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A Note on αgrw-Closed Sets

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Abstract. In this paper, some properties of αgrw-closed sets are discussed and also some characterizations of αgrw-closed sets are studied in topological spaces.

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1. Introduction

In 2013, αgrw-closed sets are introduced and studied by Selvanayaki and Gnanambal Ilango [14] and some basic properties of αgrw-closed sets are investigated. The class of αgrw-closed sets properly lies between the class of rw-closed sets and the class of $gprw$-closed sets. In 2007, Benchalli and Wali [1] have introduced a new type of Kernel known as regular semi kernel.The aim of this paper is to study some properties of αgrw-closed sets and some characterizations of it.

Throughout this paper, space (X,τ) (or simply X) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space $X, cl(A), int(A)$ and $X-A$ (or A^c)denote the closure of A, the interior of A and the complement of A in X, respectively.

2. Preliminaries

Definition 1. *A subset A of a topological space (X,τ) is called*

(i) regular open [15] if $A=int(cl(A))$ and regular closed if $A=cl(int(A))$.

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(ii) *semi-open [7] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.*

(iii) *α-open [13] if $A \subseteq int(cl(int(A)))$ and α-closed [12] if $cl(int(cl(A))) \subseteq A$.*

Definition 2 ([2])**.** *A subset A of a space (X, τ) is called regular semi-open if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$. The family of all regular semi-open sets of X is denoted by $RSO(X)$.*

Definition 3 (Noiri [10])**.** *A subset A of a space (X, τ) is said to be semi-regular open if it is both semi-open and semi-closed.*

The family of all semi-regular open sets of X is denoted by $SR(X)$. On other hand, Maio and Noiri defined a subset A of X to be semi-regular open if $A = sint(scl(A))$. However, these three notions are equivalent, which is given in the following theorem.

Theorem 1 ([10])**.** *For a subset A of a space X, the followings are equivalent:*

(i) $A \in SR(X)(= RSO(X))$,

(ii) $A = sint(scl(A))$,

(iii) *there exists a regular open set U of X such that $U \subseteq A \subseteq cl(U)$.*

Definition 4. *A subset A of a topological space (X, τ) is called*

(i) *generalized closed (briefly g-closed) [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

(ii) *α-generalized closed (briefly αg-closed)[11] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

(iii) *α-generalized regular weakly closed (briefly αgrw-closed)[14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X.*

The set of all αgrw-closed sets in (X, τ) is denoted by $\alpha grwC(X)$.

Definition 5 ([9])**.** *A topological space (X, τ) is said to be s-normal if for each pair of disjoint closed sets A and B, there exists disjoint semi-open sets U, V such that $A \subseteq U$ and $B \subseteq V$.*

Definition 6 ([1])**.** *The intersection of all regular semi-open subsets of (X, τ) containing A is called the regular semi-kernel of A and is denoted by $rsker(A)$.*

Theorem 2 ([3])**.** *If A is open and S is semi-open in a topological space X, then $A \cap S$ is semi-open in X.*

Lemma 1 ([1])**.** *Let $A \subseteq Y \subseteq X$, where X is a topological space and Y is an open subspace of X. If $A \in RSO(X)$, then $A \in RSO(Y)$.*

Lemma 2 ([1])**.** *Let Y be regular open in X and U be a subset of Y. Then U is regular semi-open in X if and only if U is regular semi-open in the subspace Y.*

Lemma 3 ([6])**.** *Let x be a point of (X, τ). Then $\{x\}$ is either nowhere dense or pre-open.*

Lemma 4 ([5])**.** *If A is regular semi-open in (X, τ), then $X - A$ is also regular semi-open.*

Lemma 5 ([1])**.** *For any subset A of (X, τ), $A \subseteq rsker(A)$.*

3. αgrw-Closed Sets

Proposition 1. *In a space (X,τ), if $RSO(X)=\{\emptyset,X\}$, then every subset of X is an αgrw-closed set.*

Proof. Let $RSO(X)=\{\emptyset,X\}$ and A be any subset of X. Suppose $A=\emptyset$, then A is an αgrw-closed set in X. Suppose $A\neq\emptyset$, then X is the only regular semi-open set containing A and so $\alpha cl(A)\subseteq X$. Hence A is αgrw-closed. □

Remark 1. *The converse of the above proposition need not be true as seen from the following example.*

Example 1. *Let $X=\{a,b,c\}$ with topology $\tau=\{\emptyset,\{a\},\{b,c\},X\}$. Then every subset of X is αgrw-closed in X but $RSO(X)=\{\emptyset,\{a\},\{b,c\},X\}$.*

Proposition 2. *Every subset of (X,τ) is αgrw-closed if and only if*

$$RSO(X,\tau)\subseteq\{F\subseteq X: F^c\in\tau_\alpha\},$$

where τ_α is the topology generated by the α-open sets in (X,τ).

Proof. Suppose that every subset of (X,τ) is αgrw-closed. Let $U\in RSO(X,\tau)$. Since $U\subseteq U$ and U is αgrw-closed, we have $\alpha cl(U)\subseteq U$. Thus $U\in\{F\subseteq X: F^c\in\tau_\alpha\}$ and hence $RSO(X,\tau)\subseteq\{F\subseteq X: F^c\in\tau_\alpha\}$.

Conversely, assume that $RSO(X,\tau)\subseteq\{F\subseteq X: F^c\in\tau_\alpha\}$. Let A be any subset of (X,τ) such that $A\subseteq U$, where U is regular semi-open. Thus U is α-closed and so $\alpha cl(A)\subseteq U$. Hence A is αgrw-closed in X. □

Proposition 3. *If A is both open and g-closed in X, then it is αgrw-closed in X.*

Proof. Let A be open and g-closed in X. Let $A\subseteq U$ and U be regular semi-open in X. Now $A\subseteq A$, we have $cl(A)\subseteq A$. This implies $\alpha cl(A)\subseteq U$. Hence A is αgrw-closed in X. □

Remark 2. *If A is both open and αgrw-closed in X, then A need not be g-closed in X.*

Example 2. *Let $X=\{a,b,c\}$ with topology $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$.Then $A=\{a,b\}$ is both open and αgrw-closed but not g-closed.*

Proposition 4. *If A is regular semi-open and αgrw-closed, then A is α-closed.*

Proof. Suppose A is regular semi-open and αgrw-closed. We have $\alpha cl(A)\subseteq A$. Since $A\subseteq\alpha cl(A)$ always, $\alpha cl(A)=A$. Hence A is α-closed. □

Example 3. *In Example 2, the set $\{b,c\}$ is α-closed and αgrw-closed but is not regular semi-open.*

Corollary 1. *Let A be regular semi-open and αgrw-closed in X. Then $A\cap F$ is αgrw-closed in X, where F is α-closed.*

Proof. Since A is regular semi-open and αgrw-closed then by Proposition 4, we have A is α-closed. Therefore $A \cap F$ is α-closed, since F is α-closed. Hence $A \cap F$ is αgrw-closed. □

Proposition 5. *If A is both open and αg-closed, then A is αgrw-closed.*

Proof. Let A be an open and αg-closed. Let $A \subseteq U$ and U be regular semi-open. Now $A \subseteq A$ and by hypothesis $\alpha cl(A) \subseteq A$. Therefore $\alpha cl(A) \subseteq U$. Hence A is αgrw-closed. □

Remark 3. *If A is both open and αgrw-closed, then A need not be αg-closed.*

Example 4. *Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then the subsets $\{a, b\}$ and $\{a, b, c\}$ are αgrw-closed and open but not αg-closed.*

Remark 4. *Difference of two αgrw-closed sets is not generally αgrw-closed.*

Example 5. *Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then the sets $A = \{a, c, d\}$ and $B = \{c, d\}$ are αgrw-closed but $A - B = \{a\}$ is not αgrw-closed.*

Proposition 6. *Let $B \subseteq A \subseteq X$. If A is open in X, then $A \in \alpha grwC(X)$ implies $A \in \alpha grwC(Y)$.*

Proof. Let A be αgrw-closed in X and let $A \subseteq G$ where G is regular semi-open in Y. Then $G = U \cap Y$, where U is regular semi-open in X by Lemma 1. This implies $A \subseteq U$. Since A is αgrw-closed in X, $\alpha cl(A) \subseteq U$ and so $\alpha cl(A) \cap Y \subseteq U \cap Y$. Therefore $\alpha cl_Y(A) \subseteq G$. Hence $A \in \alpha grwC(Y)$. □

Proposition 7. *Suppose $B \subseteq A \subseteq X, B$ is αgrw-closed relative to A and A is both regular open and αgrw-closed subset of X. Then B is αgrw-closed in X.*

Proof. Let $B \subseteq U$ and U be regular semi-open in X. Then we have $B \subseteq A \cap U$. Since A is open and U is semi-open in X by Theorem 2, $A \cap U$ is semi-open in X. Since every regular-open set is regular semi-open and every regular semi-open set is semi-closed, A and U are semi-closed. Therefore $A \cap U$ is semi-closed in X. Thus $A \cap U$ is regular semi-open in X. Also $A \cap U \subseteq A \subseteq X$ and A is open subspace of X by Lemma 1, $A \cap U$ is regular semi-open in A. Since B is αgrw-closed relative to A, $\alpha cl_A(B) \subseteq A \cap U$. But $\alpha cl_A(B) = A \cap \alpha cl(B)$. This implies $A \cap \alpha cl(B) \subseteq A \cap U$ and we have $A \cap \alpha cl(B) \subseteq U$. Since A is regular open and αgrw-closed by Proposition 4, $\alpha cl(A) = A$ and so $\alpha cl(B) \subseteq A$. Thus $\alpha cl(B) \subseteq U$ and hence B is αgrw-closed in X. □

Proposition 8. *If a subset A of (X, τ) is αgrw-closed, then $\alpha cl(A) - A$ contains no non-empty regular closed set.*

Proof. Suppose that A is αgrw-closed in (X, τ) and F be a regular closed subset of $\alpha cl(A)$–A. Then $A \subseteq F^c$. Since every regular open set is regular semi-open and A is αgrw-closed, $\alpha cl(A) \subseteq F^c$. Consequently $F \subseteq [\alpha cl(A)]^c$. Thus $F \subseteq \alpha cl(A) \cap [\alpha cl(A)]^c = \emptyset$. Hence $\alpha cl(A) - A$ contains no non-empty regular closed set. □

Remark 5. *The converse of the above proposition need not be true. In Example 2, let* $A = \{a\}$*. Then* $\alpha cl(A) - A = \{c\}$ *does not contain non-empty regular closed set, but* A *is not an* αgrw*-closed set.*

Proposition 9. *Let* $A \subseteq Y \subseteq X$ *and* Y *is regular open in* X *then* A *is* αgrw*-closed in* Y *whenever* A *is* αgrw*-closed in* X*.*

Proof. Let A be αgrw-closed in X and Y be regular open subset of X. Let U be any regular semi-open set in Y such that $A \subseteq U$. By Lemma 2, U is regular semi-open in X. Then we have $\alpha cl(A) \subseteq U$. That is $Y \cap \alpha cl(A) \subseteq Y \cap U = U$. Thus $\alpha cl_Y(A) \subseteq U$ and hence A is αgrw-closed in Y. □

Proposition 10. *A subset* A *of* (X, τ) *is* αgrw*-closed if and only if* $\alpha cl(A) \subseteq rsker(A)$*.*

Proof. Suppose that A is αgrw-closed. Let $x \in \alpha cl(A)$. Suppose $x \notin rsker(A)$, then there is a regular semi-open set U containing A such that $x \notin U$. Since U is regular semi-open containing A, we have $x \notin \alpha cl(A)$, which is a contradiction. Thus $\alpha cl(A) \subseteq rsker(A)$.

Conversely, let $\alpha cl(A) \subseteq rsker(A)$. If U is any regular semi-open set containing A, then $\alpha cl(A) \subseteq rsker(A) \subseteq U$. Therefore A is αgrw-closed. □

Remark 6 ([4])**.** *In the notion of Lemma 3, we may consider the following decomposition of a given topological space* (X, τ)*, namely* $X = X_1 \cup X_2$*, where* $X_1 = \{x \in X : \{x\}$ *is nowhere dense*$\}$ *and* $X_2 = \{x \in X : \{x\}$ *is preopen*$\}$*.*

Proposition 11. *For any subset* A *of* (X, τ)*,* $X_2 \cap \alpha cl(A) \subseteq rsker(A)$*.*

Proof. Let $x \in X_2 \cap \alpha cl(A)$ and suppose that $x \notin rsker(A)$. Then there is a regular semi-open set U containing A such that $x \notin U$. If $F = X - U$, then F is regular semi-closed and so F is semi-closed. We have $scl(\{x\}) = \{x\} \cup int(cl(\{x\})) \subseteq F$. Since $\alpha cl(\{x\}) \subseteq \alpha cl(A)$, we have $int(cl(\{x\})) \subseteq A \cup int(cl(A))$. Again since $x \in X_2$, we have $x \notin X_1$ and so $int(cl(\{x\})) \neq \emptyset$. Therefore there has to be some point $y \in A \cap int(cl(\{x\}))$ and hence $y \in F \cap A$, a contradiction. Thus $x \in rsker(A)$. Hence $X_2 \cap \alpha cl(A) \subseteq rsker(A)$. □

Proposition 12. *For any subset* A *of* (X, τ)*, if* $X_1 \cap \alpha cl(A) \subseteq A$*, then* A *is* αgrw*-closed in* X*.*

Proof. Suppose that $X_1 \cap \alpha cl(A) \subseteq A$. Then $X_1 \cap \alpha cl(A) \subseteq rsker(A)$, since $A \subseteq rsker(A)$. Now $\alpha cl(A) = X \cap \alpha cl(A) = (X_1 \cup X_2) \cap \alpha cl(A) = (X_1 \cap \alpha cl(A)) \cup (X_2 \cap \alpha cl(A)) \subseteq rsker(A)$, since $X_1 \cap \alpha cl(A) \subseteq rsker(A)$ and by Proposition 11. Thus A is αgrw-closed by Proposition 10. □

Proposition 13. *Let* X *be a regular space in which every regular semi-open subset is open. If* A *is compact subset of* X*, then* A *is* αgrw*-closed.*

Proof. Let $A \subseteq U$ and U be regular semi-open. By assumption U is open in X. Since A is a compact subset of a regular space X, then there exists a closed set V such that $A \subseteq V = cl(V) \subseteq U$. Thus $cl(V) \subseteq U$ and so $\alpha cl(A) \subseteq U$. Hence A is αgrw-closed. □

Proposition 14. *If (X, τ) is s-normal and $F \cap A = \emptyset$, where F is regular semi-open and A is αgrw-closed , then there exist disjoint semi-open sets S_1 and S_2 such that $A \subseteq S_1$ and $F \subseteq S_2$.*

Proof. Since F is regular semi-open and $F \cap A = \emptyset$. Then $A \subseteq F^c$ and so $\alpha cl(A) \subseteq F^c$. Thus $\alpha cl(A) \cap F = \emptyset$. Since $\alpha cl(A)$ and F are semi-closed and X is s-normal, there exist semi-open sets S_1 and S_2 such that $\alpha cl(A) \subseteq S_1$ and $F \subseteq S_2$. This implies $A \subseteq S_1$ and $F \subseteq S_2$. □

Remark 7. *Disjoint αgrw-closed sets in a semi-normal space cannot be separated by semi-open sets. In Example 1, the space (X, τ) is s-normal, but $\{a, b\}$ and $\{c\}$ are disjoint αgrw-closed sets which cannot be separated by disjoint semi-open sets.*

Proposition 15. *If (X, τ) is normal in which every α-closed set is closed and $F \cap A = \emptyset$, where F is regular closed and A is αgrw-closed then there exist disjoint open sets O_1 and O_2 such that $A \subseteq O_1$ and $F \subseteq O_2$.*

Proof. Similar to Proposition 14. □

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