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On Some New Operations In Probabilistic Soft Set Theory

Çiğdem Gunduz(Aras)*, Hande Poşul

Department of Mathematics, Kocaeli University,Kocaeli 41380, Turkey

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Abstract. In this paper, we study the theory of probabilistic soft sets introduced by [7]. We define equality of two probabilistic soft sets, subset, complement of a probabilistic soft set with examples. We also introduce the operations of union, intersection, difference and symmetric difference. We prove that certain De Morgan's laws hold in probabilistic soft set theory with respect to these new definitions.

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1. Introduction

In theory, for formal modeling, reasoning, and computing we have traditional tools such as crisp, deterministic, and precise in character but in practical way we see that data in economics, engineering, environment, social science, medical science, etc. are not always all crisp and classical methods because of various types of uncertainties present in these problems can not be used, successfully. There are some theories like theory of probability, theory of fuzzy sets and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. According to Molodtsov [5], since all these theories have their inherent difficulties the concept of soft set theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties has been initiated in [5]. Soft set theory has a rich potential for applications in several directions [1–4, 6]. Zhu and Wen [7] have proposed the notion of probabilistic soft sets incorporated Molodtsov's soft set theory with probability theory and introduced three operations with probabilistic soft sets the conditional probabilistic soft set. In the present paper, we make a theoretical study of the "Probabilistic soft set theory" in more detail.

*Corresponding author.

Email address: `caras@kocaeli.edu.tr` (Ç. Gunduz(Aras))

2. Preliminary

Definition 1. *Let U be a universe. A probabilistic set X over U is a set defined by a function μ_X representing a mapping*

$$\mu_X : U \to I = [0,1]$$

satisfying the following conditions:

(i) For each $\widetilde{U} \subset U$, $\sum_{u \in \widetilde{U}} \mu_X(u) \leq 1$

(ii) If $\widetilde{U} = U$, then $\sum_{u \in \widetilde{U}} \mu_X(u) = 1$ or $\sum_{u \in \widetilde{U}} \mu_X(u) = 0$

μ_X is called the the probabilistic membership function of X, and the value $\mu_X(u)$ is called the probabilistic grade of membership of $u \in U$. Thus a probabilistic set X over U can be represented as follows:

$$X = \{(\mu_X(u)/u) : u \in U\}.$$

Note that the set of all the probabilistic sets over U will be denoted by $\Pr(U)$.

Example 1. *Let $U = \{u_1, u_2, u_3, u_4\}$ be a universal set. Then*

$$X = \{(0.4/u_1), (0.1/u_2), (0.2/u_3), (0.3/u_4)\}$$

is a probabilistic set over U.

Definition 2. *A probabilistic set X over U is called empty probabilistic set if its membership function is zero everywhere in U and denoted by $\emptyset$. i.e,*

$$\mu_X : U \to I, \mu_X(u) = 0.$$

Example 2. *Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set. Then*

$$X = \{(0/u_1), (0/u_2), (0/u_3), (0/u_4), (0/u_5)\} = \emptyset$$

is an empty probabilistic set.

3. Probabilistic Soft Set

In this section, we define probabilistic soft sets and their operations. From now on, we will use $\Gamma_A^P, \Gamma_B^P, \ldots$ etc, for probabilistic soft sets and $\gamma_A^P, \gamma_B^P, \ldots$ etc. for their probabilistic approximate functions, respectively.

Throughout this work, U refers to an initial universe, E is a set of parameters and $A \subset E$.

Definition 3. *A probabilistic soft set (prs-set) Γ_A^P over U is a set defined by a function γ_A^P representing a mapping*

$$\gamma_A^P : E \to \Pr(U) \text{ such that } \gamma_A^P(x) = \emptyset \text{ if } x \notin A.$$

Here, γ_A^P is called probabilistic approximate function of the probabilistic soft set Γ_A^P. Hence probabilistic soft set Γ_A^P over U can be represented by the set of ordered pairs

$$\Gamma_A^P = \{(x, \gamma_A^P(x)) : x \in E, \gamma_A^P(x) \in \Pr(U)\}.$$

Note that the set of all probabilistic soft set Γ_A^P over U will be denoted by $\Pr S(U)$.

Example 3. *Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters. If $A = \{x_1, x_3, x_4\}$,*

$$\begin{aligned}
\gamma_A^P(x_1) &= \{0.9/u_2, 0.1/u_4\}\\
\gamma_A^P(x_3) &= \{0.2/u_1, 0.2/u_2, 0.2/u_3, 0.2/u_4, 0.2/u_5\}\\
\gamma_A^P(x_4) &= \{0.2/u_1, 0.4/u_3, 0.4/u_5\}
\end{aligned}$$

then the prs-set Γ_A^P is written

$$\begin{aligned}
\Gamma_A^P = \{&(x_1, \{0.9/u_2, 0.1/u_4\}),\\
&(x_3, \{0.2/u_1, 0.2/u_2, 0.2/u_3, 0.2/u_4, 0.2/u_5\}),\\
&(x_4, \{0.2/u_1, 0.4/u_3, 0.4/u_5\})\}.
\end{aligned}$$

Definition 4. *Let $\Gamma_A^P \in \Pr S(U)$. If $\gamma_A^P(x) = \emptyset$ for all $x \in A$ then Γ_A^P is called A—impossible prs-set, denoted by Γ_Φ^P.*

Example 4. *Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters. If $A = \{x_1, x_2\}$, and $\gamma_A^P(x_1) = \emptyset$, $\gamma_A^P(x_2) = \emptyset$, then probabilistic soft set Γ_A^P is an impossible prs-set, i.e. $\Gamma_A^P = \Gamma_\Phi^P$.*

Definition 5. *Let $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$. Then Γ_A^P is a prs-subset of Γ_B^P, denoted by $\Gamma_A^P \tilde{\subseteq} \Gamma_B^P$, if $A \subset B$ and $\gamma_A^P(x) \subseteq \gamma_B^P(x)$ for all $x \in A$.*

Remark 1. *As in the definition of the classical subset, $\Gamma_A^P \tilde{\subseteq} \Gamma_B^P$ does not imply that every element of Γ_A^P is an element of Γ_B^P.*

Example 5. *Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3\}$ is a set of all parameters.*

$$\begin{aligned}
x_1 &\to \{0.4/u_1, 0.2/u_2, 0.4/u_5\}\\
x_2 &\to \{0.2/u_1, 0.4/u_2, 0.1/u_3, 0.1/u_4, 0.2/u_5\}\\
x_3 &\to \{1/u_3\}
\end{aligned}$$

If $A = \{x_1\}, B = \{x_1, x_2\}$, then

$$\begin{aligned}
\gamma_A^P(x_1) &= \{0.4/u_1, 0.2/u_2\}\\
\gamma_B^P(x_1) &= \{0.4/u_1, 0.2/u_2, 0.4/u_5\}
\end{aligned}$$

$$\gamma_B^P(x_2) = \{0.2/u_1, 0.4/u_2, 0.1/u_3\}.$$

Hence

$$\Gamma_A^P = \{(x_1, \{0.4/u_1, 0.2/u_2\})\}$$
$$\Gamma_B^P = \{(x_1, \{0.4/u_1, 0.2/u_2, 0.4/u_5\}), (x_2, \{0.2/u_1, 0.4/u_2, 0.1/u_3\})\}$$

Then for all $x \in E$, $\gamma_A^P(x) \subseteq \gamma_B^P(x)$, *hence* $\Gamma_A^P \tilde{\subseteq} \Gamma_B^P$. *But it is clear that* $(x_1, \{0.4/u_1, 0.2/u_2\}) \in \Gamma_A^P$, *but* $(x_1, \{0.4/u_1, 0.2/u_2\}) \notin \Gamma_B^P$.

Proposition 1. *Let* $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$. *Then*

(i) $\Gamma_A^P \tilde{\subseteq} \Gamma_A^P$

(ii) $\Gamma_A^P \tilde{\subseteq} \Gamma_B^P$ *and* $\Gamma_B^P \tilde{\subseteq} \Gamma_C^P \Rightarrow \Gamma_A^P \tilde{\subseteq} \Gamma_C^P$.

Proof. They can be proved easily by using the probabilistic approximate function of prs-set. □

Definition 6. *Let* $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$. *Then* Γ_A^P *and* Γ_B^P *are prs-equal set written as* $\Gamma_A^P = \Gamma_B^P$, *if* Γ_A^P *is a prs-subset of* Γ_B^P *and* Γ_B^P *is a prs-subset of* Γ_A^P .

Proposition 2. *Let* $\Gamma_A^P, \Gamma_B^P, \Gamma_C^P \in \Pr S(U)$. *Then*

(i) $\Gamma_A^P = \Gamma_B^P$ *and* $\Gamma_B^P = \Gamma_C^P \Rightarrow \Gamma_A^P = \Gamma_C^P$

(ii) $\Gamma_A^P \tilde{\subseteq} \Gamma_B^P$ *and* $\Gamma_B^P \tilde{\subseteq} \Gamma_A^P \Leftrightarrow \Gamma_A^P = \Gamma_B^P$.

Proof. The proofs are straightforward. □

Definition 7. *Let* $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$. *Then the difference of* Γ_A^P *and* Γ_B^P, *denoted by* $\Gamma_A^P \tilde{\setminus} \Gamma_B^P$, *is defined by its probabilistic approximate functions:*

$$\gamma_{A \setminus B}^P(x) = \gamma_A^P(x) \setminus \gamma_B^P(x), \textit{ for all} x \in E.$$

Definition 8. *Let* $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$ *and* $\Gamma_A^P \tilde{\subseteq} \Gamma_B^P$. *Then the complement of* Γ_A^P *on* Γ_B^P, *denoted by* $(\Gamma_A^P)^c_{\Gamma_B^P}$, *is defined by*

$$(\gamma_A^P)^c_{\gamma_B^P}(x) = \gamma_B^P(x) \setminus \gamma_A^P(x), \textit{ for all } x \in E.$$

Example 6. *Let us consider Example 5. Then,*

$$(\Gamma_A^P)^c_{\Gamma_B^P} = \{(x_1, \{0.4/u_5\}), (x_2, \{0.2/u_1, 0.4/u_2, 0.1/u_3)\}.$$

Definition 9. *Let* $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$. *Then the union of* Γ_A^P *and* Γ_B^P, *denoted by* $\Gamma_A^P \tilde{\cup} \Gamma_B^P$, *is defined by its probabilistic approximate functions:*

$$\gamma_{A \cup B}^P(x) = \gamma_A^P(x) \cup \gamma_B^P(x), \textit{ for all } x \in E.$$

Example 7. *Assume that* $U = \{u_1, u_2, u_3, u_4, u_5\}$ *is a universal set and* $E = \{x_1, x_2, x_3, x_4\}$ *is a set of all parameters.*

$$\begin{aligned} x_1 &\rightarrow \{0.4/u_1, 0.2/u_2, 0.4/u_5\} \\ x_2 &\rightarrow \{0.2/u_1, 0.4/u_2, 0.1/u_3, 0.1/u_4, 0.2/u_5\} \\ x_3 &\rightarrow \{1/u_3\} \\ x_4 &\rightarrow \{0.2/u_1, 0.1/u_2, 0.3/u_3, 0.2/u_4, 0.2/u_5\} \end{aligned}$$

If $A = \{x_1, x_2\}, B = \{x_1, x_2, x_4\}$, *then*

$$\begin{aligned} \gamma_A^P(x_1) &= \{0.2/u_2, 0.4/u_5\} \\ \gamma_A^P(x_2) &= \{0.2/u_1, 0.1/u_4\} \\ \gamma_B^P(x_1) &= \{0.4/u_1, 0.2/u_2, 0.4/u_5\} \\ \gamma_B^P(x_2) &= \{0.4/u_2, 0.1/u_3\} \\ \gamma_B^P(x_4) &= \{0.2/u_4, 0.2/u_5\} \end{aligned}$$

Hence

$$\begin{aligned} \Gamma_A^P &= \{(x_1, \{0.2/u_2, 0.4/u_5\}), (x_2, \{0.2/u_1, 0.1/u_4\})\} \\ \Gamma_B^P &= \{(x_1, \{0.4/u_1, 0.2/u_2, 0.4/u_5\}), (x_2, \{0.4/u_2, 0.1/u_3\}), (x_4, \{0.2/u_4, 0.2/u_5\})\} \end{aligned}$$

It is clear that $A \cup B = \{x_1, x_2, x_4\}$ *and*

$$\begin{aligned} \gamma_{A\cup B}^P(x_1) &= \gamma_A^P(x_1) \cup \gamma_B^P(x_1) = \{0.4/u_1, 0.2/u_2, 0.4/u_5\} \\ \gamma_{A\cup B}^P(x_2) &= \gamma_A^P(x_2) \cup \gamma_B^P(x_2) = \{0.2/u_1, 0.4/u_2, 0.1/u_3, 0.1/u_4\} \\ \gamma_{A\cup B}^P(x_4) &= \gamma_A^P(x_4) \cup \gamma_B^P(x_4) = \{0.2/u_4, 0.2/u_5\} \end{aligned}$$

i.e.

$$\Gamma_A^P \widetilde{\cup} \Gamma_B^P = \{(x_1, \{0.4/u_1, 0.2/u_2, 0.4/u_5\}), (x_2, \{0.2/u_1, 0.4/u_2, 0.1/u_3, 0.1/u_4\}) (x_4, \{0.2/u_4, 0.2/u_5\})\}.$$

Proposition 3. *Let* $\Gamma_A^P, \Gamma_B^P, \Gamma_C^P \in \Pr S(U)$. *Then*

(i) $\Gamma_A^P \widetilde{\cup} \Gamma_A^P = \Gamma_A^P$

(ii) $\Gamma_A^P \widetilde{\cup} \Gamma_B^P = \Gamma_B^P \widetilde{\cup} \Gamma_A^P$

(iii) $(\Gamma_A^P \widetilde{\cup} \Gamma_B^P) \widetilde{\cup} \Gamma_C^P = \Gamma_A^P \widetilde{\cup} (\Gamma_B^P \widetilde{\cup} \Gamma_C^P)$.

Proof. The proofs can be proved easily by using the Definition 9. □

Definition 10. *Let* $\Gamma_A^P, \Gamma_B^P \in \Pr S(U)$. *Then the intersection of* Γ_A^P *and* Γ_B^P, *denoted by* $\Gamma_A^P \widetilde{\cap} \Gamma_B^P$, *is defined by its probabilistic approximate functions:*

$$\gamma_{A\cap B}^P(x) = \gamma_A^P(x) \cap \gamma_B^P(x), \text{ for all } x \in A \cap B,\ A \cap B \neq \emptyset.$$

Example 8. *Let us consider Example 7. Then* $A \cap B = \{x_1, x_2\}$ *and*

$$\gamma^P_{A\cap B}(x_1) = \gamma^P_A(x_1) \cap \gamma^P_B(x_1) = \{0.2/u_2, 0.4/u_5\}$$
$$\gamma^P_{A\cap B}(x_2) = \gamma^P_A(x_2) \cap \gamma^P_B(x_2) = \emptyset$$

Hence

$$\Gamma^P_A \widetilde{\cap} \Gamma^P_B = \{(x_1, \{0.2/u_2, 0.4/u_5\})\}$$

is obtained.

Proposition 4. *Let* $\Gamma^P_A, \Gamma^P_B, \Gamma^P_C \in \Pr S(U)$. *Then*

(i) $\Gamma^P_A \widetilde{\cap} \Gamma^P_A = \Gamma^P_A$

(ii) $\Gamma^P_A \widetilde{\cap} \Gamma^P_B = \Gamma^P_B \widetilde{\cap} \Gamma^P_A$

(iii) $\left(\Gamma^P_A \widetilde{\cap} \Gamma^P_B\right) \widetilde{\cap} \Gamma^P_C = \Gamma^P_A \widetilde{\cap} \left(\Gamma^P_B \widetilde{\cap} \Gamma^P_C\right)$.

Proof. The proofs can be proved easily by using the Definition 10. □

Proposition 5. *Let* $\Gamma^P_A, \Gamma^P_B, \Gamma^P_C \in \Pr S(U)$ *and* $\Gamma^P_A, \Gamma^P_B \widetilde{\subseteq} \Gamma^P_C$. *Then, De Morgan's laws for* Γ^P_A, Γ^P_B *are valid as follows:*

(i) $\left(\Gamma^P_A \widetilde{\cap} \Gamma^P_A\right)^c_{\Gamma^P_C} = \left(\Gamma^P_A\right)^c_{\Gamma^P_C} \widetilde{\cup} \left(\Gamma^P_B\right)^c_{\Gamma^P_C}$

(ii) $\left(\Gamma^P_A \widetilde{\cup} \Gamma^P_B\right)^c_{\Gamma^P_C} = \left(\Gamma^P_A\right)^c_{\Gamma^P_C} \widetilde{\cap} \left(\Gamma^P_B\right)^c_{\Gamma^P_C}$

Proof. The proofs can be proved easily by using the respective probabilistic approximate functions. So, we only prove *(i)* case. For all $x \in E$,

$$\begin{aligned}\left(\gamma^P_{A\cap B}\right)^c_{\gamma^P_C}(x) &= \left(\gamma^P_A \cap \gamma^P_B\right)^c_{\gamma^P_C}(x) = \gamma^P_C(x) \setminus \left(\gamma^P_A \cap \gamma^P_B\right)(x) \\ &= \left(\gamma^P_C(x) \setminus \gamma^P_A(x)\right) \cup \left(\gamma^P_C(x) \setminus \gamma^P_B(x)\right) \\ &= \left(\gamma^P_A\right)^c_{\gamma^P_C}(x) \cup \left(\gamma^P_B\right)^c_{\gamma^P_C}(x).\end{aligned}$$

□

Definition 11. *Let* $\Gamma^P_A, \Gamma^P_B \in \Pr S(U)$. *Then the symmetric difference of* Γ^P_A *and* Γ^P_B, *denoted by* $\Gamma^P_A \widetilde{\Delta} \Gamma^P_B$, *is defined by its probabilistic approximate functions:*

$$\gamma^P_A(x) \Delta \gamma^P_B(x) = \left(\gamma^P_A(x) \setminus \gamma^P_B(x)\right) \cup \left(\gamma^P_B(x) \setminus \gamma^P_A(x)\right), \text{ for all } x \in E.$$

Example 9. *Let us consider Example 7. Then*

$$\Gamma^P_A \widetilde{\Delta} \Gamma^P_B = \left\{(x_1, \{0.4/u_1\}), (x_2, \{0.2/u_1, 0.4/u_2, 0.1/u_3, 0.1/u_4\}) (x_4, \{0.2/u_4, 0.2/u_5\})\right\}.$$

is obtained.

Proposition 6. *Let* $\Gamma_A^P, \Gamma_B^P, \Gamma_C^P \in \Pr S(U)$*. The following conditions are satisfied:*

(i) $\Gamma_A^P \widetilde{\Delta} \Gamma_B^P = \Gamma_B^P \widetilde{\Delta} \Gamma_A^P$

(ii) $\left(\Gamma_A^P \widetilde{\Delta} \Gamma_B^P\right) \widetilde{\Delta} \Gamma_C^P = \Gamma_A^P \widetilde{\Delta} \left(\Gamma_B^P \widetilde{\Delta} \Gamma_C^P\right)$

(iii) $\Gamma_A^P = \Gamma_B^P \Leftrightarrow \Gamma_A^P \widetilde{\Delta} \Gamma_B^P = \Gamma_\Phi^P$.

Proof. The proofs can be proved easily by using the respective probabilistic approximate functions. So, we only prove *(i)* case. For all $x \in E$,

$$\begin{aligned} \gamma_A^P(x) \Delta \gamma_B^P(x) &= \left(\gamma_A^P(x) \setminus \gamma_B^P(x)\right) \cup \left(\gamma_B^P(x) \setminus \gamma_A^P(x)\right) \\ &= \left(\gamma_B^P(x) \setminus \gamma_A^P(x)\right) \cup \left(\gamma_A^P(x) \setminus \gamma_B^P(x)\right) \\ &= \gamma_B^P(x) \Delta \gamma_A^P(x) \end{aligned}$$

i.e., $\Gamma_A^P \widetilde{\Delta} \Gamma_B^P = \Gamma_B^P \widetilde{\Delta} \Gamma_A^P$ is obtained. □

4. Conclusion

In this paper, we study the theory of probabilistic soft sets. We give some operations such as union, intersection, difference and symmetric difference. We prove that certain De Morgan's laws hold in probabilistic soft set theory with respect to these new definitions.

In addition, this theory not only provides a significant addition to existing theories for handling uncertainties, but also leads to potential areas of further research.

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