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Binary Soft Set Theory

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Abstract. In this paper, we study the concept of soft sets theory which introduced by Molodtsov [9] and the theory of soft sets which studied by Maji *et al.* [6]. The authors introduce a binary soft set on two initial universal sets and a parameter set. Also subset of a binary soft set, soft super set of a binary soft set, equality of two binary soft sets, complement of a binary soft set, null binary soft set, absolute binary soft set, union of two binary soft sets, intersection of two binary soft sets, difference and symmetric difference of two binary soft sets, *AND* and *OR* operations via two binary soft sets are defined by authors in the present paper. Also some properties of binary soft sets are investigated.

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1. Introduction

Some set theories such as theory of fuzzy sets [13], rough sets [10], intuitionistic fuzzy sets [3], vague sets [5] etc. can be deal with unclear notions. But, these theories are not sufficient to solve some difficulties and problems. There are some vague problems in economics, medical science, social science, finance etc. Then, what is the reason of vague problems and difficulties? It is possible the insufficiency of the parametrization tool of the theories. In 1999, Molodtsov [9] introduced the idea of soft set theory as a general mathematical tool for coping with these difficulties. In 2001, Maji, Biswas and Roy [7] defined the concept of a fuzzy soft set and [8] an intuitionistic fuzzy soft set. In 2003, Maji *et al.* [6] studied the theoretical concepts of the soft set theory. In 2009, Ali *et al.* [1] investigated several operations on soft sets and defined some new notions such as the restricted union etc. In 2010, Xu *et al.* [12] introduced vague soft sets and studied some properties of them. In 2010, Feng *et al.* [4] studied soft sets combined with fuzzy sets and rough sets as a tentative approach. In 2011, Shabir *et al.* [2] introduced algebraic structures of soft sets via new notions. In 2011, Naz *et al.* [11] defined some notions such as soft topological space, soft interior, soft closure etc.

The aim of this present paper is to introduce the binary soft set on two initial universal sets and investigated some properties. This paper is organized as below: In Section 2 we

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give known definitions and a proposition related soft set theory. In Section 3 we introduce the concept of a binary soft set and the operations on binary soft sets such as binary subset, binary equality, union of two binary soft sets, intersection of two binary soft sets, difference of two binary soft sets, symmetric difference of two binary soft sets etc. In Section 4 we give a characteristic function of binary soft set. The last section is the conclusion.

2. Preliminaries

In this section we recall some definitions and a proposition.

Definition 1 ([9]). *A pair (F,A) is called a soft set over X as follows:*

$$F : A \to P(X).$$

Definition 2 ([6]). *Let (F,A) and (G,B) be two soft sets. Then (F,A) is said to be a soft subset of (G,B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. It is denoted by $(F,A)\widetilde{\subseteq}(G,B)$. (F,A) is said to be soft equal to (G,B) if $(F,A)\widetilde{\subseteq}(G,B)$ and $(G,B)\widetilde{\subseteq}(F,A)$. It is denoted by $(F_1,A) = (F_2,A)$.*

Definition 3 ([1]). *The complement of a soft set (F,A) is defined as $(F,A)^c = (F^c,A)$, where $F^c(e) = (F(e))^c = X - F(e)$, for all $e \in A$.*

Definition 4 ([11]). *The difference of two soft sets (F,A) and (G,A) is defined by $(F,A)-(G,A) = (F-G,A)$, where $(F-G)(e) = F(e)-G(e)$, for all $e \in A$.*

Definition 5 ([11]). *Let (F,A) be a soft set over X and $x \in X$. x is said to be in the soft set (F,E) denoted by $x \in (F,A)$ if $x \in F(e)$ for all $e \in A$.*

Definition 6 ([6]). *A soft set (F,A) over X is said to be a null soft set if $F(e) = \emptyset$, for all $e \in A$. This is denoted by $\widetilde{\emptyset}$.*

Definition 7 ([6]). *A soft set (F,A) over X is said to be an absolute soft set if $F(e) = X$, for all $e \in A$. This is denoted by $\widetilde{X}$.*

Definition 8 ([6]). *The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$ and $H(e) = F(e)$ if $e \in A-B$ or $H(e) = G(e)$ if $e \in B-A$ or $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$ for all $e \in C$.*

Definition 9 ([6]). *The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.*

Definition 10 ([6]). *Let $E = \{e_i : 1 \leq i \leq n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E = \{\rceil e_i : 1 \leq i \leq n\}$ where $\rceil e_i =$ not e_i for each i.*

Proposition 1 ([6]). *Let $A, B \subseteq E$ be parameter sets.*

(i) $\rceil(\rceil A) = A$.

(ii) $\rceil(A \cup B) = \rceil A \cup \rceil B$.

(iii) $\rceil(A \cap B) = \rceil A \cap \rceil B$.

Definition 11 ([6]). *Let (F,A) and (G,B) be two soft sets. Then "$(F,A) AND (G,B)$" denoted by $(F,A) \wedge (G,B)$ is defined by $(F,A) \wedge (G,B) = (H, A \times B)$, where $H(e,f) = F(e) \cap G(e)$ for each $(e,f) \in A \times B$.*

Definition 12 ([6]). *Let (F,A) and (G,B) be two soft sets. Then "$(F,A) OR (G,B)$" denoted by $(F,A) \vee (G,B)$ is defined by $(F,A) \vee (G,B) = (O, A \times B)$, where $O(e,f) = F(e) \cup G(e)$ for each $(e,f) \in A \times B$.*

3. Binary Soft Sets

Let U_1, U_2 be two initial universe sets and E be a set of parameters. Let $P(U_1), P(U_2)$ denote the power set of U_1, U_2, respectively. Also, let $A, B, C \subseteq E$.

Definition 13. *A pair (F,A) is said to be a binary soft set over U_1, U_2, where F is defined as below:*

$$F : A \to P(U_1) \times P(U_2),$$
$$F(e) = (X, Y) \text{ for each } e \in A \text{ such that } X \subseteq U_1, Y \subseteq U_2.$$

Example 1. *Consider the following sets:*

$$U_1 = \{t_1, t_2, t_3, t_4, t_5\} \text{ is the set of trousers.}$$
$$U_2 = \{b_1, b_2, b_3, b_4, b_5\} \text{ is the set of blouses.}$$
$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}.$$

E is the set of parameters, where e_1: expensive, e_2: cheap, e_3: sport, e_4: classic, e_5: colorful, e_6: plain, e_7: small, e_8: large.

The binary soft set (F,A) describes "the special features of both the trousers and the blouses" which Mrs. X is going to buy, where $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. (F,A) is a binary soft set over U_1, U_2 defined as follows:

$$F(e_1) = (\{t_1, t_2\}, \{b_1, b_3\}),$$
$$F(e_2) = (\{t_3, t_4\}, \{b_2, b_4, b_5\}),$$
$$F(e_3) = (\{t_2, t_3, t_5\}, \{b_1, b_5\}),$$
$$F(e_4) = (\{t_1, t_4\}, \{b_2, b_3\}).$$

So, we can say the binary soft set

$$\begin{aligned}(F,A) = \{&\text{expensive trousers, blouses : resp. } \{t_1, t_2\}, \{b_1, b_3\};\\ &\text{cheap trousers, blouses : resp. } \{t_3, t_4\}, \{b_2, b_4, b_5\};\\ &\text{sport trousers, blouses : resp. } \{t_2, t_3, t_5\}, \{b_1, b_5\};\\ &\text{classic trousers, blouses : resp. } \{t_1, t_4\}, \{b_2, b_3\}\}.\end{aligned}$$

We denote the binary soft set (F,A) as below:

$$(F,A) = \{(e_1,(\{t_1,t_2\},\{b_1,b_3\})),(e_2,(\{t_3,t_4\},\{b_2,b_4,b_5\})),(e_3,(\{t_2,t_3,t_5\},\{b_1,b_5\})),\\ (e_4,(\{t_1,t_4\},\{b_2,b_3\}))\}.$$

In this example, we can see the views of Mrs. X who wants to buy both trousers and blouses under the same parameters.

Definition 14. *Let (F,A), (G,B) be two binary soft sets over the common U_1, U_2. (F,A) is called a binary soft subset of (G,B) if*

(i) $A \subseteq B$,

(ii) $X_1 \subseteq X_2$ *and* $Y_1 \subseteq Y_2$ *such that* $F(e) = (X_1,Y_1)$, $G(e) = (X_2,Y_2)$ *for each* $e \in A$.

We denote it $(F,A)\,\widetilde{\widetilde{\subseteq}}\,(G,B)$, *briefly.*

(F,A) *is called a binary soft super set of* (G,B) *if* (G,B) *is a binary soft subset of* (F,A). *We write* $(F,A)\,\widetilde{\widetilde{\supseteq}}\,(G,B)$.

Example 2. *Let* $U_1 = \{t_1,t_2,t_3,t_4,t_5\}$, $U_2 = \{b_1,b_2,b_3,b_4,b_5\}$ *and* $E = \{e_1,e_2,e_3,e_4,e_5\}$. *Let* $A = \{e_1,e_2,e_3\} \subseteq E$ *and* $B = \{e_1,e_2,e_3,e_4\} \subseteq E$.

$(F,A),(G,B)$ *are two binary soft sets over* U_1,U_2 *defined as follows:*

$$(F,A) = \{(e_1,(\{t_1,t_2\},\{b_1\})),(e_2,(\{t_3\},\{b_3,b_4\})),(e_3,(\{t_1,t_4\},\{b_1,b_2\}))\},$$
$$(G,B) = \{(e_1,(\{t_1,t_2,t_3\},\{b_1\})),(e_2,(\{t_1,t_3\},\{b_3,b_4,b_5\})),(e_3,(\{t_1,t_3,t_4\},U_2)),(e_4,(U_1,U_2))\}.$$

Therefore, $(F,A)\,\widetilde{\widetilde{\subseteq}}\,(G,B)$.

Definition 15. *Let (F,A), (G,B) be two binary soft sets over the common U_1, U_2. (F,A) is called a binary soft equal of (G,B) if (F,A) is a binary soft subset of (G,B) and (G,B) is a binary soft subset of (F,A). We denote it $(F,A) = (G,B)$.*

Definition 16. *The complement of a binary soft set (F,A) is denoted by $(F,A)^c$ and is defined $(F,A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to P(U_1) \times P(U_2)$ is a mapping given by $F^c(e) = (U_1 - X, U_2 - Y)$ such that $F(e) = (X,Y)$. Clearly, $((F,A)^c)^c = (F,A)$.*

Example 3. *Consider Example 1. Then*

$$(F,A)^c = \{\text{not expensive trousers, blouses: resp.}\{t_3,t_4,t_5\},\{b_2,b_4,b_5\};\\ \text{not cheap trousers, blouses: resp.}\{t_1,t_2,t_5\},\{b_1,b_3\};\\ \text{not sport trousers, blouses: resp.}\{t_1,t_4\},\{b_2,b_3,b_4\};\\ \text{not classic trousers, blouses: resp.}\{t_2,t_3,t_5\},\{b_1,b_4,b_5\}\}.$$

Definition 17. *A binary soft set (F,A) over U_1, U_2 is called a binary null soft set denoted by $\widetilde{\widetilde{\emptyset}}$ if $F(e) = (\emptyset,\emptyset)$ for each $e \in A$.*

Example 4. *Consider the following sets:*

$$\begin{aligned} U_1 =& \{j_1, j_2, j_3\} \text{ is the set of jeans,}\\ U_2 =& \{t_1, t_2, t_3, t_4\} \text{ is the set of t-shirts,}\\ A =& \{e_1 = \text{expensive, } e_2 = \text{smart, } e_3 = \text{beautiful }\}, \end{aligned}$$

Where A is the set of parameters. Let (F,A) be a binary soft set as follows:

$$(F,A) = \{(e_1,(\emptyset,\emptyset)),(e_2,(\emptyset,\emptyset)),(e_3,(\emptyset,\emptyset))\}.$$

Therefore, (F,A) is a binary null soft set.

Definition 18. *A binary soft set (F,A) over U_1, U_2 is called a binary absolute soft set denoted by $\widetilde{\widetilde{A}}$ if $F(e) = (U_1, U_2)$ for each $e \in A$.*

Example 5. *Let U_1, U_2 and A be sets as in Example 4. Let (F,A) be a binary soft set as follows:*

$$(F,A) = \{(e_1,(U_1,U_2)),(e_2,(U_1,U_2)),(e_3,(U_1,U_2))\}.$$

Therefore, (F,A) is a binary absolute soft set. Clearly, $(\widetilde{\widetilde{A}})^c = \widetilde{\widetilde{\emptyset}}$ and $(\widetilde{\widetilde{\emptyset}})^c = \widetilde{\widetilde{A}}$.

Definition 19. *Union of two binary soft sets (F,A) and (G,B) over the common U_1, U_2 is the binary soft set (H,C), where $C = A \cup B$, and for each $e \in C$,*

$$H(e) = \begin{cases} (X_1, Y_1), & e \in A - B \\ (X_2, Y_2), & e \in B - A \\ (X_1 \cup X_2, Y_1 \cup Y_2), & e \in A \cap B \end{cases}$$

such that $F(e) = (X_1,Y_1)$ for each $e \in A$ and $G(e) = (X_2,Y_2)$ for each $e \in B$. We denote it $(F,A)\widetilde{\widetilde{\cup}}(G,B) = (H,C)$.

Example 6. *Consider the following sets:*

$$\begin{aligned} U_1 =& \{s_1,s_2,s_3,s_4,s_5,s_6\} \text{ is the set of shoes,}\\ U_2 =& \{p_1,p_2,p_3,p_4\} \text{ is the set of purses,}\\ E =& \{e_1 = \text{expensive, } e_2 = \text{cheap, } e_3 = \text{black,}\\ & e_4 = \text{brown, } e_5 = \text{leather, } e_6 = \text{sport, } e_7 = \text{classic, } e_8 = \text{smart}\}. \end{aligned}$$

Let $A = \{e_1, e_3, e_5\} \subseteq E$ and $B = \{e_3, e_4, e_6, e_8\} \subseteq E$. Let $(F,A),(G,B)$ be two binary soft sets as follows:

$$\begin{aligned} (F,A) =& \{(e_1,(\{s_1,s_2\},\{p_2\})),(e_3,(\{s_4,s_5,s_6\},\{p_1,p_3\})),(e_5,(\{s_2,s_4,s_6\},\{p_2,p_4\}))\},\\ (G,B) =& \{(e_3,(\{s_4,s_5\},\{p_1,p_4\})),(e_4,(\{s_1\},\{p_2\})),(e_6,(\{s_1,s_2\},\{p_4\})),(e_8,(\{s_5\},\{p_1\}))\}. \end{aligned}$$

Then $(H,C) = (F,A)\widetilde{\widetilde{\cup}}(G,B)$ is the binary soft set as below such that $C = A \cup B$:

$$\begin{aligned} (H,C) =& \{(e_1,(\{s_1,s_2\},\{p_2\})),(e_3,(\{s_4,s_5,s_6\},\{p_1,p_3,p_4\})),(e_4,(\{s_1\},\{p_2\})),\\ & (e_5,(\{s_2,s_4,s_6\},\{p_2,p_4\})),(e_6,(\{s_1,s_2\},\{p_4\})),(e_8,(\{s_5\},\{p_1\}))\}. \end{aligned}$$

Definition 20. *Intersection of two binary soft sets (F,A) and (G,B) over the common U_1, U_2 is the binary soft set (H,C), where $C = A \cap B$, and $H(e) = (X_1 \cap X_2, Y_1 \cap Y_2)$ for each $e \in C$ such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each $e \in B$. We denote it $(F,A)\tilde{\tilde{\cap}}(G,B) = (H,C)$.*

Example 7. *In the Example 6, intersection of two binary soft sets (F,A) and (G,B) is the binary soft set (H,C), where $C = A \cap B = \{e_3\}$ and $(H,C) = \{(e_3, (\{s_4, s_5\}, \{p_1\}))\}$.*

Proposition 2. *Let (F,A), (G,B) and (H,C) be three binary soft sets. Then we have the following results:*

(i) $(F,A)\tilde{\tilde{\cup}}(F,A) = (F,A)$.

(ii) $(F,A)\tilde{\tilde{\cup}}(G,B) = (G,B)\tilde{\tilde{\cup}}(F,A)$.

(iii) $(F,A)\tilde{\tilde{\cup}}((G,B)\tilde{\tilde{\cup}}(H,C)) = ((F,A)\tilde{\tilde{\cup}}(G,B))\tilde{\tilde{\cup}}(H,C)$.

(iv) $(F,A)\tilde{\tilde{\cup}}\tilde{\tilde{\emptyset}} = (F,A)$.

(v) $(F,A)\tilde{\tilde{\cup}}\tilde{\tilde{A}} = \tilde{\tilde{A}}$.

(vi) $(F,A)\tilde{\tilde{\subseteq}}(F,A)\tilde{\tilde{\cup}}(G,B)$ *and* $(G,B)\tilde{\tilde{\subseteq}}(F,A)\tilde{\tilde{\cup}}(G,B)$.

(vii) $(F,A)\tilde{\tilde{\cup}}(G,B) = \tilde{\tilde{\emptyset}}$ *if and only if* $(F,A) = \tilde{\tilde{\emptyset}}$ *and* $(G,B) = \tilde{\tilde{\emptyset}}$.

(viii) $(F,A)\tilde{\tilde{\subseteq}}(G,B)$ *if and only if* $(F,A)\tilde{\tilde{\cup}}(G,B) = (G,B)$.

Proof. It is obvious from Definition 19. □

Proposition 3. *Let (F,A), (G,B) and (H,C) be three binary soft sets. Then we have the following results:*

(i) $(F,A)\tilde{\tilde{\cap}}(F,A) = (F,A)$.

(ii) $(F,A)\tilde{\tilde{\cap}}(G,B) = (G,B)\tilde{\tilde{\cap}}(F,A)$.

(iii) $(F,A)\tilde{\tilde{\cap}}((G,B)\tilde{\tilde{\cap}}(H,C)) = ((F,A)\tilde{\tilde{\cap}}(G,B))\tilde{\tilde{\cap}}(H,C)$.

(iv) $(F,A)\tilde{\tilde{\cap}}\tilde{\tilde{\emptyset}} = \tilde{\tilde{\emptyset}}$.

(v) $(F,A)\tilde{\tilde{\cap}}\tilde{\tilde{A}} = (F,A)$.

(vi) $(F,A)\tilde{\tilde{\cap}}(G,B)\tilde{\tilde{\subseteq}}(F,A)$ *and* $(F,A)\tilde{\tilde{\cup}}(G,B)\tilde{\tilde{\subseteq}}(G,B)$.

(vii) $(F,A)\tilde{\tilde{\cap}}(G,B) = \tilde{\tilde{\emptyset}}$ *if and only if* $(F,A) = \tilde{\tilde{\emptyset}}$ *or* $(G,B) = \tilde{\tilde{\emptyset}}$.

(viii) $(F,A)\tilde{\tilde{\subseteq}}(G,B)$ *if and only if* $(F,A)\tilde{\tilde{\cap}}(G,B) = (F,A)$.

Proof. It is obvious from Definition 20. □

Proposition 4. *Let (F,A) and (G,B) be two binary soft sets. Then we have the following results:*

(i) $(F,A)\widetilde{\widetilde{\cup}}(F,A)^c = \widetilde{\widetilde{A}}$.

(ii) $(F,A)\widetilde{\widetilde{\cap}}(F,A)^c = \widetilde{\widetilde{\emptyset}}$.

(iii) $(F,A)\widetilde{\widetilde{\subseteq}}(G,B)$ *if and only if* $(G,B)^c\widetilde{\widetilde{\subseteq}}(F,A)^c$.

(iv) $((F,A)\widetilde{\widetilde{\cup}}(G,B))^c = (F,A)^c\widetilde{\widetilde{\cup}}(G,B)^c$.

(v) $((F,A)\widetilde{\widetilde{\cap}}(G,B))^c = (F,A)^c\widetilde{\widetilde{\cap}}(G,B)^c$.

Proof.

(i) It is obvious from Definitions 16, 19, and 20.

(ii) It is obvious from Definitions 16, 19, and 20.

(iii) It is obvious from Definitions 16, 19, and 20.

(iv) Let $(F,A)\widetilde{\widetilde{\cup}}(G,B) = (H,A\cup B)$, where for each $e \in A\cup B$

$$H(e) = \begin{cases} (X_1,Y_1), & e \in A-B \\ (X_2,Y_2), & e \in B-A \\ (X_1\cup X_2, Y_1\cup Y_2), & e \in A\cap B \end{cases}$$

such that $F(e) = (X_1,Y_1)$ for each $e \in A$ and $G(e) = (X_2,Y_2)$ for each $e \in B$. Hence, $((F,A)\widetilde{\widetilde{\cup}}(G,B))^c = (H,A\cup B)^c = (H^c, \rceil A\cup\rceil B)$.

Now, $H^c(\rceil e) = (U_1 - X, U_2 - Y)$ for each $\rceil e \in \rceil A\cup\rceil B$ such that $H(e) = (X,Y)$. Therefore,

$$H^c(\rceil e) = \begin{cases} (U_1-X_1, U_2-Y_1), & \rceil e \in \rceil A-\rceil B \\ (U_1-X_2, U_2-Y_2), & \rceil e \in \rceil B-\rceil A \\ (U_1-(X_1\cup X_2), U_2-(Y_1\cup Y_2)), & \rceil e \in \rceil A\cap\rceil B \end{cases} \quad (1)$$

Now, $(F,A)^c\widetilde{\widetilde{\cup}}(G,B)^c = (F^c, \rceil A)\widetilde{\widetilde{\cup}}(G^c, \rceil B) = (K, \rceil A\cup\rceil B)$, where

$$K(\rceil e) = \begin{cases} (U_1-X_1, U_2-Y_1), & \rceil e \in \rceil A-\rceil B \\ (U_1-X_2, U_2-Y_2), & \rceil e \in \rceil B-\rceil A\,. \\ (U_1-(X_1\cup X_2), U_2-(Y_1\cup Y_2)), & \rceil e \in \rceil A\cap\rceil B \end{cases} \quad (2)$$

Finally, H^c and K are same. So, proof is completed.

(v) It is proved by a similar way.

□

Proposition 5. *Let (F,A), (G,B) and (H,C) be three binary soft sets. Then we have the following results:*

(i) $(F,A)\tilde{\tilde{\cup}}((G,B)\tilde{\tilde{\cap}}(H,C)) = ((F,A)\tilde{\tilde{\cup}}(G,B))\tilde{\tilde{\cap}}((F,A)\tilde{\tilde{\cup}}(H,C))$.

(ii) $(F,A)\tilde{\tilde{\cap}}((G,B)\tilde{\tilde{\cup}}(H,C)) = ((F,A)\tilde{\tilde{\cap}}(G,B))\tilde{\tilde{\cup}}((F,A)\tilde{\tilde{\cap}}(H,C))$.

Proof. It is obvious from Definitions 7 and 8. □

Definition 21. *The difference of two binary soft sets (F,A) and (G,A) over the common U_1, U_2 is the binary soft set (H,A), where $H(e) = (X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that $(F,A) = (X_1, Y_1)$ and $(G,A) = (X_2, Y_2)$.*

Example 8. *Consider the following sets:*

$$U_1 = \{c_1, c_2, c_3, c_4, c_5\} \text{ is the set of computers.}$$
$$U_2 = \{m_1, m_2, m_3, m_4, m_5\} \text{ is the set of mobile phones.}$$
$$E = \{e_1 = \text{expensive},\ e_2 = \text{outlook},\ e_3 = \text{functions}\}.$$

Let (F,E), (G,E) be two binary soft sets as follows:

$$(F,E) = \{(e_1,(\{c_1,c_3\},\{m_2,m_3\})),(e_2,(\{c_4\},\{m_1,m_5\})),(e_3,(\{c_3,c_4\},\{m_2\}))\}.$$
$$(G,E) = \{(e_1,(\{c_1,c_4\},\{m_1\})),(e_2,(\{c_4\},\{m_2,m_5\})),(e_3,(\{c_4\},\{m_2\}))\}.$$

Then $(H,E) = \{(e_1,(\{c_3\},\{m_2,m_3\})),(e_2,(\emptyset,\{m_1\})),(e_3,(\{c_3\},\emptyset))\}$.

Definition 22. *The symmetric difference of two binary soft sets (F,A) and (G,A) over the common U_1, U_2 is the binary soft set (H,A) defined $(H,A) = ((F,A)-(G,A))\tilde{\tilde{\cup}}((G,A)-(F,A))$. We denote it $(H,A) = (F,A)\Delta(G,A)$.*

Example 9. *In the Example 8, symmetric difference of two binary soft sets (F,E) and (G,E) is the binary soft set (H,E) as follows:*

$$(H,E) = \{(e_1,(\{c_3,c_4\},\{m_1,m_2,m_3\})),(e_2,(\emptyset,\{m_1,m_2\})),(e_3,(\{c_3\},\emptyset))\}.$$

Proposition 6. *Let (F,A), (G,A) and (H,A) be three binary soft sets. Then we have the following results:*

(i) $\tilde{\tilde{A}} - \tilde{\tilde{\emptyset}} = \tilde{\tilde{A}}$ *and* $\tilde{\tilde{A}} - \tilde{\tilde{A}} = \tilde{\tilde{\emptyset}}$.

(ii) $\tilde{\tilde{A}} - (F,A)^c = (F,A)$.

(iii) $(F,A)\tilde{\tilde{\subseteq}}(G,A)$ *if and only if* $(G,A)^c\tilde{\tilde{\subseteq}}(F,A)^c$.

(iv) $(F,A)\tilde{\tilde{\cap}}(G,A)$ *if and only if* $(F,A)\tilde{\tilde{\subseteq}}(G,A)^c$ *if and only if* $(G,A)\tilde{\tilde{\subseteq}}(F,A)^c$.

(v) $(F,A)\tilde{\tilde{\cup}}(G,A)=\tilde{\tilde{A}}$, $(F,A)\tilde{\tilde{\cap}}(G,A)=\tilde{\tilde{\emptyset}}$ *if and only if* $(F,A)=(G,A)^c$.

(vi) $((F,A)\tilde{\tilde{\cup}}(G,A))^c=(F,A)^c\ \tilde{\tilde{\cap}}\ (G,A)^c$.

(vii) $((F,A)\tilde{\tilde{\cap}}(G,A))^c=(F,A)^c\ \tilde{\tilde{\cup}}\ (G,A)^c$.

(viii) *If* $(F,A)\tilde{\tilde{\subseteq}}(G,A)$, $(F,A)\tilde{\tilde{\cup}}(H,A)\ \tilde{\tilde{\subseteq}}\ (G,A)\tilde{\tilde{\cup}}(H,A)$.

(ix) *If* $(F,A)\tilde{\tilde{\subseteq}}(G,A)$, $(F,A)\tilde{\tilde{\cap}}(H,A)\ \tilde{\tilde{\subseteq}}\ (G,A)\tilde{\tilde{\cap}}(H,A)$.

(x) *If* $(F,A)\tilde{\tilde{\subseteq}}(G,A)$ *and* $(F,A)\tilde{\tilde{\subseteq}}(H,A)$, $(F,A)\tilde{\tilde{\subseteq}}(G,A)\tilde{\tilde{\cap}}(H,A)$.

(xi) *If* $(F,A)\tilde{\tilde{\subseteq}}(G,A)$ *and* $(F,A)\tilde{\tilde{\subseteq}}(H,A)$, $(F,A)\tilde{\tilde{\cup}}(G,A)\tilde{\tilde{\subseteq}}(H,A)$.

(xii) $(F,A)-((G,A)-(H,A))=(F,A)-((G,A)\tilde{\tilde{\cup}}(H,A))$.

(xiii) $(F,A)-((G,A)\tilde{\tilde{\cap}}(H,A))=((F,A)-(G,A))\ \tilde{\tilde{\cup}}\ (F,A)-(H,A))$.

(xiv) $(F,A)\Delta\tilde{\tilde{\emptyset}}=(F,A)$, $(F,A)\Delta(F,A)=\tilde{\tilde{\emptyset}}$, $(F,A)\Delta(G,A)=(G,A)\Delta(F,A)$.

(xv) $(F,A)\Delta((G,A)\Delta(H,A))=((F,A)\Delta(G,A))\Delta(H,A)$.

(xvi) $(F,A)\tilde{\tilde{\cap}}((G,A)\Delta(H,A))=((F,A)\tilde{\tilde{\cap}}(G,A))\ \Delta\ ((F,A)\tilde{\tilde{\cap}}(H,A))$.

Proof. It is obvious from Definitions 16, 19, 20, 21, and 22. □

Definition 23. *If (F,A) and (G,B) are two binary soft sets then "$(F,A)AND(G,B)$" denoted by $(F,A)\tilde{\tilde{\wedge}}(G,B)$ is defined by $(F,A)\tilde{\tilde{\wedge}}(G,B)=(H,A\times B)$, where $H(e,f)=(X_1\cap X_2,Y_1\cap Y_2)$ for each $(e,f)\in A\times B$ such that $F(e)=(X_1,Y_1)$ and $G(e)=(X_2,Y_2)$.*

Example 10. *In the Example 6, $(H,A\times B)=(F,A)\tilde{\tilde{\wedge}}(G,B)$ is the binary soft set as follows:*

$$\begin{aligned}(H,A\times B)=\{&((e_1,e_3),(\emptyset,\emptyset)),((e_1,e_4),(\{s_1\},\{p_2\})),((e_1,e_6),(\{s_1,s_2\},\emptyset)),((e_1,e_8),(\emptyset,\emptyset)),\\&((e_3,e_3),(\{s_4,s_5\},\{p_1\})),((e_3,e_4),(\emptyset,\emptyset)),((e_3,e_6),(\emptyset,\emptyset)),((e_3,e_8),(\{s_5\},\{p_1\})),\\&((e_5,e_3),(\{s_4\},\{p_4\})),((e_5,e_4),(\emptyset,\{p_2\})),((e_5,e_6),(\{s_2\},\{p_4\})),((e_5,e_8),(\emptyset,\emptyset))\}.\end{aligned}$$

Definition 24. *If (F,A) and (G,B) are two binary soft sets then "$(F,A)OR(G,B)$" denoted by $(F,A)\tilde{\tilde{\vee}}(G,B)$ is defined by $(F,A)\tilde{\tilde{\vee}}(G,B)=(O,A\times B)$, where $O(e,f)=(X_1\cup X_2,Y_1\cup Y_2)$ for each $(e,f)\in A\times B$ such that $F(e)=(X_1,Y_1)$ and $G(e)=(X_2,Y_2)$.*

Example 11. *In the Example 6, $(O,A\times B)=(F,A)\tilde{\tilde{\vee}}(G,B)$ is the binary soft set as follows:*

$$\begin{aligned}(O,A\times B)=\{&((e_1,e_3),(\{s_1,s_2,s_4,s_5\},\{p_1,p_2,p_4\})),((e_1,e_4),(\{s_1,s_2\},\{p_1,p_2\})),\\&((e_1,e_6),(\{s_1,s_2\},\{p_2,p_4\})),((e_1,e_8),(\{s_1,s_2,s_5\},\{p_1,p_2\})),((e_3,e_3),\\&(\{s_4,s_5,s_6\},\{p_1,p_3,p_4\})),((e_3,e_4),(\{s_1,s_4,s_5,s_6\},\{p_1,p_2,p_3\})),((e_3,e_6),\end{aligned}$$

$(\{s_1,s_2,s_4,s_5,s_6\},\{p_1,p_3,p_4\})),((e_3,e_8),(\{s_4,s_5,s_6\},\{p_1,p_3\})),((e_5,e_3),$
$(\{s_2,s_4,s_5,s_6\},\{p_1,p_2,p_4\})),((e_5,e_4),(\{s_1,s_2,s_4,s_6\},\{p_2,p_4\})),((e_5,e_6),$
$(\{s_1,s_2,s_4,s_6\},\{p_2,p_4\})),((e_5,e_8),(\{s_2,s_4,s_5,s_6\},\{p_1,p_2,p_4\}))\}.$

Proposition 7. *Let* (F,A), (G,B) *and* (H,C) *be three binary soft sets. Then we have the following results:*

(i) $((F,A)\tilde{\tilde{\vee}}(G,B))^c = (F,A)^c \,\tilde{\tilde{\wedge}}\, (G,B)^c$.

(ii) $((F,A)\tilde{\tilde{\wedge}}(G,B))^c = (F,A)^c \,\tilde{\tilde{\vee}}\, (G,B)^c$.

(iii) $((F,A)\tilde{\tilde{\vee}}(G,B))\tilde{\tilde{\vee}}(H,C) = (F,A)\tilde{\tilde{\vee}}((G,B)\tilde{\tilde{\vee}}(H,C))$.

(iv) $((F,A)\tilde{\tilde{\wedge}}(G,B))\tilde{\tilde{\wedge}}(H,C) = (F,A)\tilde{\tilde{\wedge}}((G,B)\tilde{\tilde{\wedge}}(H,C))$.

(v) $(F,A)\tilde{\tilde{\vee}}((G,B)\tilde{\tilde{\wedge}}(H,C) = ((F,A)\tilde{\tilde{\vee}}(G,B)) \,\tilde{\tilde{\wedge}}\, ((F,A)\tilde{\tilde{\vee}}(H,C))$.

(vi) $(F,A)\tilde{\tilde{\wedge}}((G,B)\tilde{\tilde{\vee}}(H,C) = ((F,A)\tilde{\tilde{\wedge}}(G,B)) \,\tilde{\tilde{\vee}}\, ((F,A)\tilde{\tilde{\wedge}}(H,C))$.

Proof. It is obvious from Definitions 23 and 24. □

4. A Characteristic Function of The Binary Soft Set

In this section, we give a characteristic function of the binary soft set. We can easily see elements of initial universal sets providing parameter properties. The characteristic function is as follows:

Consider the following sets:

$$\begin{aligned} U_1 =&\{h_j : 1 \le j \le n\},\\ U_2 =&\{t_k : 1 \le k \le m\},\\ E =&\{e_i : 1 \le i \le p\}. \end{aligned}$$

Let $(F,E) = \{(e_i,(X_i,Y_i)) : 1 \le i \le p, X_i \subseteq U_1, Y_i \subseteq U_2\}$ is the binary soft set.

The characteristic functions of (F,E) binary soft set as below:

$$f_E(e_i)_j = \begin{cases} 1, & h_j \in X_i \\ 0, & h_j \notin X_i \end{cases}$$

and

$$g_E(e_i)_k = \begin{cases} 1, & t_k \in Y_i \\ 0, & t_k \notin Y_i \end{cases}.$$

We can form a table which shows the elements providing parameters properties.

Table 1: The Elements of Initial Universal Sets Providing Parameter Properties.

$E/U_1 - U_2$	h_1	h_2	$\dots$	h_n	t_1	t_2	$\dots$	t_m
e_1	$f_E(e_1)_1$	$f_E(e_1)_2$	$\dots$	$f_E(e_1)_n$	$g_E(e_1)_1$	$g_E(e_1)_2$	$\dots$	$g_E(e_1)_m$
e_2	$f_E(e_2)_1$	$f_E(e_2)_2$	$\dots$	$f_E(e_2)_n$	$g_E(e_2)_1$	$g_E(e_2)_2$	$\dots$	$g_E(e_2)_m$
e_1	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
e_p	$f_E(e_p)_1$	$f_E(e_p)_2$	$\dots$	$f_E(e_p)_n$	$g_E(e_p)_1$	$g_E(e_p)_2$	$\dots$	$g_E(e_p)_m$

Now we investigate a following example:

Example 12. *Consider the following sets:*

$$U_1 = \{h_1, h_2, h_3\} \text{ is the set of houses,}$$
$$U_2 = \{c_1, c_2, c_3, c_4\} \text{ is the set of cars,}$$
$$E = \{e_1 = \text{expensive}, \ e_2 = \text{sport}, \ e_3 = \text{beautiful}, \ e_4 = \text{cheap}\ \}$$

where E is the set of parameters. Let (F,E) is a binary soft sets as follows:

$$(F,E) = \{(e_1,(X_1 = \{h_1\}, Y_1 = \{c_2\})), (e_2,(X_2 = \emptyset, Y_2 = \{c_2,c_3\})), (e_3,(X_3 = \{h_1,h_3\}, Y_3 = \{c_2,c_4\})), (e_4,(X_4 = \{h_2\}, Y_4 = \{c_1\}))\}.$$

Then, for $1 \leq i \leq 4$, $1 \leq j \leq 3$ and $1 \leq k \leq 4$,

$$f_E(e_i)_j = \begin{cases} 1, & h_j \in X_i \\ 0, & h_j \notin X_i \end{cases}$$

and

$$g_E(e_i)_k = \begin{cases} 1, & c_k \in Y_i \\ 0, & c_k \notin Y_i \end{cases}.$$

Table 2: Houses and Cars Providing Parameter Properties. In the above table, we see some houses and cars providing parameter properties. For example, the first house is expensive but the second and third houses are not expensive. Similarly, the first car is cheap but the other cars are not cheap.

$E/U_1 - U_2$	h_1	h_2	h_3	c_1	c_2	c_3	c_4
e_1	1	0	0	0	1	0	0
e_2	0	0	0	0	1	1	0
e_3	1	0	1	0	1	0	1
e_4	0	1	0	1	0	0	0

5. Conclusion

In this paper, we give a definition of binary soft set on two initial universal sets and a parameter set. Also we define the binary soft subset, the binary soft equality, the binary null soft set, the binary absolute soft set, the union of two binary soft sets, the intersection of two binary soft sets, the difference of two binary soft sets, the symmetric difference of two binary soft sets, *AND* operation and *OR* operation via two binary soft sets. We investigate some properties of binary soft sets with defined operations (union, intersection etc.) Definition of a soft set can be given on n - dimension initial universal sets and a parameter set as follows:

$$F : A \to \prod_{i=1}^{n} P(U_i)$$

where U_i are initial universal sets for $1 \leq i \leq n$ and A is a parameter set.

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