



Optimal Orchestration of Packaging for Sustainability

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Abstract. This study proposes two new mathematical models. These models aim to determine optimal ratio of using package for minimum waste in the environment. The models determine package usage ratios which minimize the packaging wastes when packaging amount is at maximum level in the environment. Models are distinguished from each other ignoring or not ignoring decay of each package in nature themselves. Proposed theoretical mathematical models have real world applications based on genuine data. These models easily can used by any enterprise which is concerned with environment and struggles to minimize the packaging waste.

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1. Introduction

Sustainability is a familiar topic in different fields due to its different dimensions such as economic, environmental and social. Sustainability's topics - economic, environment and social (equity) - can be referred as "three Es" that constitutes the "triple bottom line" [9]. The notion of the three pillars - social, environmental, economic - as symbolized by the summit motto "People, Planet, Prosperity" in the World Summit on Sustainable Development, 2002 [12, 13].

Meaning of sustainability can be defined as; "talk about maintenance, sustenance, continuity of a certain resource, system, condition, relationship" [8]. Sustainable development is defined as "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs." and emphasized as, "It contains within it two key concepts; the concept of 'needs', in particular the essential needs of the world's poor, to which overriding priority should be given; and the idea of limitations imposed by the state of technology and social organization on the environment's ability to meet present and future needs." in the Our Common Future: Report of the World Commission on Environment and Development, United Nations 1987 [15].

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Companies explore sustainability as an area for growth and development. So marketing concept is extending towards fulfilling the needs of future generations, which further means that creating, communicating and delivering sustainability based value to customer [11]. Sustainable businesses look for balanced and integrated performances in the three sustainability triple bottom line: social, economic and environmental [1].

There are several terms that can describe relationship between the marketing discipline, the public policy process and the natural environment such as ecological marketing, environmental marketing, green marketing, sustainable marketing and greener marketing [16]. Kirchgeorg and Winn [10] define sustainability marketing as “the planning, coordination, implementation and controlling of all market transactions in such a way that a sustained satisfaction of the needs of current and potential customers toward the achievement of corporate objectives is granted, while at the same time contributing toward reducing ecological and social impacts and restoring social and ecological health” [10].

Packaging is significant factor in the marketing discipline. There are a lot of functions of packages. Among the most important functions of them are; protection, promotion, information, convenience, unitization, handling, waste reduction and recycling and reuse of by-products [7].

North American packaging markets, including the USA, Canada and Mexico, accounted for 32% of global packaging which is the largest share of it, ahead of western Europe 26% and Asia 26%, in 2003 [17]. In the same report of WPO (2008), the figure “World packaging consumption by region, 2003-09”, Asia is first with between 160,000-180,000 US \$ million in packaging consumption, North America second with between 140,000-160,000 US \$ million in packaging consumption, Western Europe is third with between 120,000-140,000 US \$ million in packaging consumption in 2009 [17].

There are a lot of reports about packaging waste management. Reports give opinion about importance of packaging waste. One of them is, “European Packaging Waste Management Systems Main Report” from European Commission in February 2001. In the report, there is a definition of its objectives. The objectives of this report defines in it as “to provide an overview of the different management systems in operation in each Member State, covering the managerial, technical and economic aspects involved in packaging waste management systems, and to draw up potential scenarios for each Member State for the years 2006 and 2011” [5]. People may understand from this definition that the management of packaging waste a very serious work.

Packaging usage is indispensable for marketers because of its functions. This situation may cause more packaging wastes and more problems for environment. They should carefully manage wastes to protect environment. From this objective, this study proposes two mathematical models to optimize usage of packaging ratios between their types and minimize packaging wastes when they are maximum level in the environment.

2. Materials and Methods

2.1. Developing the Models

The goal of the models is to find the optimal ratio of packaging usage, that is; to minimize the packaging waste when it is at maximum level in the environment.

2.1.1. Developing the First Model

The mathematical programming model is as follows.

$j = 1, 2, \dots, n$ be the number of types of packages,

x_{jT} be total amount of package j in the environment (before recycle or recovery),

x_j be amount of waste package of package j after subtracting recycled (or recovered) packages,

t be time unit,

P_j be usage ratio of package j ,

c be a constant,

k_j be coefficient of package j in the differential equation,

B_0 be growth rate,

r be discount rate,

The first mathematical model begins with a differential equation giving waste package amount with respect to time,

$$\frac{dx_j}{dt} = -k_j(x_{jT} - x_j) \quad (1)$$

We assume there is initially no package in the environment. That is, $x_j(0) = 0$ for $t = 0$. The solution of the differential equation with initial condition yields,

$$x_j = x_{jT}(1 - e^{-k_j t}) \quad (2)$$

This result gives net amount of package amount (after subtracting recycled / recovered amount) in the environment at any time t .

By discounting to get the present value, the objective function which is minimizing the packaging waste in the environment at time t becomes;

$$\min \sum_{j=1}^n P_j(x_{jT}(1 - e^{-k_j t}))e^{-rt} \quad (3)$$

with decision variables are P_j 's.

This objective function minimizes the packaging waste at time t , but our problem is the minimizing when it is maximum amount in the environment. Because of that, we should find the time t^* , when the packages amount is at the maximum level.

Rewriting the above function,

$$R(t) = P_j(x_{jT}(1 - e^{k_j t})) \tag{4}$$

$$T(t) = R(t)e^{-rt} \tag{5}$$

When the amount of each type of package is maximum level (at that time t^*), the present value of the sum of waste amount will be at a maximum level. Now, our objective is the minimizing the packaging waste at that time t^* . In the light of this considerations, the objective function becomes,

$$\text{minmax} \sum_{j=1}^n T(t) \tag{6}$$

By maximizing the $T(t)$, t^* can be found;

$$\text{max} T(t) = \text{max} R(t)e^{-rt} \tag{7}$$

By differentiating with respect to t to get the necessary condition, $T'(t) = 0$;

$$T'(t) = (R'(t) - rR(t))e^{-rt} \tag{8}$$

then, the expression (8) equals to 0 if and only if $R'(t) - rR(t) = 0$. Then, the optimal solution,

$$t^* = \left(\frac{1}{k_j}\right) \ln\left(\frac{r}{(r - k_j)}\right) \tag{9}$$

For the sufficient condition $T''(t) < 0$;

$$T''(t) = (R''(t) - rR'(t))e^{-rt} - r(R'(t) - rR(t))e^{-rt} \tag{10}$$

By using first and second order differentiation of equation (4) and using eq. (9),

$$T''(t) = \left[-k_j^2 \left(\frac{r}{(r - k_j)}\right) + 2rk_j \left(\frac{r}{(r - k_j)}\right) + r^2 \left(1 - \left(\frac{r}{(r - k_j)}\right)\right) \right] P_j x_{jT} \left(\frac{r}{(r - k_j)}\right)^{\frac{(-r)}{k_j}} \tag{11}$$

Packaging waste amount is maximum level, when time is t^* and $T''(t) < 0$. By using (9), we can revisit and rewrite the objective function of the first mathematical model,

$$\begin{aligned} \text{min} \sum_{j=1}^n P_j \left[x_{jT} \left(\frac{-k_j}{(r - k_j)}\right) \right] \left(\frac{r}{(r - k_j)}\right)^{\frac{(-r)}{k_j}} \\ \sum_{j=1}^n P_j = 1 \\ 0 \leq \varepsilon \leq P_j \leq 1 \end{aligned} \tag{12}$$

The rate of growth of the objective function can be calculated as [4],

$$B_o = \frac{T'(t)}{T(t)} \quad (13)$$

Thus, we have,

$$B_o = \frac{((r - k_j)e^{k_j t} - r)}{1 - e^{k_j t}} \quad (14)$$

2.1.2. Developing the Second Model

Second model is similar to the first model until equation (6). The second model alter after the equation (6) when calculating t^* by adding decay of packages at the environment.

$j = 1, 2, \dots, n$ be the number of types of packages,

x_{jT} be total amount of package j in the environment (before recycle or recovery),

x_j be amount of waste package of package j after subtracting recycled (or recovered) packages,

t be time unit,

P_j be usage ratio of package j ,

c be a constant,

k_j be coefficient of package j in the differential equation,

B_o be growth rate,

r be discount rate,

m_j be decay amount of package j within unit time,

After equation (6), $\min \max \sum_{j=1}^n T(t)$, second model is as follows. Since packaging decays at the environment, which depends on t , we write the net present amount of decaying waste packaging at time t in the environment by the same continuous discount rate as [3, 4]

$$\int_0^t m_j e^{-rt} dt = \frac{m_j}{-r} (e^{-rt} - 1) \quad (15)$$

By adding equation (15) to the $T(t)$, $(R(t)e^{-rt} - \frac{m_j}{-r}(e^{-rt} - 1))$, and maximizing

$$\max T(t) = \max (R(t)e^{-rt} + \frac{m_j}{r}(e^{-rt} - 1)) \quad (16)$$

By differentiating with respect to t to get the necessary condition,

$$T'(t) = (R'(t) - rR(t) - m_j)e^{-rt} = 0 \quad (17)$$

then, the expression (17) equals 0 if and only if $R'(t) - rR(t) - m_j = 0$. This gives,

$$t^* = \left(\frac{1}{k_j}\right) \ln\left(\frac{rP_jx_{jT} + m_j}{(r - k_j)P_jx_{jT}}\right) \quad (18)$$

For the sufficient condition, $T''(t) < 0$,

$$T''(t) = (R''(t) - 2rR'(t) + r^2R(t) + rm_j)e^{-rt} \quad (19)$$

By using first and second order differentiation of equation (4) and using (18),

$$\begin{aligned} T''(t) = & \left(-k_j^2 \left(\frac{rP_jx_{jT} + m_j}{r - k_j}\right) + 2rk_j \left(\frac{rP_jx_{jT} + m_j}{r - k_j}\right) + r^2 \left(\frac{-k_jP_jx_{jT} - m_j}{r - k_j}\right) + rm_j\right) \\ & \times \left(\frac{rP_jx_{jT} + m_j}{(r - k_j)P_jx_{jT}}\right)^{\frac{(-r)}{k_j}} \end{aligned} \quad (20)$$

We conclude that the packaging waste amount is at maximum level at t^* . By using (18), we can rewrite the objective function and build the second mathematical model,

$$\begin{aligned} \min & \sum_{j=1}^n P_jx_{jT} \left(\frac{-k_jP_jx_{jT} - m_j}{(r - k_j)P_jx_{jT}}\right) \left(\frac{rP_jx_{jT} + m_j}{(r - k_j)P_jx_{jT}}\right)^{\frac{(-r)}{k_j}} \\ & \sum_{j=1}^n P_j = 1 \\ & 0 < \varepsilon \leq P_j \leq 1 \end{aligned} \quad (21)$$

Note that, $P_j \neq 0$ and $\varepsilon \neq 0$. The rate of growth of objective function can calculate as,

$$B_o = \frac{(P_jx_{jT}e^{k_jt}(r - k_j) - rP_jx_{jT} - m_j)e^{-rt}}{(P_jx_{jT}(1 - e^{k_jt}) + \frac{m_j}{r})e^{-rt} - \frac{m_j}{r}} \quad (22)$$

3. An Illustrative Real World Application

We apply our mathematical derivation to the real world data. The data is taken from the "Packaging Industry Report of Turkey 2012" [2]. This is an annual report about the packaging industry in the Turkey. The produced packaging amount from 2007 to 2011 is published in the report whereas that from 2006 to 2010 published in the 2011 report [6].

We employ our mathematical models for the packaging amount of 2011 in Table 1. We know the produced packaging amount from the 2012 report. The recovery ratios in Table 2, are taken from the Regulation of Packaging Wastes Control from the Ministry of Environment and Forestry of Turkey, published in the gazette whose date is 24/08/2011 and volume is 28035 [14].

The discount rate can be found find as

$$r = \frac{1}{t} \ln \frac{V}{A} \quad (23)$$

where V is the future value, t time, and A is present value of a capital. Analogously, we use this approach for packaging, as V is future and A is present amounts of packaging material, respectively.

The discount rate from 2006 to 2011 is computed by the root mean square of the produced packaging amounts. Since we should satisfy the packaging demand, we add some assumptions to models as constraints. These are at least usage ratios amount (ε_j) of each packaging material.

Table 1: Types of packaging and their production amount in Turkey in 2011.

Types of packaging	Production amount (tonne)
Paper packaging	106,200
Board packaging	564,000
Plastics packaging	2,012,700
Metal packaging	364,000
Glass packaging	772,000

Table 2: Recovery ratio goals of each packaging according to the regulation.

Year	Glass packaging	Plastics packaging	Metal packaging	Paper/Board packaging
2011	38%	38%	38%	38%

3.1. Application of the First Model

We implement the first model based on the data as follows.

- $x_{1T} = 106,200$
- $x_{2T} = 564,000$
- $x_{3T} = 2,012,700$
- $x_{4T} = 364,000$
- $x_{5T} = 772,000$
- $\varepsilon_1 = 0.025$
- $\varepsilon_2 = 0.05$
- $\varepsilon_3 = 0.4$

- $\varepsilon_4 = 0.05$
- $\varepsilon_5 = 0.15$
- $r = 0.091$
- $n = 5$

$$\begin{aligned}
 & \min \sum_{j=1}^5 P_j \left[x_{jT} \left(\frac{-k_j}{(r - k_j)} \right) \right] \left(\frac{r}{(r - k_j)} \right)^{\frac{(-r)}{k_j}} \\
 & \sum_{j=1}^5 P_j = 1 \\
 & 0.025 \leq P_1 \leq 1 \\
 & 0.05 \leq P_2 \leq 1 \\
 & 0.4 \leq P_3 \leq 1 \\
 & 0.05 \leq P_4 \leq 1 \\
 & 0.15 \leq P_5 \leq 1
 \end{aligned} \tag{24}$$

Values of k_j can compute by solving differential equation. Values of k_j are given in Table 3.

Table 3: Values of k_j .

k_1	-0.00265
k_2	-0.00265
k_3	-0.00265
k_4	-0.00265
k_5	-0.00265

This mathematical program is solved by the Microsoft Excel addin, Solver, which is a powerful nonlinear optimization tool. Results are given in Table 4, Table 5 and Table 6.

Table 4: The optimal values of decision variables and the objective function of the first model.

P_1	P_2	P_3	P_4	P_5	Value of objective function
0.35	0.05	0.4	0.05	0.15	10610.190

Table 5: The optimal time at which the packaging amount is maximum of the first model.

Types of packaging	t^* value of each type of packaging
Paper packaging	10.83119
Board packaging	10.83119
Plastics packaging	10.83119
Metal packaging	10.83119
Glass packaging	10.83119

Table 6: The values of the second order derivatives of the first model.

T_1''	T_2''	T_3''	T_4''	T_5''
-3.34612	-2.53862	-72.4748	-1.6384	-10.4245

Since second order derivatives are negative values in Table 6 (the sufficient condition), t^* values are optimal at which the packaging amount is maximum.

According to the Table 7, the first model proposes more usage of paper packaging. Board, plastics, glass and metal packaging ratios are binding. According to these results we can say, under the assumptions, the first model proposes to use much more paper packaging. But other packaging materials should be used much less to protect the environment in an optimal way.

Table 7: Comparison of the usage ratios of packaging with the first model's suggested ratios.

Types of packaging	Usage ratios of packaging	The first model's suggested ratios
Paper packaging	0.027809	0.35
Board packaging	0.147687	0.05
Plastics packaging	0.527037	0.4
Metal packaging	0.095315	0.05
Glass packaging	0.202152	0.15

The packaging waste amount was 2,367,718.00 tonnes in 2011 in Turkey. If our model suggestions should have been applied in the country, the packaging waste amount would be 312,221.19 tonnes. This means that 86.8% less waste in the environment.

The model applied to packaging amount of 2011. The data is one-year (365 days) data. The objective function vs. time and the growth rate of objective function vs. time are sketched in Figure 1 and Figure 2, respectively.

Note that around the 10th day, the packaging amount is increasing in the environment, and then reducing in Figure 1. The parallel behavior of the growth rate is seen in Figure 2.

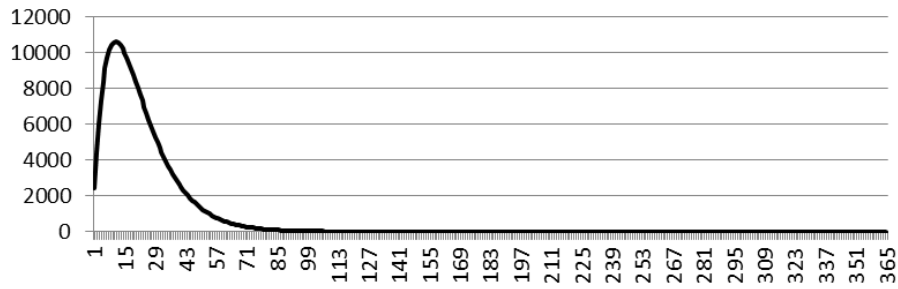


Figure 1: The First Model: Objective Function vs. Time.

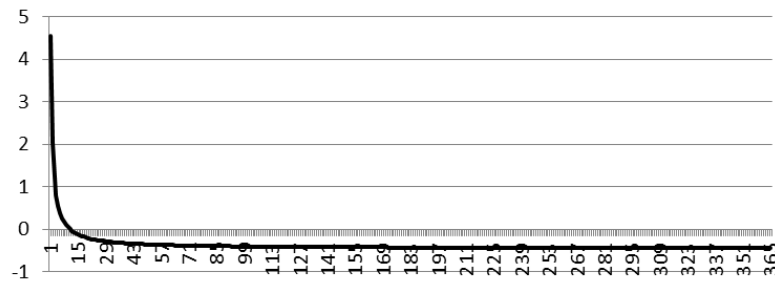


Figure 2: The First Model: Growth Rate of the Objective Function vs. Time.

3.2. Application of the Second Model

In addition to first model, there is decay rates assumption of each packaging type in the second model. These rates can be determined by special experiments for each material. Since the decay rates of each material may change along its own environmental conditions and other factors. The following data of application is general accepted information about produced packaging amount and there is no data about their raw materials. Thus, we assume that the decay rates as in Table 8. By using decay rates, decay amount of packaging can be calculated as in Table 9.

According to the given data at the tables, the second mathematical model be applied as follows.

- $x_{1T} = 106,200$
- $x_{2T} = 564,000$
- $x_{3T} = 2,012,700$

- $x_{4T} = 364,000$
- $x_{5T} = 772,000$
- $\varepsilon_1 = 0.025$
- $\varepsilon_2 = 0.05$
- $\varepsilon_3 = 0.4$
- $\varepsilon_4 = 0.05$
- $\varepsilon_5 = 0.15$
- $r = 0.091$
- $n = 5$

$$\min \sum_{j=1}^5 P_j x_{jT} \left(\frac{-k_j P_j x_{jT} - m_j}{(r - k_j) P_j x_{jT}} \right) \left(\frac{r P_j x_{jT} + m_j}{(r - k_j) P_j x_{jT}} \right)^{\frac{(-r)}{k_j}}$$

$$\sum_{j=1}^5 P_j = 1$$

$$0.025 \leq P_1 \leq 1$$

$$0.05 \leq P_2 \leq 1$$

$$0.4 \leq P_3 \leq 1$$

$$0.05 \leq P_4 \leq 1$$

$$0.15 \leq P_5 \leq 1$$
(25)

Table 8: Decay rates of packaging types of the second model.

Types of packaging	Decay rates (per year)
Paper packaging	1×10^{-18}
Board packaging	4×10^{-19}
Plastics packaging	9×10^{-21}
Metal packaging	5×10^{-20}
Glass packaging	9×10^{-23}

Table 9: Decay amount of packaging types of the second model.

Types of packaging	m_j
Paper packaging	6.58×10^{-14}
Board packaging	1.40×10^{-13}
Plastics packaging	1.12×10^{-14}
Metal packaging	1.13×10^{-14}
Glass packaging	4.31×10^{-17}

Table 10: The optimal values of the decision variables and objective function of the second model.

P_1	P_2	P_3	P_4	P_5	Value of objective function
0.35	0.05	0.4	0.05	0.15	10610.19

This mathematical program is again solved by Solver, and the results are given in Table 10, Table 11 and Table 12. Since the second order derivatives are negative values in Table 12 (the sufficient condition), t^* values are optimal values.

Table 11: The optimal time at which the packaging amount is maximum of the second model.

Types of packaging	t^* value of each type of packaging
Paper packaging	10.832
Board packaging	10.832
Plastics packaging	10.832
Metal packaging	10.832
Glass packaging	10.832

Table 12: The values of second order derivatives of the second model.

T''_1	T''_2	T''_3	T''_4	T''_5
-3.34611	-2.53862	-72.4748	-1.6384	-10.4245

Table 13: Comparison of the usage ratios of packaging with the second model's suggested ratios.

Types of packaging	Usage ratios of packaging	The second model's suggested ratios
Paper packaging	0.0278	0.35
Board packaging	0.1477	0.05
Plastics packaging	0.5270	0.4
Metal packaging	0.0953	0.05
Glass packaging	0.2022	0.15

According to Table 13, the second model proposes much more usage of the paper packaging, as in the first model. According to these results we can say, under the assumptions, the second model proposes use of much more paper packaging whereas much less use of the other packaging types to protect the environment.

The results of the both models are very close since the data is only 365 days data-the decay factor may be ignored for 1-year data. If we are in a case of, say, 100 years data, the results would be significantly different from each other.

The objective function and the growth rate of objective function vs. time are sketched in Figure 3 and Figure 4 for 365 days, respectively. We observe that the second model behaves in a similar way as in the first model experiments, given in the Figure 1 and Figure 2.

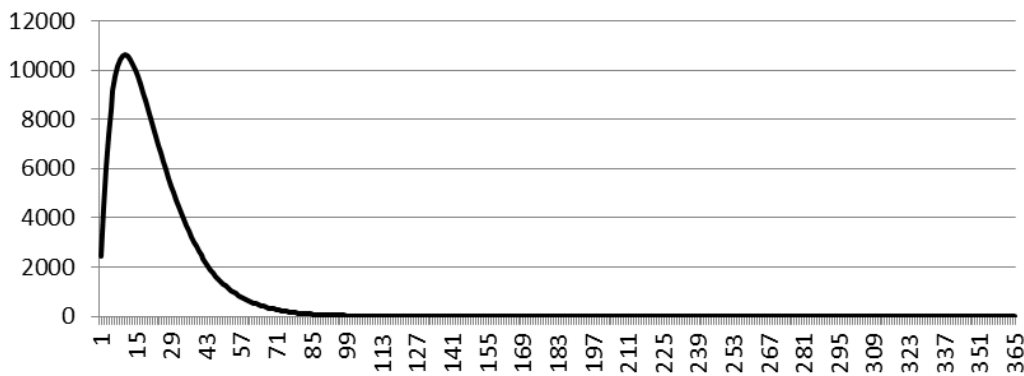


Figure 3: The Second Model: The Objective Function vs. Time.

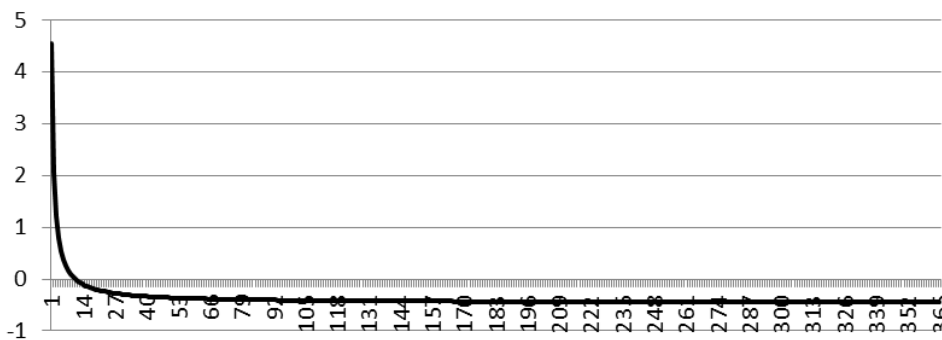


Figure 4: The Second Model: The Growth Rate of the Objective Function vs. Time.

4. Conclusion

In this study, we propose two mathematical models that optimize the packaging usage; that is, minimizing the packaging waste when it is at the maximum level in the environment. The real world application is on optimizing the packaging usage in Turkey under the several assumptions. The models suggest nearly 86.8% less packaging waste in Turkey.

Although the applications are on optimizing the packaging usage of a country, the models can be used in the different areas in industry. For example, a beverage firm uses packaging such as glass, board, metal bottles or boxes. Managers can determine amount of used packaging in a period and the at least needed amount of each packaging such as for transportation and logistics, protection, promotion, information, selling and so on of the company. The company managers can use this mathematical models for optimizing packaging usage of their companies.

The models also offer optimization routines for the companies for the usage of packaging and protecting environment to achieve a sustainable medium.

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