A NUMERICAL SOLUTION OF ONE-DIMENSIONAL NEUTRON TRANSPORT EQUATION WITH CHEBYSHEV QUADRATURES

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ABSTRACT

The numerical solution of neutron scalar flux for one-group and one dimensional slab-geometry with isotropic scattering is studied by using Chebyshev quadrature sets and Spectral Green's Function (SGF) method. The unknowns in the method are the cell-edge and cell-average angular fluxes, and numerical values for these quantities are obtained by using two different quadrature sets. Finally, tabulated numerical results of cell-average scalar fluxes are provided.

TEK-BOYUTLU NÖTRON TRANSPORT DENKLEMİNİN CHEBYSHEV QUADRARÜRLERİ İLE BİR SAYISAL ÇÖZÜMÜ

ÖZET

Spektral Green Fonksiyonu (SGF) metodu ve Chebyshev quadratürü kullanılarak tek-guruplu ve tek-boyutlu izotropik saçılmalı dilim-gometride nötron skalar akı- sının sayısal çözümlemesi yapıldı. Metottaki bilinmeyenler hücre-kenarı ve hücre-ortalama açısal akıları olup bunların sayısal değerleri farklı iki quadratür seti kullanılarak elde edilmiştir. Sonuçta, hücre-ortalama skalar akıları için sayısal sonuçlar tablolar halinde verilmiştir.

INTRODUCTION

Up to now, various methods are developed and widely used for the solution of S_N transport equations. One of them is the source iteration (SI) method that is extensively used to solve the neutron transport equation. This method converges rapidly for optically thin problems and converges slowly for optically thick problem, see (1,2). Even with modern computers, computer storage limitations often require the use of spatial meshes that are not optically thin in order to obtain accurate results. DeBarros and Larsen (3) developed a new method for general one-group slab-geometry discrete ordinates problems with linearly anisotropic scattering. This method produces solutions that are completely free from spatial truncation errors.

In this paper, implementation and development of the solution methods with Chebyshev quadratures to S_N transport equations in slab-geometry are reported. An outline of the remainder of this paper follows. In section II, first, the S_N transport equation is evaluated to a form for which the numerical solution with Chebyshev quadrature sets is obtainable. Then, the discrete ordinates equations and separation of variables procedure for obtaining the analytic solution set of these equations in a homogeneous domain are described. So, by implementing the SGF method to spatially discretized neutron balance equation an iterative solution algorithm is derived. In section III, numerical results are given and a brief discussion is given in section IV.

METHOD

Let us consider the following transport equation in a homogeneous slab;

$$\mu\frac{d\psi(x,\mu)}{dx}+\sigma_T(x)\psi(x,\mu)=\int_0^{2\pi}\int_{-1}^{1}\sigma_S(x,\mu_0)\psi(x,\mu')d\mu' d\varphi'+\frac{1}{2}Q_0(x),$$
$$-1\leq\mu\leq 1,\quad 0\leq x\leq a \tag{1}$$

where a is the thickness of the slab. In Eq.(1) scattering function is expanded in Chebyshev polynomials (4), by writing

$$\sigma(x,\mu_0)=\frac{\sigma_{S0}(x)}{4\pi}T_0(\mu_0)+\sum_{n=1}^{\infty}\frac{\sigma_{Sn}(x)}{2\pi}T_n(\mu_0) \tag{2}$$

((2)

where μ_0 is the cosine of the scattering angle and given by

$$\mu_0=\mu\mu'+\sqrt{1-\mu^2}\sqrt{1-\mu'^2}\,Cos(\varphi-\varphi'),\quad 1<\mu,\ \mu'<1,\ 0<\varphi,\ \varphi'<2\pi \tag{3}$$

Upon insertion of Eq.(2) and Eq.(3) into Eq.(1) and carrying out the integration over φ', the result for only isotropic source and isotropic scattering is

$$\mu\frac{d\psi(x,\mu)}{dx}+\sigma_T\psi(x,\mu)=\frac{\sigma_{S0}(x)}{2}\int_{-1}^{1}\psi(x,\mu')d\mu'+\frac{Q_0(x)}{2},\quad 0\leq x\leq a \tag{4}$$

Now, it is desired to solve numerically by using Chebyshev quadratures. To do this, the first term on the right hand side of Eq.(4) may be evaluated as

$$\int_{-1}^{1}\psi(x,\mu')d\mu'=\int_{-1}^{1}\sqrt{1-\mu'^2}\,\psi(x,\mu')\frac{d\mu'}{\sqrt{1-\mu'^2}}\cong\sum_{n=1}^{N}\sqrt{1-\mu_n^2}\,\psi_n\omega_n$$

and by using this evaluated expression in Eq.(4), S_N equation is obtained for which a numerical solution is obtainable by using Gauss - Chebyshev quadrature sets;

$$\mu_m\frac{d\psi_m(x)}{dx}+\sigma_T\psi_m(x)=\frac{\sigma_{S0}(x)}{2}\sum_{n=1}^{N}\sqrt{1-\mu_n^2}\,\psi_n(x)\omega_n+\frac{Q_0(x)}{2},$$

$\psi_m(0)=f_m$ for $\mu_m>0$, $\psi_m(a)=g_m$ for $\mu_m<0$, $0\leq x\leq a$, $m=1,...,N$ (5)

Here f_m and g_m are specificated incident fluxes on the outer boundaries of the slab. Used notations are standard, see (5); ψ_m (x) is the angular flux of particles travelling in the discrete ordinates direction μ_m , ω_m is the quadrature weights or weighting factor for direction μ_m, see (6), σ_T (x) is the total cross section, σ_{S0} (x) is the zero'th component of the differential scattering cross section and Q_0 (x) is the zero'th component of the interior source. All of these quantities are assumed to be piecewise constant in space. In this paper, even-order Gauss-Chebyshev quadrature sets are used, then μ_m are the roots of N'th order Chebyshev polynomials and these roots have symmetric values on the interval $-1 \leq \mu \leq 1$. These roots and weighting factors can be calculated easily from the following equations, respectively;

$$\mu_m = \text{Cos}(\,(2m-1)\,\pi / 2N\,), \quad \omega_m = \pi / N \qquad m = 1, \ldots.. N$$

General solution of Eq.(5) can be written as

$$\psi_m(x) = \psi_m^p(x) + \psi_m^h(x) \tag{6}$$

where $\psi_m^p(x)$ and $\psi_m^h(x)$ denote the particular and homogeneous solutions of Eq.(5), respectively. It is easy to verify that the spatially constant particular solution of Eq.(5) is given by

$$\psi_m^p(x) = \frac{Q_0(x)}{2(\sigma_T(x) - \sigma_{S0}(x))}, \quad 0 \leq x \leq a, \quad 1 \leq m \leq N \tag{7}$$

In this paper it is chosen to be $\sigma_T > \sigma_{S0}$ in all case. This assumption means physically that each spatial cell consists of material for which an infinite system is subcritical.

Particular solution, that is, $\psi_m^p(x)$, was found, it is needed to determine the homogeneous solution $\psi_m^h(x)$. For this solution, it is customary to use the method of se- paration of variables. Thus the solution of homogeneous part of Eq.(5) is the form of

$$\psi_m^h(x) = H_m(\nu)\exp(\sigma_T x / \nu), \qquad 0 \leq x \leq a, \quad 1 \leq m \leq N \tag{8}$$

Substituting Eq.(8) into the homogeneous part of Eq.(5), it is easy to obtain an expression for $H_m(\nu)$

$$H_m(\nu) = \frac{\nu C_0}{2(\nu + \mu_m)} \sum_{n=1}^{N} \sqrt{1-\mu_n^2}\, H_n(\nu)\omega_n, \quad C_0 = \sigma_{S0} / \sigma_T, \quad 1 \leq m \leq N \tag{9}$$

To obtain an equation for the ν eigenvalues, Eq.(9) is multiplied by $\sqrt{1-\mu_m{}^2}\,\omega_m$ and summed over all m. The result is the following equation for ν ;

$$\frac{\nu C_0}{2}\sum_{m=1}^{N}\frac{\sqrt{1-\mu_m^2}\,\omega_m}{\nu+\mu_m}=1, \quad \nu \neq -\mu_m \tag{10}$$

The roots ν_k, $1 \leq k \leq N$ of Eq.(10) are the eigenvalues of the S_N equations. Due to symmetry of Gauss-Chebyshev quadrature set, the roots ν_k which are obtained from the resulting equation has also only symmetric values.

To conclude, Eq.(6) can be written as

$$\psi_m(x)=\psi_m^p(x)+\sum_{k=1}^{N}\beta_k H_m(\nu_k)\exp(\sigma_T x/\nu_k), \; 1\leq m\leq N \tag{11}$$

where $\psi_m^p(x)$ is given by Eq.(7) and β_k, $1 \leq k \leq N$ are arbitrary constants. Eq.(11) represents the general solution of Eq.(5) in given domain. In the following procedure, it is derived a numerical method in which a generalization is made for the solution of S_N equations in slab geometry by using the boundary conditions and the continuity conditions.

It is driven Spectral Green's Function (SGF) equations for one spatial cell and described iterative scheme for solving these equations on system consisting of many cells. Let us begin by integrating Eq.(5) over an arbitrary cell which has constant cross sections and constant interior source. The result is the familiar spatial balance equation;

$$\frac{\mu_m}{h_i}(\psi_{m,i+1/2}-\psi_{m,i-1/2})+\sigma_{T,i}\psi_{m,i}=\frac{\sigma_{S0,i}}{2}\sum_{n=1}^{N}\sqrt{1-\mu_n^2}\,\psi_{n,i}\omega_n+\frac{Q_{0,i}}{2}, \; 1\leq m\leq N, \tag{12}$$

where $\psi_{m,i+1/2}$ and $\psi_{m,i-1/2}$ are the cell-edge-average angular fluxes, $\psi_{m,i}$ are the cell-average angular fluxes and $\sigma_{S0,i}$ and $Q_{0,i}$ are the cross section and interior source for i'th cell, respectively. In Eq.(12), $\psi_{m,i}$ cell-average angular flux is defined by

$$\psi_{m,i}=\frac{1}{h_i}\int_{x_{i-1/2}}^{x_{i+1/2}}\psi_m(x)dx \tag{13}$$

where h_i is the cell width, $h_i = x_{i+1/2} - x_{i-1/2}$. The balance equation (i.e. Eq.(12)) relates the cell-average angular fluxes to the cell-edge-average angular fluxes. The spatial balance equation, combined with the boundary conditions imposed on the outer boundaries of the slab

$$\psi_{m,1/2} = f_m \qquad \mu_m > 0\,, \qquad 1 \le m \le N/2 \tag{14.a}$$
$$\psi_{m,I+1/2} = g_m \qquad \mu_m < 0\,, \qquad N/2 < m \le N \tag{14.b}$$

work, see where f_m and g_m are prescribed boundary conditions and I is the number of spatial cells in the slab. As shown in Eq.(12) and Eq.(14), the resulting system still has more unknowns than equations. Therefore, additional equations are needed to obtain the same number of equations as unknowns. Here, it is followed earlier study (3), and imposed the auxiliary equations as additional equations;

$$\psi_{m,i} = \sum_{n=1}^{N/2} \theta_{m,n}\psi_{n,i-1/2} + \sum_{n=\frac{N}{2}+1}^{N} \theta_{m,n}\psi_{n,i+1/2} + G_{m,i} \tag{15}$$

where $G_{m,i}$ represents the contribution to the cell-average angular flux $\psi_{m,i}$ due to interior source $Q_{0,i}$ and $\theta_{m,n}$ ' s play the role of the Green's function in discretized space. To determine $\theta_{m,n}$ first, let's substitute Eq.(11) into Eq.(13) and get

$$\psi_{m,i} = \psi^{p}_{m,i} + \sum_{k=1}^{N} \beta_k \left\{ \frac{\nu_k H_m(\nu_k)}{h\sigma_T} \left[\exp(\frac{\sigma_T x_{i+1/2}}{\nu_k}) - \exp(\frac{\sigma_T x_{i-1/2}}{\nu_k}) \right] \right\} \tag{16}$$

and also from Eq.(11) cell-edge-average angular flux can be written in the form of

$$\psi_{m,i\pm1/2} = \psi^{p}_{m} + \sum_{k=1}^{N} \beta_k H_m(\nu_k) \exp(\frac{\sigma_T x_{i\pm1/2}}{\nu_k}) \tag{17}$$

To proceed further, Eq.(16) and Eq.(17) are introduced into Eq.(15) to obtain

$$G_m = \left(1 - \sum_{n=1}^{N} \theta_{m,n} \right) \psi^{p}_{m} \tag{18}$$

$$\frac{2\nu_k H_m(\nu_k)}{h\sigma_T} \mathrm{Sinh}\left(\frac{h\sigma_T}{2\nu_k} \right) = \exp(-\frac{h\sigma_T}{2\nu_k}) \sum_{n=1}^{N/2} H_n(\nu_k)\theta_{m,n} + \exp(\frac{h\sigma_T}{2\nu_k}) \sum_{n=N/2+1}^{N} H_n(\nu_k)\theta_{m,n} \tag{19}$$

Thus, Eq.(19) represents a linear system of N^2 equations in the N^2 unknowns $\theta_{m,n}$. By using Eq.(15) in Eq.(12), equations which are soluble with simple iteration technique are obtained

$$\psi_{m,i-1/2} = \sum_{n=1}^{N/2} A_{m,n}\psi_{n,i-1/2} + \sum_{n=N/2+1}^{N} A_{m,n}\psi_{n,i+1/2} \qquad 1 \le m \le N/2 \tag{20}$$

$$\psi_{m,i+1/2} = \sum_{n=1}^{N/2} A_{m,n}\psi_{n,i-1/2} + \sum_{n=N/2+1}^{N} A_{m,n}\psi_{n,i+1/2} \quad N/2+1 \le m \le N \tag{21}$$

where $A_{m,n}$ are the elements of iteration matrix which includes the values of μ_m , C_0 , $\theta_{m,n}$, ω_m and h. From computed values of cell-edge-average angular flux, average cell-edge scalar flux are computed by summing over all angles.

$$\Phi_{i+1/2} = \frac{1}{2}\sum_{n=1}^{N} \sqrt{1-\mu_n^2}\,\psi_{n,i+1/2}\,\omega_n \tag{22}$$

NUMERICAL RESULTS

Let us first consider a homogeneous slab with $\sigma_T = 0.9$ cm^{-1}, $\sigma_{S0} = 0.6$ cm^{-1}, a = 50 cm, $Q_0 = 0$ and the boundary conditions $\psi_{m,1/2} = 1.0$ for $\mu_m > 0$ and $\psi_{m,I+1/2} = 0$ for $\mu_m < 0$. Solution of the problem is made by using the standard S_2 , S_4 and S_{12} Gauss - Chebyshev quadrature sets, with different number of spatial cells. The cell-edge scalar fluxes at x (cm) = 0, 25, and 50 for each run are presented in Tables I, II and III. In each Table the number of iterations (ITN) required to achieve a convergence criterion of $\in = 0$ are also given. In these problem, iteration is continued up to the difference between two consecutive iterations equals to zero. Thus, because the iteration number is given, it is not needed to compute the spectral radius. This problem is also solved by using Gauss - Legendre quadrature sets at the same conditions. The results of these solutions are also presented at the right hand side of Tables 1,2 and 3.

Table 1. S_2 Solution of Homogeneous Problem

Number of cells	with Gauss-Chebyshev quadrature sets				with Gauss-Legendre quadrature sets			
	$\Phi(0)$	$\Phi(25)$	$\Phi(50)$	ITN	$\Phi(0)$	$\Phi(25)$	$\Phi(50)$	ITN
2	$0.736\text{x}10^{0}$	$0.671\text{x}10^{-8}$	$0.413\text{x}10^{-14}$	51	$0.634\text{x}10^{0}$	$0.107\text{x}10^{-9}$	$0.133\text{x}10^{-19}$	45
4	$0.736\text{x}10^{0}$	$0.671\text{x}10^{-8}$	$0.413\text{x}10^{-14}$	59	$0.634\text{x}10^{0}$	$0.107\text{x}10^{-9}$	$0.133\text{x}10^{-19}$	57
10	$0.736\text{x}10^{0}$	$0.671\text{x}10^{-8}$	$0.413\text{x}10^{-14}$	67	$0.634\text{x}10^{0}$	$0.107\text{x}10^{-9}$	$0.133\text{x}10^{-19}$	63
20	$0.736\text{x}10^{0}$	$0.671\text{x}10^{-8}$	$0.413\text{x}10^{-14}$	95	$0.634\text{x}10^{0}$	$0.107\text{x}10^{-9}$	$0.133\text{x}10^{-19}$	82
50	$0.736\text{x}10^{0}$	$0.671\text{x}10^{-8}$	$0.413\text{x}10^{-14}$	207	$0.634\text{x}10^{0}$	$0.107\text{x}10^{-9}$	$0.133\text{x}10^{-19}$	160

Table 2. S_4 Solution of Homogeneous Problem

Number of cells	with Gauss-Chebyshev quadrature sets				with Gauss-Legendre quadrature sets			
	$\Phi(0)$	$\Phi(25)$	$\Phi(50)$	ITN	$\Phi(0)$	$\Phi(25)$	$\Phi(50)$	ITN
2	$0.657x10^{0}$	$0.466x10^{-8}$	$0.344x10^{-16}$	45	$0.634x10^{0}$	$0.146x10^{-8}$	$0.349x10^{-17}$	43
4	$0.657x10^{0}$	$0.466x10^{-8}$	$0.344x10^{-16}$	53	$0.634x10^{0}$	$0.146x10^{-8}$	$0.349x10^{-17}$	52
10	$0.657x10^{0}$	$0.466x10^{-8}$	$0.344x10^{-16}$	58	$0.634x10^{0}$	$0.146x10^{-8}$	$0. 349x10^{-17}$	57
20	$0.657x10^{0}$	$0.466x10^{-8}$	$0.344x10^{-16}$	76	$0.634x10^{0}$	$0.146x10^{-8}$	$0. 349x10^{-17}$	72
50	$0.657x10^{0}$	$0.466x10^{-8}$	$0.344x10^{-16}$	156	$0.634x10^{0}$	$0.146x10^{-8}$	$0. 349x10^{-17}$	150

Table 3. S_{12} Solution of Homogeneous Problem

Number of cells	with Gauss-Chebyshev quadrature sets				with Gauss-Legendre quadrature sets			
	$\Phi(0)$	$\Phi(25)$	$\Phi(50)$	ITN	$\Phi(0)$	$\Phi(25)$	$\Phi(50)$	ITN
2	$0.637x10^{0}$	$0.178x10^{-8}$	$0.557x10^{-17}$	44	$0.634x10^{0}$	$0.161x10^{-8}$	$0.460x10^{-17}$	42
4	$0.637x10^{0}$	$0.178x10^{-8}$	$0.557x10^{-17}$	50	$0.634x10^{0}$	$0.161x10^{-8}$	$0.460x10^{-17}$	51
10	$0.637x10^{0}$	$0.178x10^{-8}$	$0.557x10^{-17}$	56	$0.634x10^{0}$	$0.161x10^{-8}$	$0. 460x10^{-17}$	56
20	$0.637x10^{0}$	$0.178x10^{-8}$	$0.557x10^{-17}$	72	$0.634x10^{0}$	$0.161x10^{-8}$	$0. 460x10^{-17}$	71
50	$0.637x10^{0}$	$0.178x10^{-8}$	$0.557x10^{-17}$	148	$0.634x10^{0}$	$0.161x10^{-8}$	$0. 460x10^{-17}$	147

Second problem in this paper is the heterogeneous slab, 50cm thick, consisting of three regions. The first (leftmost) region, 10cm thick, has $\sigma_T = 0.9cm^{-1}$, $\sigma_{S0} = 0.8cm^{-1}$ and $Q_0 = 0$. The second (middle) region, 30cm thick, has $\sigma_T = 0.8cm^{-1}$, $\sigma_{S0} = 0.4cm^{-1}$ and $Q_0 = 0$. The third (rightmost) region, 10cm thick, has the same material properties as the first region. The boundary conditions are $\psi_{m,1/2} = 1.0$ for $\mu_m > 0$ and $\psi_{m,I+1/2} = 0$ for $\mu_m < 0$. As in the first problem, solution of this problem is made by using the standard S_2 and S_4 Gauss - Chebyshev quadrature sets, with different number of spatial cells. Tables 4 and 6 show the results obtained for different number of cells. The first column of each table in which the number of cells in each subdomain is also presented shows the total number of cells, next four columns show the cell-edge scalar fluxes at the interfaces or at the points x(cm)= 0, 10, 40 and 50, the last columns show the number of iterations required to achieve the convergence criterion of $\in=0$. This problem is also solved by using Gauss-Legendre quadrature sets at the same conditions. The results of this solution are presented in Tables 5 and 7 for different number of spatial cells.

Table 4. S_2 Solution of Heterogeneous Problem with Gauss-Chebyshev Quadrature Sets

Number of Cells	Φ(0)	Φ(10)	Φ(40)	Φ(50)	ITN
1 x 3 x 1	$0.9898x10^{0}$	$0.7107x10^{-1}$	$0.1778x10^{-10}$	$0.8989x10^{-12}$	67
2 x 6 x 2	$0.9898x10^{0}$	$0.7107x10^{-1}$	$0.1778x10^{-10}$	$0.8989x10^{-12}$	200
4 x 12 x 4	$0.9898x10^{0}$	$0.7107x10^{-1}$	$0.1778x10^{-10}$	$0.8989x10^{-12}$	359
10 x 30 x 10	$0.9898x10^{0}$	$0.7107x10^{-1}$	$0.1778x10^{-10}$	$0.8989x10^{-12}$	849

Table 5. S_2 Solution of Heterogeneous Problem with Gauss-Legendre Quadrature Sets

Number of Cells	Φ(0)	Φ(10)	Φ(40)	Φ(50)	ITN
1 x 3 x 1	$0.7500x10^{0}$	$0.2661x10^{-2}$	$0.6206x10^{-15}$	$0.1718x10^{-17}$	61
2 x 6 x 2	$0.7500x10^{0}$	$0.2661x10^{-2}$	$0.6206x10^{-15}$	$0.1718x10^{-17}$	112
4 x 12 x 4	$0.7500x10^{0}$	$0.2661x10^{-2}$	$0.6206x10^{-15}$	$0.1718x10^{-17}$	158
10 x 30 x 10	$0.7500x10^{0}$	$0.2661x10^{-2}$	$0.6206x10^{-15}$	$0.1718x10^{-17}$	348

Table 6. S_4 Solution of Heterogeneous Problem with Gauss-Chebyshev Quadrature Sets

Number of Cells	Φ(0)	Φ(10)	Φ(40)	Φ(50)	ITN
1 x 3 x 1	$0.7915x10^{0}$	$0.5473x10^{-2}$	$0.1071x10^{-11}$	$0.8144x10^{-14}$	56
2 x 6 x 2	$0.7915x10^{0}$	$0.5473x10^{-2}$	$0.1071x10^{-11}$	$0.8144x10^{-14}$	105
4 x 12 x 4	$0.7915x10^{0}$	$0.5473x10^{-2}$	$0.1071x10^{-11}$	$0.8144x10^{-14}$	157
10 x 30 x 10	$0.7915x10^{0}$	$0.5473x10^{-2}$	$0.1071x10^{-11}$	$0.8144x10^{-14}$	343

Table 7. S_4 Solution of Heterogeneous Problem with Gauss-Legendre Quadrature Sets

Number of Cells	Φ(0)	Φ(10)	Φ(40)	Φ(50)	ITN
1 x 3 x 1	$0.7500x10^{0}$	$0.2867x10^{-2}$	$0.1514x10^{-12}$	$0.6384x10^{-15}$	55
2 x 6 x 2	$0.7500x10^{0}$	$0.2867x10^{-2}$	$0.1514x10^{-12}$	$0.6384x10^{-15}$	96
4 x 12 x 4	$0.7500x10^{0}$	$0.2867x10^{-2}$	$0.1514x10^{-12}$	$0.6384x10^{-15}$	137
10 x 30 x 10	$0.7500x10^{0}$	$0.2867x10^{-2}$	$0.1514x10^{-12}$	$0.6384x10^{-15}$	291

CONCLUSION

In this paper, implementation of SGF method to the numerical solution of S_N transport equations with Chebyshev quadrature sets is given. A more detailed information about SGF method is given in the earlier work (3). Here, it is not informed again. The source Q_0 (x) in each region is accepted to be constant. However, in the method, any functional form of the source Q_0 (x) can be used and an analytic particular solution analogous to Eq.(7) can be obtained. In the work, the method is applied up to N=12 for both Legendre and Chebyshev quadrature sets. But it can also be applied to quadrature sets of arbitrary order N>12. In this case, as shown in Eq.(10), to calculate ν_k, $1\leq k \leq N$ eigenvalues N'th order polynomial equation must be solved. Then, when N become large, finite arithmetic and roundoff errors may occur. Besides, to determine $\theta_{m,n}$, N system of N linear equations must be solved; as N becomes large, the coefficients of unknowns $\theta_{m,n}$ become more dispersed. So, this situation imposes a practical limitation on the order N for the accuracy of the method. As shown in the tables, cell's width corresponding to h (cm) = 1, 5/2, 5, 25/2 and 25 give the same results; that is, this method is free from truncation error, as in earlier work(3). It is also noted here, that the number of iterations necessary to achieve the convergence of $\in = 0$ become smaller when the cell's width becomes coarser. In all calculations, convergence criteria is chosen to be $\in = 0$. As seen from the tables the scalar flux at the end of the slab (x = 50 cm) changes from $\sim 10^{-12}$ to $\sim 10^{-19}$. If it had been chosen to be $\in > 10^{-12}$ -10^{-19}, different results were obtained for different numbers of spatial cells. For more general situation it is taken to be $\in = 0$ for each run. Another interesting feature of this paper is that, as shown in table III, when the quadrature order N increases, the results obtained for both Legendre and Chebyshev quadrature sets approach to same results, as expected. For Gauss-Legendre quadrature sets, the summation of ω_m over all m exactly equal to 2 for any order N. But Gauss-Chebyshev quadrature sets, the summation of $\sqrt{1-\mu_m^2}\,\omega_m$ over all m, see Eq.(5), converge to 2 slowly, that is, if N becomes sufficiently large then this summation converges to 2 exactly.

It is assumed that the neutrons have the same energy, that is, it is operated only on a one-group problem. It is also assumed that the scattering is isotropic. However, this method can be applied to multigroup and anisotropic scattering problems. It is planned to investigate these problems in future works.

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