset A$. We call $(X, \mu, \mathcal{H})$ a hereditary generalized topological space and briefly we denote it by HGTS.

Next we recall some known definitions, corollaries and theorems which will be used in the work.

Theorem 1. *[5] Let (X, μ) be a GTS, $\mathcal{H}$ a hereditary class on X and A be a subset of X. If A is μ^*-open, then for each $x \in A$ there exist $U \in \mu_x$ and $H \in \mathcal{H}$ such that $x \in U \setminus H \subset A$.*

Definition 1. *[17] Let A be a subset of a space (X, μ). Then A is said to be:*
(1) μ-regular closed if $A = c_\mu(i_\mu(A))$;
(2) μ-regular open if $X \setminus A$ is μ-regular closed.

Corollary 1. *[14] Let $f : (X, \mu, \mathcal{H}) \to (Y, \nu)$ be a (μ, ν)-continuous surjection. If $(X, \mu, \mathcal{H})$ is weakly $\mu\mathcal{H}$-compact, then (Y, ν) is weakly $\nu f(\mathcal{H})$-compact.*

Definition 2. *A subset A of X is said to be $\mu\mathcal{H}$-compact [2] (resp. μ-compact [7, 16]) if for every cover $\{U_\alpha : \alpha \in \Delta\}$ of A by μ-open sets of (X, μ) there exists a finite subset Λ_0 of Λ such that $A \setminus \cup\{U_\alpha : \alpha \in \Lambda_0\} \in \mathcal{H}$ (resp. $A \subseteq \cup\{U_\alpha : \alpha \in \Delta_0\}$). If $A = X$, then (X, μ) is called a $\mu\mathcal{H}$-compact (resp. μ-compact) space.*

Definition 3. *Let (X, μ) and (Y, ν) be two GTSs, then a function $f : (X, \mu) \to (Y, \nu)$ is said to be.*

(1) (μ,ν)-continuous [3] if $U \in \nu$ implies $f^{-1}(U) \in \mu$.
(2) almost (μ,ν)-continuous [13] if for each $x \in X$ and each ν-open set V containing $f(x)$, there exists a μ-open set U containing x such that $f(U) \subseteq i_\nu(c_\nu(V))$.
(3) (μ,ν)-precontinuous [11] if $f^{-1}(V) \subseteq i_\nu(c_\nu(f^{-1}(V)))$ for every ν-open set V in Y.
(4) $\delta(\mu,\nu)$-continuous [10] (resp. almost $\delta(\mu,\nu)$-continuous) if for each $x \in X$ and each ν-open set V of Y containing $f(x)$, there exists a μ-open set U of X containing x such that $f(i_\mu(c_\mu(U))) \subseteq i_\nu(c_\nu(V))$ (resp. $f(i_\mu(c_\mu(U))) \subseteq c_\nu(V)$).
(5) $\theta(\mu,\nu)$-continuous [3] (resp. strongly $\theta(\mu,\nu)$-continuous [12]) if for every $x \in X$ and every ν-open subset V of Y containing $f(x)$, there exists a μ-open subset U in X containing x such that $f(c_\mu(U)) \subseteq c_\nu(V)$ (resp. $f(c_\mu(U)) \subseteq V$).
(6) contra-(μ,ν)-continuous [1] if $f^{-1}(V)$ is μ-closed in X for every ν-open set V in Y.

3. Weakly $\mu\mathcal{H}$-Compact Spaces

Firstly, we show some basic properties for weakly $\mu\mathcal{H}$-compact spaces.

Definition 4. *[14] Let (X,μ) be a GTS with HC. An HGTS $(X,\mu,\mathcal{H})$ is said to be weakly $\mu\mathcal{H}$-compact if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of X by μ-open sets in X, there exists a finite subset Δ_0 of Δ such that $X \setminus \cup\{c_\mu(V_\alpha) : \alpha \in \Delta_0\} \in \mathcal{H}$.*

The following lemma is used in the proof of the corollary stated below.

Lemma 1. *An HGTS $(X,\mu,\mathcal{H}_f)$ is weakly μ-compact if and only if $(X,\mu,\mathcal{H}_f)$ is weakly $\mu\mathcal{H}_f$-compact.*

Proof. The necessity is clear and we prove the sufficiency. Assume that $(X,\mu,\mathcal{H}_f)$ is weakly $\mu\mathcal{H}_f$-compact. Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of X by μ-open subsets of X. Then by hypothesis, there exists a fnite subset Δ_0 of Δ such that $X \setminus \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha) \in \mathcal{H}_f$. Let $X \setminus \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha) = \{x_1, x_2, ..., x_n\}$. For each $1 \leq j \leq n$, choose V_{α_j} such that $x_j \in V_{\alpha_j}$. Hence $X = (\bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha)) \cup (\bigcup_{1\leq j\leq n} c_\mu(V_{\alpha_i}))$. This implies that (X,μ) is weakly μ-compact.

Corollary 2. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu)$ be a (μ,ν)-continuous surjection. If $(X,\mu,\mathcal{H})$ is weakly $\mu\mathcal{H}$-compact and Y is a finite space, then (Y,ν) is weakly ν-compact.*

Proof. Let f be a (μ,ν)-continuous surjection. By Corollary 1, (Y,ν) is weakly $\nu f(\mathcal{H})$-compact. Since Y is a finite space, then the HC $f(\mathcal{H})$ of finite subsets and and apply the Lemma 1.

A subset A of a GTS (X,μ) is said to be μ-nowhere dense if $i_\mu(c_\mu(A)) = \emptyset$, and we denote the HC of μ-nowhere dense sets by $\mathcal{N}(\mu)$.

Proposition 1. *If $(X,\mu,\mathcal{H})$ is weakly $\mu\mathcal{H}$-compact and $\mathcal{H}$ is μ-condense, then (X,μ) is weakly μ-compact.*

Proof. Suppose that $(X, \mu, \mathcal{H})$ is weakly $\mu\mathcal{H}$-compact and let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of X by μ-open subsets of X. There exists a finite subset Δ_0 of Δ such that $X \setminus \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha) \in \mathcal{H}$. Since $\mathcal{H}$ is μ-codense, then $i_\mu(X \setminus \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha)) = \emptyset$ which implies $X - i_\mu(X \setminus \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha)) = c_\mu(\bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha) = \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha) = X$. Hence (X, μ) is weakly μ-compact .

Theorem 2. *For a GTS (X, μ), the following properties hold.*
(1) (X, μ) is weakly μ-compact if and only if $(X, \mu, \mathcal{N}(\mu))$ is $\mu\mathcal{N}(\mu)$-compact.
(2) (X, μ) is weakly μ-compact if and only if $(X, \mu, \mathcal{H})$ is $\mu\mathcal{H}$-compact with respect to some μ-codense HC.

Proof. (1) Necessity. Assume (X, μ) is weakly μ-compact and let $\{V_\alpha : \alpha \in \Delta\}$ be a μ-open cover of X. Then by assumption there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha) = c_\mu(\bigcup_{\alpha\in\Delta_0} V_\alpha)$. Since $X \setminus c_\mu(\bigcup_{\alpha\in\Delta_0} V_\alpha) = \emptyset$ and $X \setminus \bigcup_{\alpha\in\Delta_0} V_\alpha$ is a μ-closed, then $i_\mu(X \setminus \bigcup_{\alpha\in\Delta_0} V_\alpha) = \emptyset$. This implies $X \setminus \bigcup_{\alpha\in\Delta_0} V_\alpha \in \mathcal{N}(\mu)$. Thus $(X, \mu, \mathcal{H})$ is $\mu\mathcal{N}(\mu)$-compact.
Sufficiency. Assume $(X, \mu, \mathcal{N}(\mu))$ is $\mu\mathcal{N}(\mu)$-compact and let $\{V_\alpha : \alpha \in \Delta\}$ be a μ-open cover of X. By assumption, there exists a finite subset Δ_0 of Δ such that $X \setminus \bigcup_{\alpha\in\Delta} V_\alpha \in \mathcal{N}(\mu)$. This implies that $i_\mu(X \setminus \bigcup_{\alpha\in\Delta} V_\alpha) = \emptyset$ and hence

$$X = c_\mu(\bigcup_{\alpha\in\Delta_0} V_\alpha) = \bigcup_{\alpha\in\Delta_0} c_\mu(V_\alpha).$$

(2) Necessity. From (1) $\mathcal{H}$ is a μ-codense.
Sufficiency. The proof is obvious by Proposition 1.

Theorem 3. *Let $f : (X, \mu) \to (Y, \nu, \mathcal{G})$ be a surjection onto a $\nu\mathcal{G}$-compact. If $\mu = f^{-1}(\nu)$ is the weak generalized topology on X induced by f and ν, then (X, μ) is $\mu f^{-1}(\mathcal{G})$-compact.*

Proof. Let $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ be a μ-open cover of X. Then $\{V_\alpha : \alpha \in \Delta\}$ is a ν-open cover of Y and hence there exists a finite subset Δ_0 of Δ such that $Y \setminus \bigcup_{\alpha\in\Delta} V_\alpha \in \mathcal{G}$. Now we have

$$f^{-1}\left(Y \setminus \bigcup_{\alpha\in\Delta_0} V_\alpha\right) = X \setminus \bigcup_{\alpha\in\Delta_0} f^{-1}(V_\alpha) \in f^{-1}(\mathcal{G}).$$

Hence (X, μ) is $\mu f^{-1}(\mathcal{G})$-compact.

The following lemma is used to prove the corollary which is stated below.

Lemma 2. *If $f : (X, \mu) \to (Y, \nu, \mathcal{G})$ is a surjection and $\mathcal{G}$ is ν-codense, then $f^{-1}(\mathcal{G})$ is $f^{-1}(\nu)$-codense, where $\mu = f^{-1}(\nu)$.*

Proof. Assume $f : (\mathrm{X}, \mu) \to (Y, \nu, \mathcal{G})$ is a surjection and $f^{-1}(\mathcal{G})$ is not $f^{-1}(\nu)$-codense, then there exists $G \in \mathcal{G}$ such that $f^{-1}(G) \in f^{-1}(\nu) \setminus \{\emptyset\}$, say $f^{-1}(G) = f^{-1}(V)$ where $V \in \nu \setminus \{\emptyset\}$. Then $G = V \in \nu \setminus \{\emptyset\}$ and $\mathcal{G}$ is not ν-codense. Then this contradicts to our assumption.

Corollary 3. *Let $f : (X, \mu) \to (Y, \nu, \mathcal{G})$ be a surjection and let μ denote the weak generalized topology on X induced by f and ν. If $\mathcal{G}$ is ν-codense and $(Y, \nu, \mathcal{G})$ is $\nu\mathcal{G}$-compact, then (X, μ) is weakly μ-compact*

Proof. If $\mathcal{G}$ is ν-codense and (Y, ν) is $\nu\mathcal{G}$-compact, then by Theorem 3, (X, μ) is $\mu f^{-1}(\mathcal{G})$-compact. Since $\mathcal{G}$ is ν-codense. Then by Lemma 2, $f^{-1}(\mathcal{G})$ is $\mu = f^{-1}(\nu)$-codense. By Theorem 2(2), (X, μ) is weakly μ-compact.

Next we introduce the main result and prove that the $\theta(\mu, \nu)$-continuous image of a weakly $\mu\mathcal{H}$-compact space is weakly $\nu f(\mathcal{H})$-compact. Note that if $\mathcal{H}$ is a hereditary class on a set X and $f : (X, \mu) \to (Y, \nu)$ is a function, then $f(\mathcal{H}) = \{f(H) : H \in \mathcal{H}\}$ is a HC on Y [2].

Theorem 4. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \nu)$ be a $\theta(\mu, \nu)$-continuous function. If A is a weakly $\mu\mathcal{H}$-compact subset of X, then $f(A)$ is weakly $\nu f(\mathcal{H})$-compact.*

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \Delta\}$ be a cover of $f(A)$ by ν-open subsets of Y. Let $x \in A$ and $V_{\alpha(x)}$ be a ν-open set in Y such that $f(x) \in V_{\alpha(x)}$. Since f is $\theta(\mu, \nu)$-continuous, there exists a μ-open set $U_{\alpha(x)}$ of X containing x such that $f(c_\mu(U_{\alpha(x)})) \subseteq c_\nu(V_{\alpha(x)})$. Now $\left\{U_{\alpha(x)} : x \in A\right\}$ is a cover of A by μ-open subsets of X. Since A is weakly $\mu\mathcal{H}$-compact, there exists a finite subset A_0 of A such that $A \setminus \bigcup\limits_{x \in A_0} c_\mu(U_{\alpha(x)}) \in \mathcal{H}$. Now $f(A \setminus \bigcup\limits_{x \in A_0} c_\mu(U_{\alpha(x)})) \in f(\mathcal{H})$. We know

$$f(A) \setminus f(\bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})) \subseteq f(A \setminus \bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})).$$

This implies

$$f(A) \setminus f(\bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})) = f(A) \setminus \bigcup_{x \in A_0} f(c_\mu(U_{\alpha(x)})) \in f(\mathcal{H}).$$

Since $f(c_\mu(U_{\alpha(x)})) \subseteq c_\nu(V_{\alpha(x)})$ for each $\alpha(x)$, $f(A) \setminus \bigcup\limits_{x \in A_0} c_\nu(V_{\alpha(x)}) \subseteq f(A) \setminus \bigcup\limits_{x \in A_0} f(c_\mu(U_{\alpha(x)}))$. Thus $f(A) \setminus \bigcup\limits_{x \in A_0} c_\nu(V_{\alpha(x)}) \in f(\mathcal{H})$. This implies that $f(A)$ is weakly $\nu f(\mathcal{H})$-compact.

Corollary 4. *Let $f : (X, \mu) \to (Y, \nu)$ be a $\theta(\mu, \nu)$-continuous surjection. If $(X, \mu, \mathcal{H})$ is weakly $\mu\mathcal{H}$-compact, then $(Y, \nu, f(\mathcal{H}))$ is weakly $\nu f(\mathcal{H})$-compact.*

The following lemma is used in the proofs of corollaries stated below.

Lemma 3. *If $f : (X, \mu) \to (Y, \nu)$ is almost (μ, ν)-continuous, then f is $\theta(\mu, \nu)$-continuous.*

Proof. Let f be almost (μ, ν)-continuous. Let $x \in X$ and V be a ν-open set in Y such that $f(x) \in V$. Since f is almost (μ, ν)-continuous, there exists a μ-open set U of X containing x such that $f(U) \subseteq i_\nu(c_\nu(V))$. This implies $f(U) \subseteq i_\nu(c_\nu(V)) \subseteq c_\nu(V)$. We have to show that $f(c_\mu(U)) \subseteq c_\nu(V)$. For some $x_0 \in c_\mu(U)$ let $f(x_0) \in Y \setminus c_\nu(V)$. Then by the almost (μ, ν)-continuity of f there exists a μ-open set W of X containing x_0 such that $f(W) \subseteq i_\nu(c_\nu(Y \setminus c_\nu(V)))$. But $W \cap U \neq \emptyset$ and hence $f(U) \cap i_\nu(c_\nu(Y \setminus c_\nu(V))) \neq \emptyset$. Hence, we get a contradiction to the fact that $f(U) \subseteq i_\nu(c_\nu(V)) \subseteq c_\nu(i_\nu(c_\nu(V))) \subseteq c_\nu(V)$. Thus $f(c_\mu(U)) \subseteq c_\nu(V)$. This implies that f is $\theta(\mu, \nu)$-continuous.

Corollary 5. *Let $f : (X, \mu) \to (Y, \nu)$ be an almost (μ, ν)-continuous surjection. If $(X, \mu, \mathcal{H})$ is weakly $\mu\mathcal{H}$-compact, then $(Y, \nu, f(\mathcal{H}))$ is weakly $\nu f(\mathcal{H})$-compact.*

Since every (μ, ν)-continuous function is almost (μ, ν)-continuous and by Lemma 3, we conclude the following corollary.

Corollary 6. *Weakly $\mu\mathcal{H}$-compact property is a GT property.*

By taking $\mathcal{H} = \{\emptyset\}$, we get the following corollary

Corollary 7. *Let $f : (X, \mu) \to (Y, \nu)$ be a $\theta(\mu, \nu)$-continuous function. If A is a weakly μ-compact subset of X, then $f(A)$ is weakly ν-compact.*

Proposition 2. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \nu)$ be a strongly $\theta(\mu, \nu)$-continuous function. If A is a weakly $\mu\mathcal{H}$-compact subset of X, then $f(A)$ is $\nu f(\mathcal{H})$-compact.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of $f(A)$ by ν-open subsets of Y. Let $x \in A$ and $V_{\alpha(x)}$ be a ν-open set in Y such that $f(x) \in V_{\alpha(x)}$. Since f is strongly $\theta(\mu, \nu)$-continuous, there exists a μ-open set $U_{\alpha(x)}$ of X containing x such that $f(c_\mu(U_{\alpha(x)})) \subseteq V_{\alpha(x)}$. Now $\{U_{\alpha(x)} : x \in A\}$ is a μ-open cover of the weakly $\mu\mathcal{H}$-compact set A. So there exists a finite subset A_0 of A such that $A \setminus \bigcup_{x \in A_0} c_\mu(U_{\alpha(x)}) \in \mathcal{H}$.
Now $f(A \setminus \bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})) \in f(\mathcal{H})$. We know

$$f(A) \setminus f(\bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})) \subseteq f(A \setminus \bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})).$$

This implies

$$f(A) \setminus f(\bigcup_{x \in A_0} c_\mu(U_{\alpha(x)})) = f(A) \setminus \bigcup_{x \in A_0} f(c_\mu(U_{\alpha(x)})) \in f(\mathcal{H}).$$

Since $f(c_\mu(U_{\alpha(x)})) \subseteq V_{\alpha(x)}$ for each $\alpha(x)$, $f(A) \setminus \bigcup_{x \in A_0} V_{\alpha(x)} \subseteq f(A) \setminus \bigcup_{x \in A_0} f(c_\mu(U_{\alpha(x)}))$. Thus $f(A) \setminus \bigcup_{x \in A_0} V_{\alpha(x)} \in f(\mathcal{H})$. Hence $f(A)$ is $\nu f(\mathcal{H})$-compact.

Corollary 8. *Let $f : (X, \mu) \to (X, \nu)$ be a strongly $\theta(\mu, \nu)$-continuous surjection. If $(X, \mu, \mathcal{H})$ is weakly $\mu\mathcal{H}$-compact, then $(Y, \nu, f(\mathcal{H}))$ is $\nu f(\mathcal{H})$-compact.*

By taking $\mathcal{H} = \{\emptyset\}$, we get the following corollary.

Corollary 9. *Let $f : (X, \mu) \to (X, \nu)$ be a strongly $\theta(\mu, \nu)$-continuous function. If A is a weakly μ-compact subset of X, then $f(A)$ is ν-compact.*

Proposition 3. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \nu)$ be a contra (μ, ν)-continuous and (μ, ν)-precontinuous function. If A is weakly $\mu\mathcal{H}$-compact, then $f(A)$ is $\nu f(\mathcal{H})$-compact.*

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \Delta\}$ be a cover of $f(A)$ by ν-open sets of (Y, ν). For each $x \in A$, let $V_{\alpha(x)} \in \mathcal{V}$ such that $f(x) \in V_{\alpha(x)}$. Since f is contra (μ, ν)-continuous and (μ, ν)-precontinuous, $f^{-1}(V_{\alpha(x)})$ is μ-closed in X and $f^{-1}(V_{\alpha(x)}) \subseteq i_\mu(c_\mu(f^{-1}(V_{\alpha(x)}))) = i_\mu(f^{-1}(V_{\alpha(x)}))$. So $f^{-1}(V_{\alpha(x)}) = i_\mu(f^{-1}(V_{\alpha(x)}))$. This implies $f^{-1}(V_{\alpha(x)})$ is μ-clopen and hence $\{f^{-1}(V_{\alpha(x)}) : x \in A\}$ is a μ-clopen cover of the weakly $\mu\mathcal{H}$-compact subset A. There exists a finite subset A_0 of A such that

$$A \setminus \bigcup_{x \in A_0} c_\mu(f^{-1}(V_{\alpha(x)})) = A \setminus \bigcup_{x \in A_0} f^{-1}(V_{\alpha(x)}) \in \mathcal{H}.$$

Now we have

$$f(A) \setminus \bigcup_{x \in A_0} V_{\alpha(x)} \subset f(A) \setminus f(\bigcup_{x \in A_0} f^{-1}(V_{\alpha(x)})) \subset f(A \setminus \bigcup_{x \in A_0} f^{-1}(V_{\alpha(x)})) \in f(\mathcal{H}).$$

This implies $f(A) \setminus \bigcup_{x \in A_0} V_{\alpha(x)} \in f(\mathcal{H})$. Hence $f(A)$ is $\nu f(\mathcal{H})$-compact.

Corollary 10. *Let $f : (X, \mu) \to (Y, \nu)$ be a contra (μ, ν)-continuous and (μ, ν)-precontinuous surjection. If $(X, \mu, \mathcal{H})$ is weakly $\mu\mathcal{H}$-compact, then $(Y, \nu, f(\mathcal{H}))$ is $\nu f(\mathcal{H})$-compact.*

By taking $\mathcal{H} = \{\emptyset\}$, we get the following corollary.

Corollary 11. *Let $f : (X, \mu) \to (Y, \nu)$ be a contra (μ, ν)-continuous and (μ, ν)-precontinuous function. If A is weakly μ-compact, then $f(A)$ is ν-compact.*

Theorem 5. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \nu)$ be an almost $\delta(\mu, \nu)$-continuous function. If for every cover $\{U_\alpha : \alpha \in \Delta\}$ of $A \subseteq X$ by μ-regular open sets in X there exists a finite subset Δ_0 of Δ such that $A \setminus \bigcup_{\alpha \in \Delta_0} U_\alpha \in \mathcal{H}$, then $f(A)$ is weakly $\nu f(\mathcal{H})$-compact.*

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \Delta\}$ be a cover of $f(A)$ by ν-open sets of Y. Let $x \in A$ and $V_{\alpha(x)} \in \mathcal{V}$ such that $f(x) \in V_{\alpha(x)}$. Then $i_\mu(c_\mu(V_{\alpha(x)}))$ is a μ-regularly open set in Y containing $f(x)$. Since f is almost $\delta(\mu, \nu)$-continuous, then for every $x \in A$, there exists a μ-open subset $U_{\alpha(x)}$ of X containing x such that $f(i_\mu(c_\mu(U_{\alpha(x)}))) \subseteq c_\nu(V_{\alpha(x)})$. Then

$\{i_\mu(c_\mu(U_{\alpha(x)})) : x \in A\}$ is a μ-regularly open cover of A. It follows that there exists a finite subset A_0 of A such that $A \setminus \bigcup_{x\in A_0} i_\mu(c_\mu(U_{\alpha(x)})) \in \mathcal{H}$. Now

$$f(A) \setminus f(\bigcup_{x\in A_0} i_\mu(c_\mu(U_{\alpha(x)}))) \subseteq f(A \setminus \bigcup_{x\in A_0} i_\mu(c_\mu(U_{\alpha(x)}))) \in f(\mathcal{H}).$$

This implies $f(A) \setminus \bigcup_{x\in A_0} f(i_\mu(c_\mu(U_{\alpha(x)}))) \in f(\mathcal{H})$. Therefore, we obtain

$$f(A) \setminus \bigcup_{x\in A_0} c_\nu(V_{\alpha(x)}) \subseteq f(A) \setminus \bigcup_{x\in A_0} f(i_\mu(c_\mu(U_{\alpha(x)}))).$$

This implies $f(A)\setminus \bigcup_{x\in A_0} c_\nu(V_{\alpha(x)}) \in f(\mathcal{H})$. This shows that $f(A)$ is weakly $\nu f(\mathcal{H})$-compact.

Corollary 12. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \nu)$ be an almost $\delta(\mu, \nu)$-continuous surjection. If for every cover $\{U_\alpha : \alpha \in \Delta\}$ of X by μ-regular open sets of X there exists a finite subset Δ_0 of Δ such that $X \setminus \bigcup_{\alpha\in\Delta_0} U_\alpha \in \mathcal{H}$, then $(Y, \nu, f(\mathcal{H}))$ is weakly $\nu f(\mathcal{H})$-compact.*

By taking $\mathcal{H} = \{\emptyset\}$, we get the following corollary.

Corollary 13. *Let $f : (X, \mu) \to (Y, \nu)$ be an almost $\delta(\mu, \nu)$-continuous function. If for every cover $\{U_\alpha : \alpha \in \Delta\}$ of $A \subseteq X$ by μ-regular open sets in X there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup_{\alpha\in\Delta_0} U_\alpha$, then $f(A)$ is weakly ν-compact.*

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