The Propagation Of Strong Shock Waves In Magnetohydrodynamics

A. K. Ray And S. P. Dubey
Department of Mathematics, M. G. P. G. College, 273001 Gorakhpur, UP, India
Department of Physics, M. G. P. G. College, 273001 Gorakhpur, UP, India.

ABSTRACT.

In this paper, we have studied non-self-similar gas motion in presence of magnetic field which result from the propagation of plane, cylindrical and spherical shock waves through the gas requires complicated and cumbersome calculations. An approximate method of calculation of such motions is taken from [1-3].

KEYWORDS:

Non-self similar motion, magnetic field, strong explanation.

INTRODUCTION:

The investigation of non-self-similar gas motion which results from the propagation of shock waves through the gas requires complicated and cumbersome calculations. In a few cases these have actually been carried out, e.g. on the problem of point explosion [7].

In [1-3] an approximate method of calculation of such motions is given, valid for a high gas density jump across the shock wave, i.e. for the propagation of plane, cylindrical & spherical shock waves of large intensity in a gas. This method is based on the representation of gas dynamical quantities in the form of series in a spherical form for the powers of parameter ε, which characterizes the ratio of the gas density in front of the wave to the gas density behind the wave. The successive terms of the series are found from the equations by means of quadratures. When only the two first term of the series are taken into account, the gas parameters in a disturbed region behind the shock wave are expressed in terms of the function R*(t) in [2], which treats of the law of propagation of a shock wave. For the determination of this function in the problems of motion resulting from the explosion in a gas and from the expansion of a movable boundary (piston) in a gas, a law of conservation of energy in integral form may be used, pertaining to the whole of the region of disturbed gas motion [3].

FLOW GOVERNING EQUATION:

The total energy of a moving gas (the sum of its internal and kinetic energies) at each instant must equal to the sum of the energy E which was generated by the explosion the initial energy of the gas affected by the motion, magnetic field and the work done by the piston. In the presence of magnetic field the pressure term become change which is given by $p^* = p + \dfrac{H^2}{2}$ (where p is fluid pressure and

$\frac{H^2}{2}$ is magnetic pressure) then we have used effective pressure for p*. Taking the expression $\frac{p*}{\rho(\gamma-1)}$ to be the internal energy per unit mass of a gas (where p* is the effective pressure, ρ is the density, γ is the ratio of specific heats) b, then we have

$$\int_{v^*-v^0}\left[\frac{1}{2}\left(\frac{\partial R}{\partial t}\right)^2+\frac{p^*}{\rho(\gamma-1)}\right]\rho dv = E + \int_{v^*}\frac{p^*}{\gamma-1}dv + \int_0^t p^{0*}dv^0(t).$$

(1)

Where v^*-v^0 is the volume occupied by the moving gas, v^0 is the volume displaced by the piston, p_1^* is the initial effective gas pressure p^{0*} is the effective pressure on the piston $\frac{\partial R}{\partial t}$, is the velocity of gas particles and t is the time.

To use this integral relationship along with the representation of the desired quantities R, p* and ρ in the form of a series in powers of ε, we shall also represent the function R*(t) which gives the law of propagation of shock wave in the form of a series for the function which determines the form of a low shock wave for the steady flow past a body. We shall follow wave the method of Liubimov [5] used for the case of non-stationary one dimensional motion,

$$R^*(t) = R_0(t) + \varepsilon R_1^*(t) + \ldots\ldots\ldots\ldots$$

Substituting series for R, p* and ρ in equation (1) and equating the terms on the right with the terms on the left for the same powers of ε, after appropriate transformations, we obtain a sequence of ordinary differential equations for the determination of function R_0, R_1 etc.

As will be shown below, by proper choice of the main terms in the expansions of the quantities $\frac{\partial R}{\partial t}$ and p* in powers of ε, we can obtain a satisfactorily accurate first approximation of the determination of the law of propagation of a shock wave (and evidently all parameters of the stream immediately behind it) and the effective pressure acting on the piston.

In accordance with the result of [2] let

$$\frac{\partial R}{\partial t} = \frac{2}{\gamma+2}(\dot{R}_0 - \frac{a_1^2}{R_0}) + O(\varepsilon)$$

$$\left(a_1^2 = \frac{\gamma p_1^*}{\rho_1}\right)$$

$$p^* = p_1^* + \frac{2}{\gamma+1}\rho_1\left(\dot{R}_0^2 - a_1^2\right) + \rho_1\frac{R_0\ddot{R}_0}{\nu} - \frac{\ddot{R}_0}{R_0^{\nu-1}}m + O(\varepsilon)$$

where ρ_1 is the initial gas density, m is the Lagrange coordinate which is introduced by the relation $dm = \rho_1 r^{\nu-1} dr$, where r is the initial coordinate of a particle, ν = 1, 2, 3 correspond respectively to the flow with plane, cylindrical and spherical waves. The main terms are chosen such that in the case where $R_0(t)$ is the law of propagation of a shock wave, they will yield exact values of the corresponding quantities immediately behind the shock wave, i.e. for $m = \frac{\rho_1 R_0^\nu}{\nu}$.

After substitution of the expressions for $\frac{\partial R}{\partial t}$ and p* into the integral relationship (1) for the determination of function $R_0(t)$ we obtain the following equation (index 0 is subsequently omitted) :

$$\frac{1}{2}\left[\frac{2}{\gamma+1}\left(\dot{R}-\frac{a_1^2}{\dot{R}}\right)\right]^2\frac{\rho_1 R^\nu}{\nu}+\frac{p^{0*}}{\gamma-1}\frac{R^\nu-R^{0\nu}}{\nu}$$

=

$$\frac{E}{\omega}+\frac{p_1^*}{\gamma-1}\frac{R^\nu}{\nu}+\int_0^t p^{0*}R^{0\,\nu-1}\dot{R}^0 dt$$

(2)

where

$$p^{0*}=p_1^*+\frac{2}{\gamma+1}\rho_1\left(\dot{R}^2-a_1^2\right)+\frac{\rho_1 R\ddot{R}}{\nu}$$

$\omega = 2[\pi(\nu_1 - 1) + \delta_{1\nu}]$, $\delta_{11} = 1$, $\delta_{12} = \delta_{13} = 0$.

For simplicity it is assumed that at the start the gas occupies all space.

We shall evaluate the accuracy of determination of function R(t) and p^{0*} from equation (2) by comparing the solutions of this equation with the known exact solutions of problems on self-similar gas motions.

IMPULSIVE MOTION OF PISTON

Let $R^o = ct^{n+1}$ ($n \neq -1$). For $n \neq 0$ the motion is self-similar only under the condition that $a_1 = 0$, i.e., only as long as the shock wave may be considered to be strong. Assuming E = 0 and taking R(0) = 0 from equation (2) we find

$$R = \chi^{-1/\nu}(\gamma,\theta)R^0,$$

$$\frac{p^{0*}}{p^{00*}}=1+\frac{\gamma+1}{4}\theta$$

$$\left(\theta=\frac{2n}{\nu(n+1)}\right)$$

where χ is the ratio of the volume displaced by the piston to the volume bounded by the shock wave and p^{00*} is the effective gas pressure immediately after the shock wave :

$$\chi=\left(\frac{4\gamma}{(\gamma+1)^2}+\frac{\theta}{2}\right)\Big/\left(\left[\frac{2}{\gamma+1}+\frac{\theta}{2}\right]\left[1+\frac{\gamma-1}{1+\theta}\right]\right)$$

$$p^{00*}=\frac{2}{\gamma+1}\rho_1\dot{R}^2$$

It is interesting that in the approximation under consideration the values χ and p^{0*}/p^{00*} do not depend upon each of the parameters η and ν separately, but only upon their combination θ. Graphs of these function for $\gamma = 1.4$, i.e., for $\varepsilon=\frac{(\gamma-1)}{(\gamma+1)}$ are represented in figure 1.

In this figure the values of χ and p^{0*}/p^{00*} are represented, obtained at the results of numerical integration of corresponding exact solutions for $\nu = 2$ (hollow squares, 3) and for $\nu = 3$ (hollow circles, 1) for $\nu = 1$ and $n = 0$ the approximate values, predicated upon the choice of the main terms in the ε-expansions, coincide with the exact values (hollow triangles, 6); for $\nu=1$ and $n\neq0$ the results of exact calculations are not available. Half-shaded symbols 2,5,7 for θ – 1 correspond to the exact solution of the problem of a strong explosion [5].

Finally, the black squares 4 correspond to the values obtained for the exact solution of the problem with a cylindrical piston (ν = 2), expanding according to the indicated law. This case may be considered as the limiting case of impulsive piston expansion for n → ∞.

Figure 1 shows that in all the cases enumerated approximate solutions for ε = 1/6, have a quite satisfactory accuracy.

EXPANSION OF PISTON WITH CONSTANT VELOCITY:

If R^0 = UT, then the motion will be progressive also for $a_1 \neq 0$. Substitution of this expression for R^0 into equation (2) for E = 0 leads to the relation

$$R = Dt$$

where

$$\left(\frac{U}{D}\right)^{\nu} = \frac{2}{\gamma+1}\left(1-\frac{a_1^2}{D^2}\right), \qquad \frac{p^{0*}}{p^{00*}} = 1$$

For ν = 1 these relations are exact; their curves for γ = 1.4 are represented in figure 2 by solid lines. For ν = 2 and ν =3 these relations are only of approximate validity; relations obtained for ν =3 and γ = 1.405 by numerical integration of exact equations, are represented in this figure by the dashed line. For ν =3 approximate expressions retain satisfactory accuracy up to the values $\frac{a_1}{D} \sim \frac{0.4}{0.5}$, which corresponds to ε ~ 0.3 ÷ 0.35 and up to the effective pressure ratios in the shock wave of the order 5-7.

STRONG EXPLOSION:

Assuming in equation (2) $R^0 = 0$, $p_1^* = 1$ and E ≠ 0 and presuming R(0) = 0, we find

$$R = \left(\frac{E}{\alpha\, p_1^*}\right)^{\frac{1}{2+\nu}} t^{\frac{2}{2+\nu}}$$

where

$$\alpha = \frac{4\left[\pi(\nu-1)+\delta_{1\nu}\right]}{(2+\nu)^2\nu}\,\frac{6\gamma-\gamma^2-1}{(\gamma-1)(\gamma+1)^2}, \quad \frac{p^{0*}}{p^{00*}} = \frac{3-\gamma}{4}$$

Figure 3 shows the curves of the approximate functions obtained for the quantities p^{0*}/p^{00*} and

$$Z = \frac{R^{\nu+2}}{t^2}\,\frac{p_1^*}{E}\,\frac{2\omega}{\nu(\nu+2)^2}$$

and γ functions, and the exact values of these quantities [6] for ν = 1, 2, 3 .

From figure 3 it follows that in the case of the solution of the problem of the strong explosion, the approximate expression for R and p^{0*} satisfactorily agree with the exact expressions up to the values γ ~ 1.6 ÷ 1.8, i.e., up to the values ε ~ 0.25 ÷ 0.30. (Note that the relative error in the determination of R is ν + 2 times smaller that the difference corresponding to the quantity Z between the exact and the approximate values Z in figure 3).

Equation (2) allows the computation of any non-self similar motions resulting from an explosion and from the expansion of a piston (the equation is easily modified for the cases when the initial volume of a piston is different from zero), provided the intensity of the resulting shock waves is sufficiently large, so that ε does not exceed 0.2 – 0.3.

Thus, the examples presented of comparison of the approximate and exact solutions support the conclusion that the functions R(t) and $p^{0*}(t)$, determined by equation 2, retain a satisfactory accuracy up to the values ε ~ 0.20 ÷ 0.30.

In particular, using the law of plane cross-sections, in solving this equation one may determine the form of a shock wave, which is created by the flow past a profile (ν = 1) or a body of revolution (ν = 2) of a gas with large supersonic velocity. The effective pressure distribution on the surfaces of these bodies may likewise be determined, even in the cases when the front part is some what blunt [4].

RESULT:

In present paper, we have studied the propagation of strong shock waves in magnetohydrodynamics. We did not get any significant change in nature of shock wave. Only we get change in pressure. Hence the surface of shock wave becomes smooth in presence of magnetic field. The results are shown in figures.

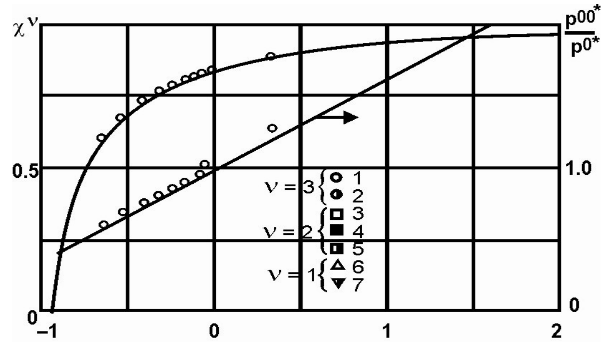

Fig. 1

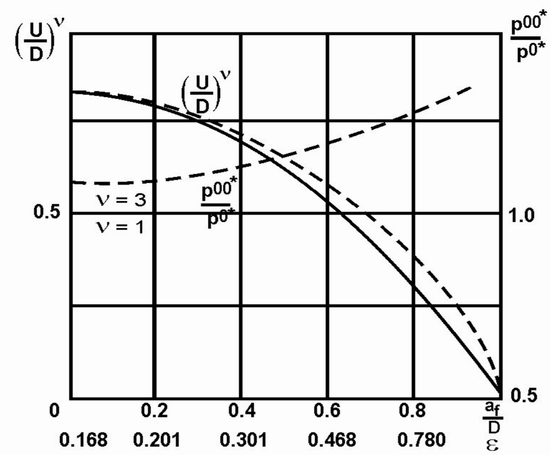

Fig. 2

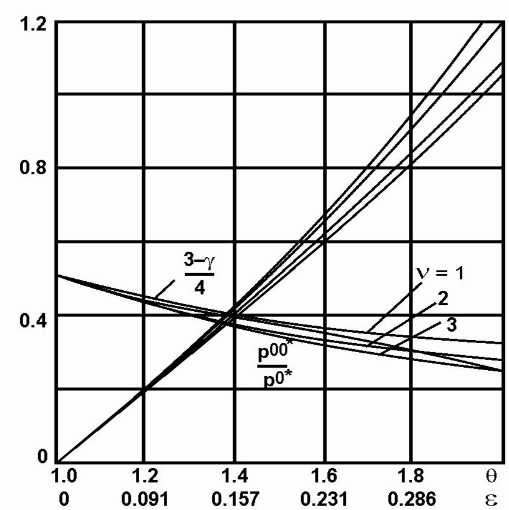

Fig. 3

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