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Peristaltic Flow of Newtonian Nanofluid through an Inclined Annulus Cylinder

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Abstract. This investigation is concerned with the peristaltic transport of Nanofluid in an inclined annulus tube. Transport equations involve the combined effects of Brownian motion and thermophoretic diffusion of nanoparticles. Mathematical modeling is carried out by utilizing long wavelength and low Reynolds number assumptions. Attention has been focused on the behaviors of Brownian motion parameter (Nb), thermophoresis parameter (Nt) and inclination of the annulus. The result indicates an appreciable increase in the temperature and nanoparticles concentration with the increase in the strength of Brownian motion effects, and the inclination angle increases the pressure rise.

Key Words and Phrases: peristaltic flow, heat transfer, Nanofluid, frictional force, Viscous fluid, Inclined Endoscopic;

1. Introduction

In the peristaltic flow, wave propagates along the length of the annulus tube, Nano fluid present in the tube get transport in the same direction where the wave propagates. peristalsis pumping technique is used by many organs such as urethra, male reproductive system and gastro-intestinal tract. Shapiro, Jaffrine and Weinberg [12] investigated the linearized problem of the peristaltic transport of the fluid in two-dimensional channel and tube contains incompressible fluid. They even considered that the wave length to be infinite. In continuation to that Jaffrine [5], investigated the effects of Reynolds number and wave number, when these are very small but not infinitesimal. Furthermore experiments of Weinberg, Eckstein, and Shapiro [19] Established that the theory of Jaffrine [5] is valid up to Reynolds number of about 10. This research was continued by Takabatake and Ayukawa [15] in which they investigated the problem of peristaltic in case of two-dimensional tube numerically and it was concluded that validity of the perturbation solutions by Jaffrine [5] is restricted within a limited range than that which he had predicted earlier. Moreover they found the reflux near the

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central axis with a large Reynolds number. Takabatake, Ayukawa, and Mori [16] studied numerically the peristaltic transport of the fluid and its efficiency in cylindrical tubes and since nano fluid has many potential applications so the new researchers paying intensive attention for references [1–4, 7, 8, 10, 17] and [13], this lead towards effective thermal capacity, for many production of heat transfer fluids in heat exchange, in plants and in automotive cooling significations, due to their extensive thermal properties, a plenty of literature available which deals with the study of nano fluid and its applications (Yoo *et al.* [20], Menca *et al.* [11], Wang and Mujumdar [18]) of the non-linear peristaltic transport of a newtonian fluid in an inclined symmetric channel considered by Kothandapani [9].

In the present paper, we investigated the peristaltic transport of Nano viscous incompressible Newtonian fluid in an inclined annulus tube under the assumptions of long wavelength and lower Reynolds number. Attention has been focused on the behaviors of Brownian motion parameter (Nb), thermophoresis parameter (Nt) and inclination of the annulus cylinder. The result indicates an appreciable increase in the temperature and Nanoparticles concentration with the increase in the strength of Brownian motion effects, and the inclination angle increases the pressure and all these also discussed through graphs.

1.1. Equations

Consider incompressible Newtonian fluid through coaxial inclined tubes such that the inner tube is rigid (endoscope or catheter) and moving with constant velocity where the outer have a sinusoidal wave traveling down its wall.

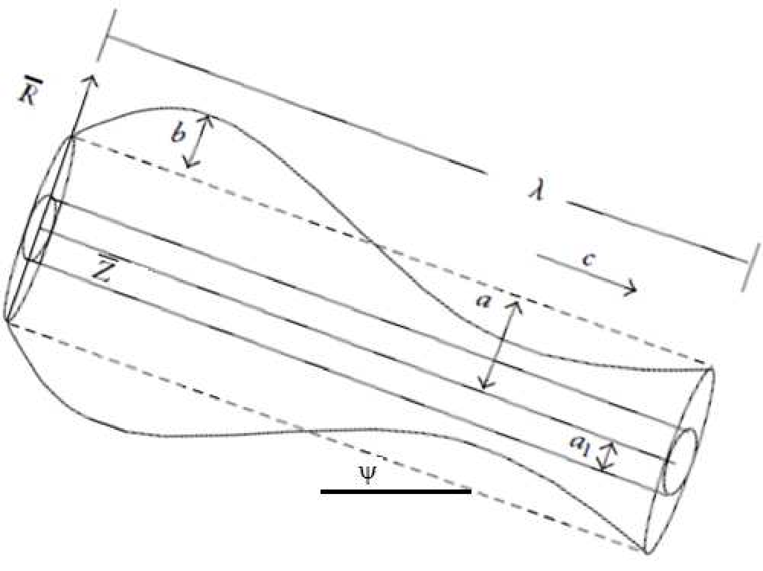


Figure 1: Simplified model geometry of the problem.

The geometry of the wall surfaces described in Figure 1 are

$$r_1' = a_1, \qquad r_2' = a' + b' \sin[\frac{2\pi}{\lambda}(z' - ct')], \tag{1}$$

where a_1, a' are the radius of the inner and the outer tubes at any axial distance z', b' are the amplitude of the wave, λ' is the wavelength, c' is the propagation velocity and t' is the time. We choose a cylindrical coordinate system (r', z'), where z'-axis lies along the centerline of the inner and outer tubes and r' is the distance measured radially.

The equations governing the problem are:

$$\bar{\nabla} \cdot \bar{\mathbf{q}} = 0, \tag{2}$$

$$\rho_f \frac{D\bar{\mathbf{q}}}{Dt} = -\bar{\nabla}\bar{p} + \mu\bar{\nabla}^2\bar{q} + \rho\mathbf{g}[\sin(\psi)\hat{z} - \cos(\psi)\hat{R}], \tag{3}$$

$$(\rho c)_f \frac{D\bar{T}}{Dt} = k\bar{\nabla}^2\bar{T} + (\rho c)_p\big[D_B(\bar{\nabla}\bar{\mathbf{C}}) \cdot (\bar{\nabla}\bar{\mathbf{T}}) + \frac{D_T}{(T_0)}(\bar{\nabla}\bar{\mathbf{T}}) \cdot (\bar{\nabla}\bar{\mathbf{T}})\big], \tag{4}$$

$$\frac{D\bar{C}}{Dt} = D_B\bar{\nabla}^2\bar{C} + \frac{D_T}{(T_0)}\bar{\nabla}^2\bar{T}, \tag{5}$$

where ρ_f is the density of the fluid, ρ_p is the density of the particle, C is the volumetric volume expansion coefficient, $\overline{q}$ is the velocity vector, T_s is the fluid temperature in static condition, d/dt represents the material time derivative, $\overline{p}$ is the pressure, $\overline{C}$ is the nanoparticle phenomena, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient.

The problem has been studied in cylindrical coordinate system (r', z'), radial, and axial coordinates respectively, introducing a wave frame (r', z') moving with velocity C away from the fixed frame (R', Z') by the transformations.

$$\frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial r'} + \frac{v'}{r'} = 0 \tag{6}$$

$$\rho\big[u'\frac{\partial u'}{\partial z'} + v'\frac{\partial u'}{\partial r'}\big] = -\frac{\partial p'}{\partial z'} + \mu\big[\frac{\partial^2 u'}{\partial z'^2} + \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'}\big] + \rho g\alpha(T' - T_0') + \rho g\alpha(C' - C_0') + \rho g \sin(\psi) \tag{7}$$

$$\rho\big[u'\frac{\partial v'}{\partial z'} + v'\frac{\partial v'}{\partial r'}\big] = -\frac{\partial p'}{\partial r'} + \mu\big[\frac{\partial^2 v'}{\partial z'^2} + \frac{\partial^2 v'}{\partial r'^2} + \frac{1}{r'}\frac{\partial v'}{\partial r'} - \frac{v'}{r'^2}\big] - \rho g \cos(\psi) \tag{8}$$

The energy equation

$$[\bar{u}\frac{\partial \bar{T}}{\partial \bar{z}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{r}}] = \alpha(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{r}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}) + \tau\Big[D_B\Big\{(\frac{\partial \bar{C}}{\partial \bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}) + (\frac{\partial \bar{C}}{\partial \bar{z}}\frac{\partial \bar{T}}{\partial \bar{z}})\Big\} + \frac{D_T}{T_0}\Big\{(\frac{\partial \bar{T}}{\partial \bar{r}})^2 + (\frac{\partial \bar{T}}{\partial \bar{z}})^2\Big\}\Big] \tag{9}$$

$$[\bar{u}\frac{\partial \bar{C}}{\partial \bar{z}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{r}}]=D_B(\frac{\partial^2 \bar{C}}{\partial \bar{r}^2}+\frac{1}{r}\frac{\partial \bar{C}}{\partial \bar{r}}+\frac{\partial^2 \bar{C}}{\partial \bar{z}^2})+\frac{D_T}{T_0}(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2}+\frac{1}{r}\frac{\partial \bar{T}}{\partial \bar{r}}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}) \tag{10}$$

where v', u' are the velocity components in r' and z'-directions respectively, ρ is the density, p' is the pressure, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the nano particle material and heat capacity of the fluid and μ is the viscosity. The boundary conditions are:

$$\begin{aligned} u' &= -1, \quad \overline{T} = \overline{T}_2, \quad \overline{C} = \overline{C}_0 \text{ at } r' = r_2', \\ u' &= -1, \quad \overline{T} = \overline{T}_1 \quad , \overline{C} = \overline{C}_0 \text{ at } r' = r_1', \end{aligned} \tag{11}$$

where V' is the velocity of the inner tube, v' and u' are the velocity components in the r'-and z' direction, respectively. We introduce the following nondimensional variables and parameters

$$\begin{aligned} & p = \frac{a^2}{\mu c \lambda} p', \; u = \frac{u'}{c}, \; v = \frac{v'}{c\delta_0}, \; t = \frac{c}{\lambda} t', \; R_e = \frac{\rho c a}{\mu}, \; r_1 = \frac{r_1'}{a} = \epsilon \\ & \phi = \frac{b}{a}, \; \delta_0 = \frac{a}{\lambda}, \; z = \frac{z'}{\lambda}, r_2 = \frac{r_2'}{a} = 1 + \frac{b'}{a}\sin(2\pi z), \\ & \overline{\theta} = \frac{\overline{T} - T_s}{T_1 - T_s}, \; N_t = \frac{(\rho c)_p D_t (T_1 - T_s)}{(\rho c)_f \alpha T_0}, \; \sigma = \frac{(\overline{C} - \overline{C_0})}{\overline{C_0}}, \; N_b = \frac{(\rho c)_p D_B \overline{C_0}}{(\rho c)_f \alpha}, \\ & \alpha = \frac{k}{(\rho c)_f}, \; G_r = \frac{g \alpha d_1^2 (T_1 - T_s)}{\upsilon c}, B_r = \frac{g \alpha d_1^2 \overline{C_0}}{\upsilon c} \end{aligned} \tag{12}$$

where ϕ is the amplitude ratio, *Re* is the Reynolds number, δ_0 is the dimensionless wave number and $F_r = \frac{c^2}{ga2}$ the Froude number. After using the above assumption, the equations of motion in the dimensionless form reduce to

$$\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial z}=0, \tag{13}$$

$$\mathrm{Re}\delta_0\left[v\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial z}+\left[\delta_0^2\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right]+G_r\overline{\theta}+B_r\sigma+\frac{R_e}{F_r}\sin(\psi), \tag{14}$$

$$\mathrm{Re}\delta_0^3\left[v\frac{\partial v}{\partial r}+u\frac{\partial v}{\partial z}\right]=-\frac{\partial p}{\partial r}+\delta_0\left[\delta_0^2\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right]-\frac{R_e}{F_r}\delta_0\cos(\psi), \tag{15}$$

$$\begin{aligned} \delta_0 p_r R_e\left[v\frac{\partial \overline{\theta}}{\partial r}+u\frac{\partial \overline{\theta}}{\partial z}\right] &= \frac{\partial^2 \overline{\theta}}{\partial r^2}+\frac{1}{r}\frac{\partial \overline{\theta}}{\partial r}+\delta_0^2\frac{\partial^2 \overline{\theta}}{\partial z^2}+N_b\left[\frac{\partial \sigma}{\partial r}\frac{\partial \overline{\theta}}{\partial r}+\delta_0^2\frac{\partial \sigma}{\partial z}\frac{\partial \overline{\theta}}{\partial z}\right] \\ &+N_t\left[(\frac{\partial \overline{\theta}}{\partial r})^2+\delta_0^2(\frac{\partial \overline{\theta}}{\partial z})^2\right] \end{aligned} \tag{16}$$

$$\delta_0 R_e p_r C_0 \tau\left[v\frac{\partial \sigma}{\partial r}+u\frac{\partial \sigma}{\partial z}\right]=\frac{\partial^2 \sigma}{\partial r^2}+\frac{1}{r}\frac{\partial \sigma}{\partial r}+\delta_0^2\frac{\partial^2 \sigma}{\partial z^2}+\frac{N_t}{N_b}\left[\frac{\partial^2 \overline{\theta}}{\partial r^2}+\frac{1}{r}\frac{\partial \overline{\theta}}{\partial r}+\delta_0^2\frac{\partial^2 \overline{\theta}}{\partial z^2}\right] \tag{17}$$

Equation (15) shows that p is not a function of r. Using the long wavelength approximation and dropping terms of order δ_0 and higher, it follows from Equations (13) to (17) that the appropriate equation describing the flow is

$$\frac{dp}{dz} - G_r\overline{\theta} - B_r\sigma = \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] + \frac{R_e}{F_r}\sin(\psi). \tag{18}$$

$$0 = \frac{\partial^2\overline{\theta}}{\partial r^2} + \frac{1}{r}\frac{\partial\overline{\theta}}{\partial r} + N_b\left[\frac{\partial\sigma}{\partial r}\frac{\partial\overline{\theta}}{\partial r}\right] + N_t\left[(\frac{\partial\overline{\theta}}{\partial r})^2\right] \tag{19}$$

$$0 = \frac{\partial^2\sigma}{\partial r^2} + \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{N_t}{N_b}\left[\frac{\partial^2\overline{\theta}}{\partial r^2} + \frac{1}{r}\frac{\partial\overline{\theta}}{\partial r}\right] \tag{20}$$

The boundary conditions are obtained from the manipulation of the viscous no-slip conditions at the boundary of the wall of the outer and inner tubes,

$$\begin{aligned} &u = 0, \overline{\theta} = 0, \quad \sigma = 0 \text{ at } r_2 = 1 + \phi\sin(2\pi z), \\ &u = 0, \overline{\theta} = 1, \quad \sigma = 1 \text{ at } r_1 = \epsilon, \end{aligned} \tag{21}$$

where $r_1 = \epsilon$, $r_2 = 1 + \phi\sin(2\pi z)$ and the wall temperature ratio.

The solution of Equations (19) and (20), valid in $r_1 \le r \le r_2$ satisfying the corresponding boundary conditions (21) is given by

$$\sigma = -\frac{N_t}{N_b}\left[\frac{r^A - r_2^A}{r_1^A - r_2^A}\right] + \left[\frac{1 + \frac{N_t}{N_b}}{Ln[\frac{r_1}{r_2}]}\right] Ln[\frac{r}{r_2}] \tag{22}$$

$$\theta = \frac{r^A - r_2^A}{r_1^A - r_2^A} \tag{23}$$

where $A = \left(N_b + N_t\right)/Ln[\frac{r_2}{r_1}]$ on substitution from the equations (22), (23) in equation (18) and solving it analytically as a second order ordinary differential equation, the solution will be of the form

$$\begin{aligned} u =& c_1 + c_2\text{Ln}[r] + \frac{r^2}{4}p + \left(\frac{\frac{r^{(A+2)}}{(A+2)^2} - \frac{r_2^A}{4}r^2}{\left(r_1^A - r_2^A\right)}\right)\left(\frac{N_t}{N_b}\text{Br} - \text{Gr}\right) \\ &- \text{Br}\left(\frac{1 + \frac{N_t}{N_b}}{\text{Ln}\left[\frac{r_1}{r_2}\right]}\right)\left(\frac{-r^2}{8} + \frac{r^2}{4}\text{Ln}\left[\frac{r}{r_2}\right] - \frac{r^2}{8}\right) - \frac{R_e}{\text{Fr}}\frac{r^2}{4}\text{Sin}[\psi] \end{aligned} \tag{24}$$

The instantaneous volume flow rate $Q(z,t)$ is given by

$$Q(z,t) = 2\pi\int_{r_1}^{r_2} r\,u\,dr \tag{25}$$

Following the analysis given by Shapiro et al. [6], the mean volume flow, $\overline{Q}$ over a period is obtained as

$$Q(z,t) = \frac{\overline{Q}}{\pi} - \frac{\phi^2}{2} + 2\phi \text{Sin}[2\pi z] + \phi^2 \text{Sin}[2\pi z]^2, \tag{26}$$

where $\overline{Q}$ is the time-average of the flow over one period of the wave.

Make combination between Eqs. (25) and (26), we get $\frac{\partial p}{\partial z}$. The pressure rise $\Delta p(t)$ and the friction force (at the wall) on the outer and inner tubes are F_2 and F_1 respectively, in a cylinder of length L, in their non-dimensional forms, are given by

$$\Delta p = \int_0^1 \frac{\partial p}{\partial z} dz, \tag{27}$$

$$F_1 = \int_0^1 {r_1}^2 \left(-\frac{\partial p}{\partial z}\right) dz \tag{28}$$

$$F_2 = \int_0^1 {r_2}^2 \left(-\frac{\partial p}{\partial z}\right) dz \tag{29}$$

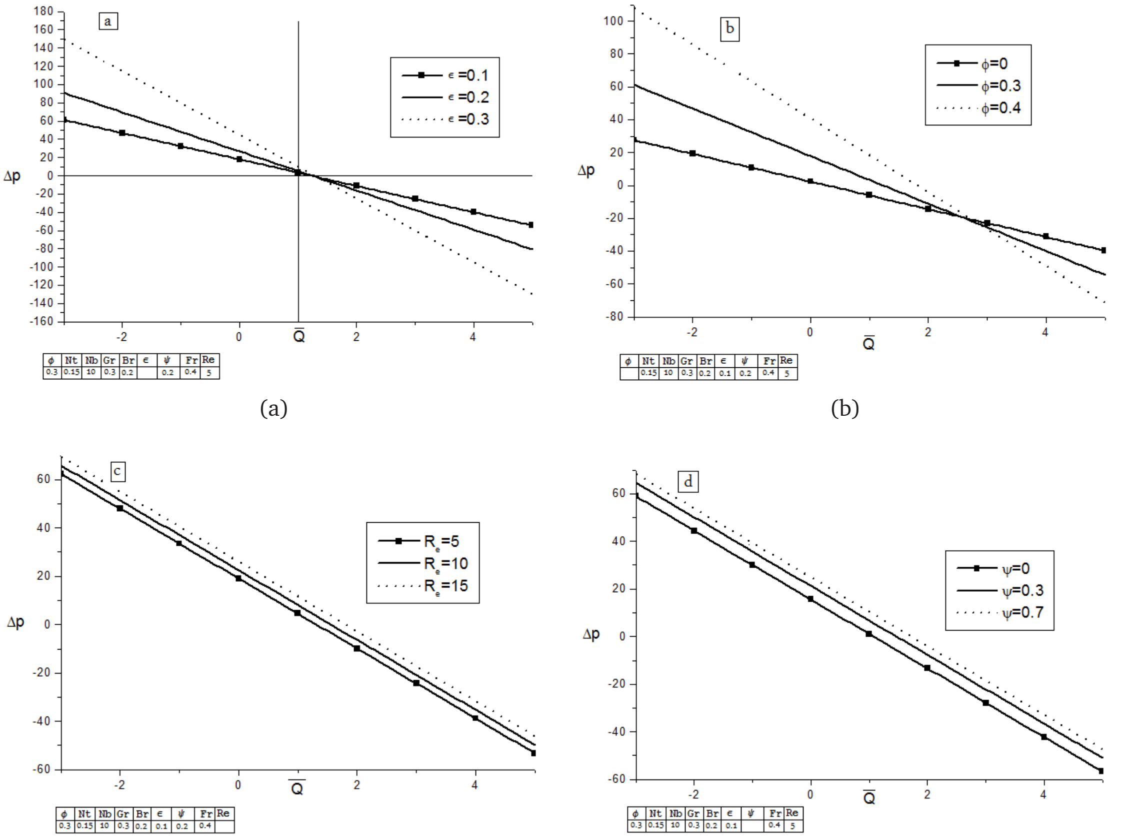
1.2. Graphical Results and Discussion

To discuss the results obtained above quantitatively, we shall assume the form of the flow rate Q, in period $(Z - t)$ as in Srivastava and Srivastava [14]:

$$Q(z,t) = \frac{\overline{Q}}{\pi} - \frac{\phi^2}{2} + 2\phi \text{Sin}[2\pi z] + \phi^2 \text{Sin}[2\pi z]^2.$$

Where $\overline{Q}$ is the time -average of the flow over one period of the wave. This form of Q has been assumed in view of the fact that the constant values of Q gives always ΔP negative, and hence there will be no pumping action. We shall now compute the dimensionless pressure rise Δp, the inner / outer frictional forces F_1, F_2(on the inner and outer surface) for various values of dimensionless flow average $\overline{Q}$, radius ratio ϵ and the velocity of the inner tube v. Following Srivastava [14], we use the values of the various parameters in equations (27) to (29) as: $a = 1.25cm$ and $L = \lambda = 8.01cm$.

Figure 2 represent the variation of pressure rise with $\overline{Q}$ for fixed thermophoresis and Brownian motion parameters N_t, N_b, the pressure rise gradually increases as the area of flow between the annulus decrease(inner radius ϵ increases). Also the pressure rise for endoscope increases as (Inclined angle of the cylinder annulus ψ increase) and the increase in both of the amplitude ratio and Reynolds number gives rise in pressure rise. The friction force have been plotted in Figure 3, which show an opposite character in comparison to pressure rise. Effects of temperature profile $\overline{\theta}$ have been shown through Figures 4a and 4c, the behaviour of temperature mainly depends on whether N_t or N_b is less than one, to illustrate it, a sudden decrease in the temperature is seen in Figures 4a and 4c when both N_t and N_b are in the range of $0.1 - 0.8$. Figures 4b and 4d here the nanopartical phenomenon decrease when there is an increase in the values of thermophoresis parameter N_t and increases gradually with an increase in Brownian motion parameter N_b.



(a) (b)

(c) (d)

Figure 2: Variation of pressure rise over the length versus $\bar{Q}$.

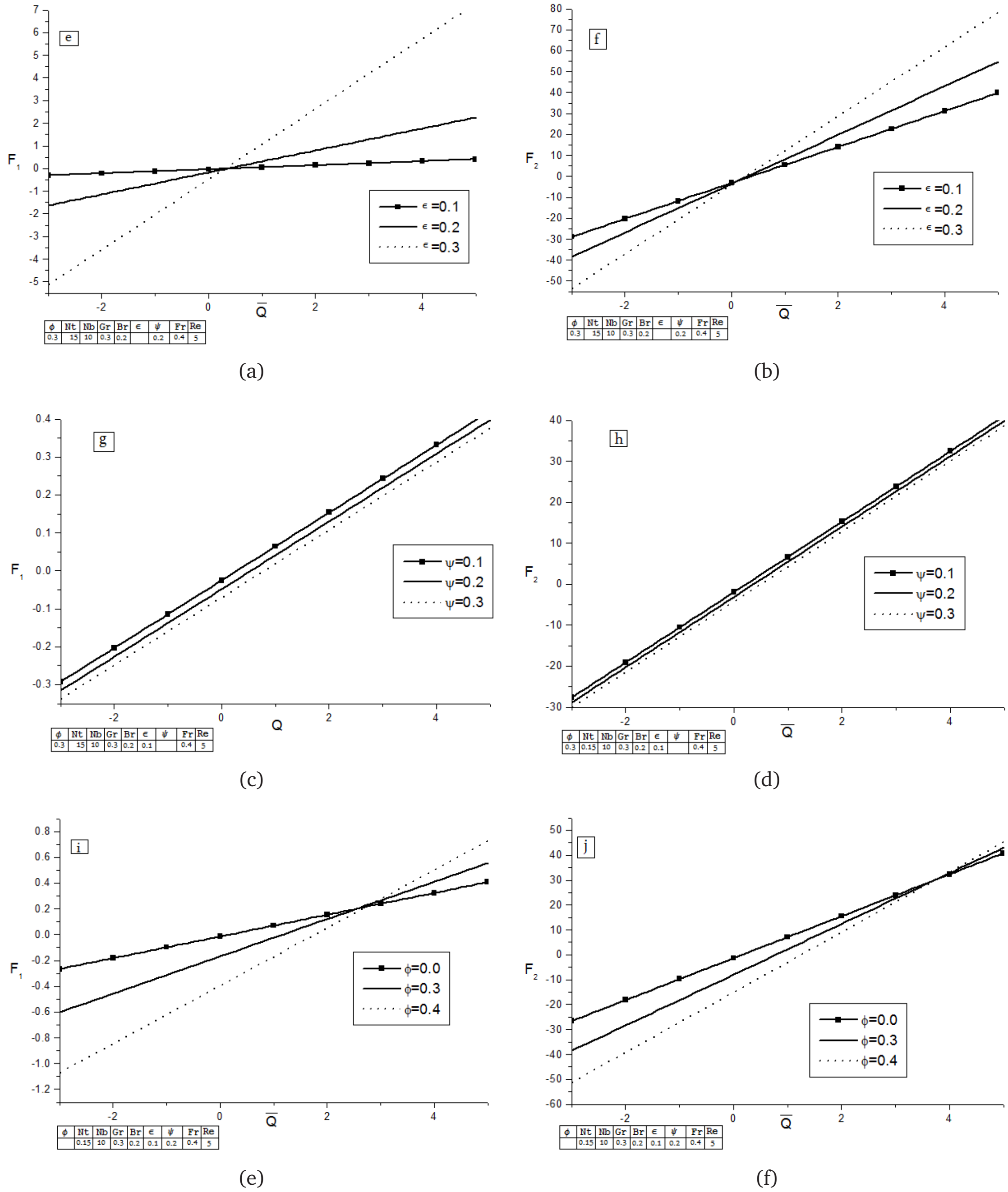


Figure 3: Variation of inner and outer frictional forces F_1, F_2 respectively.

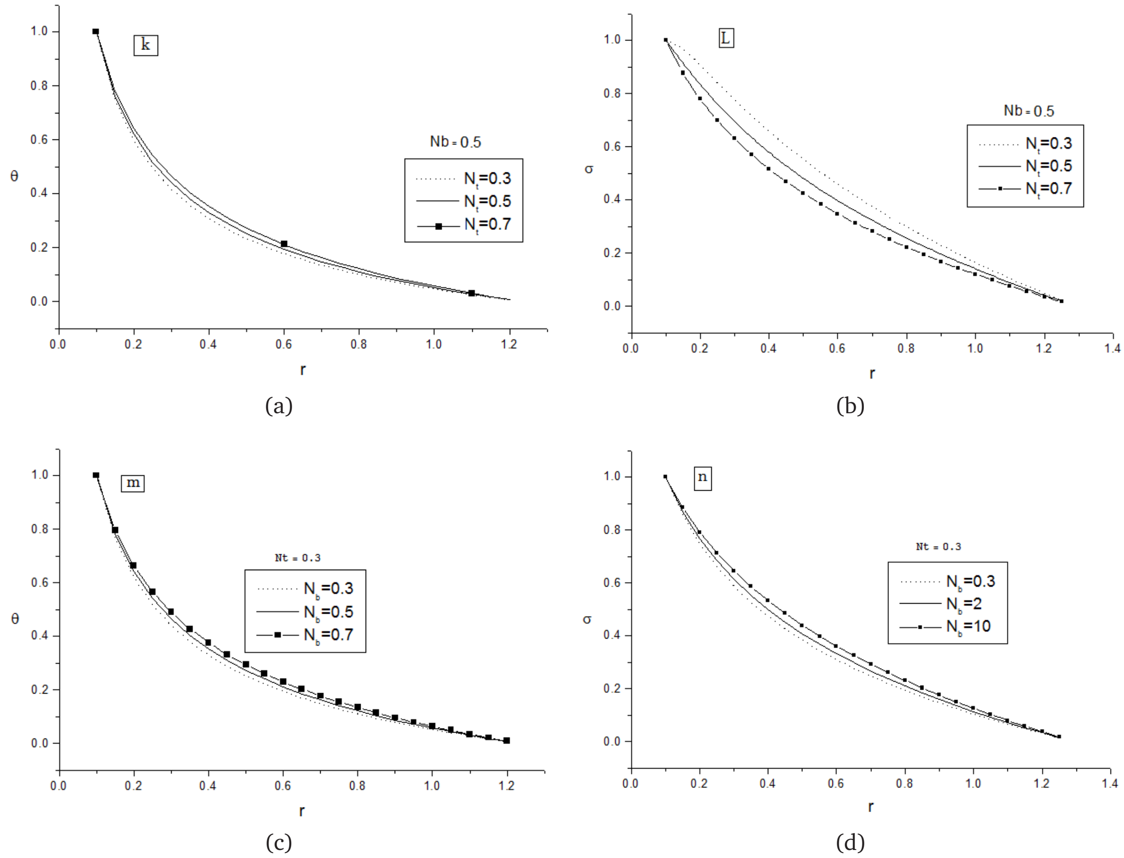


Figure 4: Concentration profile σ and heat distribution θ for different value of the parameters.

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