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Linear Damping Vibrations of Two Binding Pendulums

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The statement of fundamental problems and solution methods in a class of linear dynamic systems are studied in the present paper. Theorical results are applied to the vibrations of two binding pendulums with viscoelastic spring. The effect of viscosity coefficients on the character of the solution is investigated. The conditions of stability, asymptotic stability and instability of the solutions are obtained and the orbits of the motion for the different values of parameters are driven by using Maple 10 program.

1. INTRODUCTION

The statement of fundamental problems and solution methods in a class of linear dynamic systems are studied in the present paper. The solutions of dynamical problems are used in a number of areas of science and technology [1-3]. Obtaining the solutions of the systems are actual problem. There are some investigations of this kind in the paper, too.
Since the power of exponential kernel function is related with the viscosity coefficient, the dependence of the solutions obtained on this parameter is studied. Theorical results are applied to the motion of two binding pendulums with viscoelastic spring. Stability, asymptotic stability and instability [4-7]of the solutions are solved by using Maple 10 program for the different values of parameters (Fig. 2-5). There are some orbits in two and three dimensions. Investigating the graphs the critical points are obtained and can be used by researchers in various fields.
We take motion equations given by general dynamical systems studied in [4]. In the present paper

$$a_{11}\ddot{q}_1 + e_1\dot{q}_1 + c_{11}q_1 + c_{12}q_2 = f(t)$$

$$a_{21}\ddot{q}_1 + e_2\dot{q}_2 + c_{21}q_1 + c_{22}q_2 = g(t) \tag{1}$$

(where a_{11}, a_{21}, c_{11}, c_{12}, c_{21}, c_{22} are constants $(a_{11} \cdot a_{21} \neq 0)$, e_1 and e_2 damping coefficients, $f(t)$ and $g(t)$ are the functions depend on the time t) viscoelastic dynamical systems is studied.

2. THE STATEMENT OF THE PROBLEM

Systems (1) is adapted to the equations of motion of two binding pendulums spring to each other with the initial conditions.

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$$m_1 l^2 \ddot{\varphi} + m_1 l^2 \nu_1 \dot{\varphi} + m_1 g l \varphi + c d^2 \varphi - c d^2 \psi = e^{-\nu_1 t/2}\left(A_0 \sin t + B_0 \cos t\right)$$

$$m_2 l^2 \ddot{\psi} + m_2 l^2 \nu_2 \dot{\psi} + m_2 g l \psi + c d^2 \psi - c d^2 \varphi = e^{-\nu_2 t/2}\left(C_0 \sin t + D_0 \cos t\right) \tag{2}$$

$$t = 0: \ \varphi = \dot{\varphi} = 0, \quad \psi = \dot{\psi} = 0$$

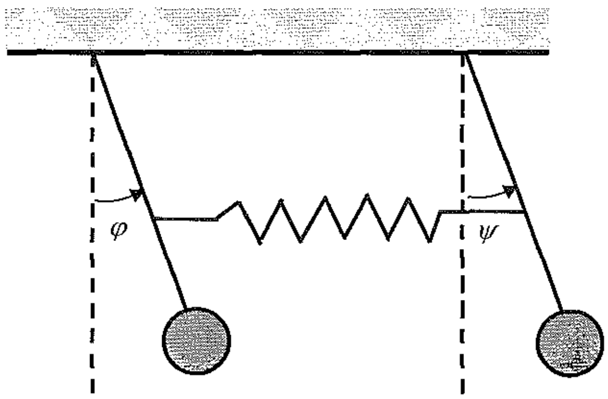


Fig. 1. Two binding pendulums with viscoelastik spring

where φ, ψ are the angels of the first and second pendulums measured from the vertical axis, respectively; m_1, m_2 are the masses of the first and second pendulums, respectively; c is the spring constant; l is the length of the pendulums; d is the length from the rotation axis to binding points of spring.

3. THE SOLUTION OF THE PROBLEM

In system (2), divide the first and second equation by $m_1 l^2$ and $m_2 l^2$, respectively.
Use the notation

$$\frac{g}{l} + \frac{cd^2}{m_1 l^2} = n_1^2, \quad \frac{g}{l} + \frac{cd^2}{m_2 l^2} = n_2^2, \quad -\frac{cd^2}{m_1 l^2} = B_1, \quad -\frac{cd^2}{m_2 l^2} = B_2$$

$$\frac{A_0}{m_1 l^2} = A, \quad \frac{B_0}{m_1 l^2} = B, \quad \frac{C_0}{m_1 l^2} = C, \quad \frac{D_0}{m_1 l^2} = D,$$

then (2) becomes as follows:

$$\ddot{\varphi} + \nu_1\dot{\varphi} + B_1\psi + n_1^2\varphi = e^{-\nu_1 t/2}\left(A\sin t + B\cos t\right)$$
$$\ddot{\psi} + \nu_2\dot{\psi} + B_2\varphi + n_2^2\psi = e^{-\nu_2 t/2}\left(C\sin t + D\cos t\right). \tag{3}$$

Substituting $\varphi = e^{-\nu_1 t/2}\Phi$, $\psi = e^{-\nu_2 t/2}\Psi$ and taking the first and second derivatives respect to t

$$\dot{\varphi} = -\frac{\nu_1}{2}e^{-\nu_1 t/2}\Phi + e^{-\nu_1 t/2}\dot{\Phi}, \quad \ddot{\varphi} = \frac{\nu_1^2}{4}e^{-\nu_1 t/2}\Phi - \nu_1 e^{-\nu_1 t/2}\dot{\Phi} + e^{-\nu_1 t/2}\ddot{\Phi}$$

$$\dot{\psi} = -\frac{\nu_2}{2}e^{-\nu_2 t/2}\Psi + e^{-\nu_2 t/2}\dot{\Psi}, \quad \ddot{\psi} = \frac{\nu_2^2}{4}e^{-\nu_2 t/2}\Psi - \nu_2 e^{-\nu_2 t/2}\dot{\Psi} + e^{-\nu_2 t/2}\ddot{\Psi}$$

in (3), we obtain

$$\ddot{\Phi} + B_1 e^{-(\nu_2-\nu_1)t/2}\Psi + \left(n_1^2 - \frac{\nu_1^2}{4}\right)\Phi = A\sin t + B\cos t$$
$$\ddot{\Psi} + B_2 e^{-(\nu_1-\nu_2)t/2}\Phi + \left(n_2^2 - \frac{\nu_2^2}{4}\right)\Psi = C\sin t + D\cos t. \tag{4}$$

Therefore under this substitution the initial conditions become

$$t = 0: \quad \Phi = \dot{\Phi} = 0, \quad \Psi = \dot{\Psi} = 0. \tag{5}$$

We are to examine the case $\nu_1 = \nu_2 = 2\nu$ in (4).

So the systems reduces to

$$\ddot{\Phi} + B_1\Psi + \left(n_1^2 - \nu^2\right)\Phi = A\sin t + B\cos t$$
$$\ddot{\Psi} + B_2\Phi + \left(n_2^2 - \nu^2\right)\Psi = C\sin t + D\cos t. \tag{6}$$

We solve the homogeneous systems

$$\ddot{\Phi} + B_1\Psi + \left(n_1^2 - \nu^2\right)\Phi = 0$$
$$\ddot{\Psi} + B_2\Phi + \left(n_2^2 - \nu^2\right)\Psi = 0 \tag{7}$$

and obtain the solution [8]

$$\Phi = a\sin\left(k_1 t + \beta_1\right) + b\sin\left(k_2 t + \beta_2\right)$$
$$\Psi = a\alpha_1 \sin\left(k_1 t + \beta_1\right) + b\alpha_2 \sin\left(k_2 t + \beta_2\right), \tag{8}$$

there a, b, β_1, β_2's are the functions of weak changed time, k_1 and k_2 $(0 < k_1 < k_2)$ are the main frequencies that are determined by the principal frequency equation

$$k^4 - \left(n_1^2 + n_2^2 - 2\nu^2\right)k^2 + \left(n_1^2 - \nu^2\right)\left(n_2^2 - \nu^2\right) - B_1 B_2 = 0,$$

also for abbreviation $\alpha_1 = \dfrac{k_1^2 + \nu^2 - n_1^2}{B_1}$, $\alpha_2 = \dfrac{k_2^2 + \nu^2 - n_1^2}{B_1}$.

To find a particular solution of the non-homogeneous systems (6), substitute

$$\Phi = a\sin t + b\cos t \tag{9}$$
$$\Psi = c\sin t + d\cos t$$

into the left members. The result must be identically equal to right side of (6), so that

$$\left(n_1^2 - \nu^2 - 1\right)a + B_1 c = A$$
$$\left(n_1^2 - \nu^2 - 1\right)b + B_1 d = B$$
$$B_2 a + \left(n_2^2 - \nu^2 - 1\right)c = C$$
$$B_2 d + \left(n_2^2 - \nu^2 - 1\right)d = D.$$

Hence

$$a = \frac{B_1 C - \left(n_2^2 - \nu^2 - 1\right)A}{B_1 B_2 - \left(n_1^2 - \nu^2 - 1\right)\left(n_2^2 - \nu^2 - 1\right)}, \quad c = \frac{B_2 A - \left(n_1^2 - \nu^2 - 1\right)C}{B_1 B_2 - \left(n_1^2 - \nu^2 - 1\right)\left(n_2^2 - \nu^2 - 1\right)},$$

$$b = \frac{B_1 D - \left(n_2^2 - \nu^2 - 1\right)B}{B_1 B_2 - \left(n_1^2 - \nu^2 - 1\right)\left(n_2^2 - \nu^2 - 1\right)}, \quad d = \frac{B_2 B - \left(n_1^2 - \nu^2 - 1\right)D}{B_1 B_2 - \left(n_1^2 - \nu^2 - 1\right)\left(n_2^2 - \nu^2 - 1\right)}.$$

The general solution of (3) is

$$\Phi = a\sin\left(k_1 t + \beta_1\right) + b\sin\left(k_2 t + \beta_2\right) + a\sin t + b\cos t \tag{10}$$
$$\Psi = a\alpha_1 \sin\left(k_1 t + \beta_1\right) + b\alpha_2 \sin\left(k_2 t + \beta_2\right) + c\sin t + d\cos t,$$

where a, b, c, d are as above.

From the substitution $\varphi = e^{-\nu}\Phi$, $\psi = e^{-\nu}\Psi$, (10) becomes

$$\varphi = e^{-\nu}\left[a\sin\left(k_1 t + \beta_1\right) + b\sin\left(k_2 t + \beta_2\right) + a\sin t + b\cos t\right] \tag{11}$$
$$\psi = e^{-\nu}\left[a\alpha_1 \sin\left(k_1 t + \beta_1\right) + b\alpha_2 \sin\left(k_2 t + \beta_2\right) + c\sin t + d\cos t\right].$$

4. EXAMPLES:

Let consider the following dynamic systems.

1. $\ddot{\varphi} + 2\nu\dot{\varphi} - 0,074\psi + 8,98\varphi = e^{-\nu t}\left(A\sin t + B\cos t\right)$ (12)

$$\ddot{\psi} + 2\nu\dot{\psi} - 0,061\varphi + 8,97\psi = e^{-\nu t}\left(C\sin t + D\cos t\right)$$

$$\varphi(0) = \psi(0) = \dot{\varphi}(0) = \dot{\psi}(0) = 0$$

We write $x(t)$ ve $y(t)$ instead of $\varphi(t)$ ve $\psi(t)$ in graphs, respectively.

In figure 2, the motion is asymptotic stable and completes each full tour in 6.2 seconds.

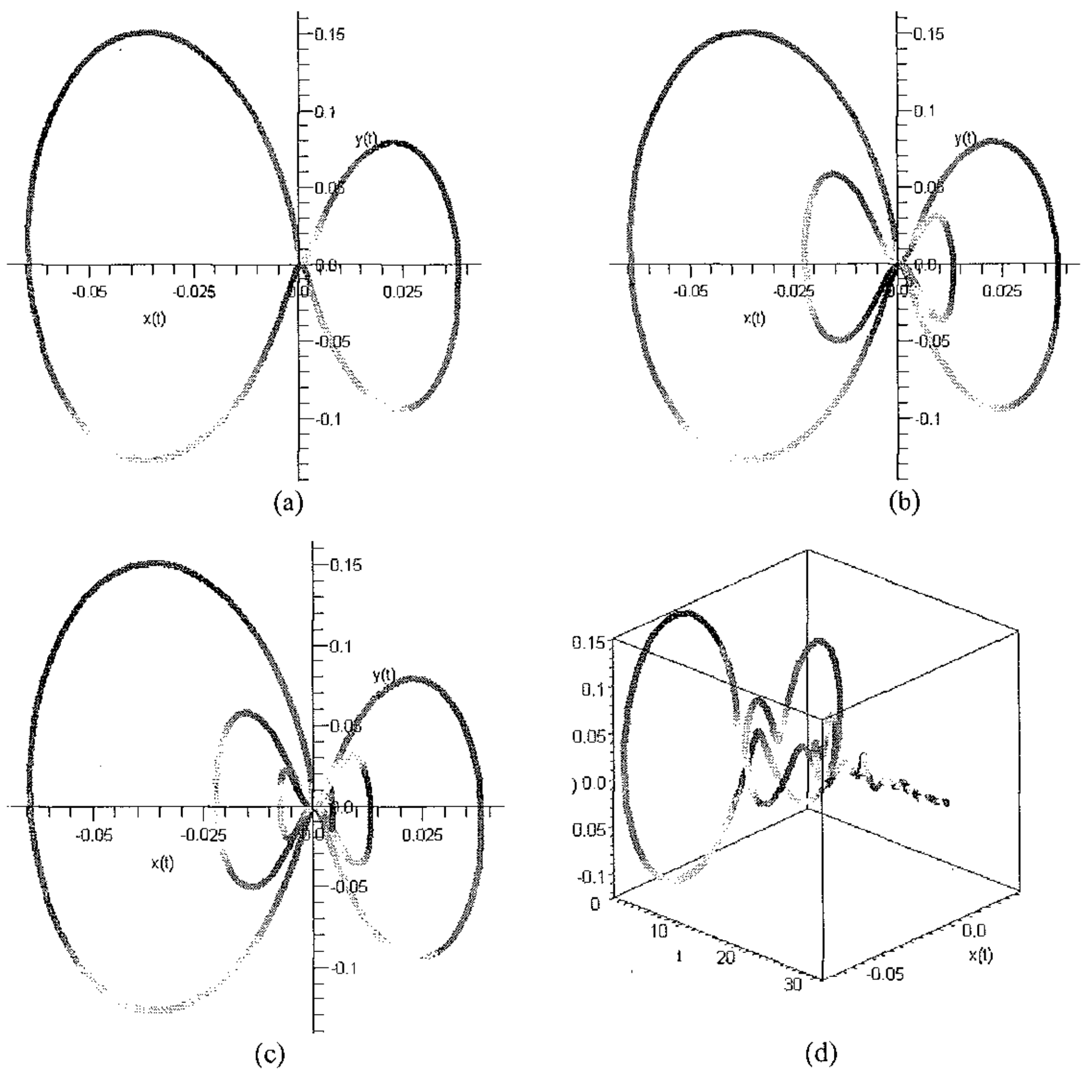


Fig. 2: $\nu = 0.15$, $A = -0.5$, $B = 0$, $C = 0$, $D = 0.9$

(a) $0 \le t \le 6.2$ (b) $0 \le t \le 12.4$ (c) $0 \le t \le 18.6$ (d) $0 \le t \le 31$, 3 − dimensions

In figure 3, the motion is asymptotic stable and completes each full tour in 6.25 seconds. Since the masses of the pendulums are equal, their orbits are almost similar.

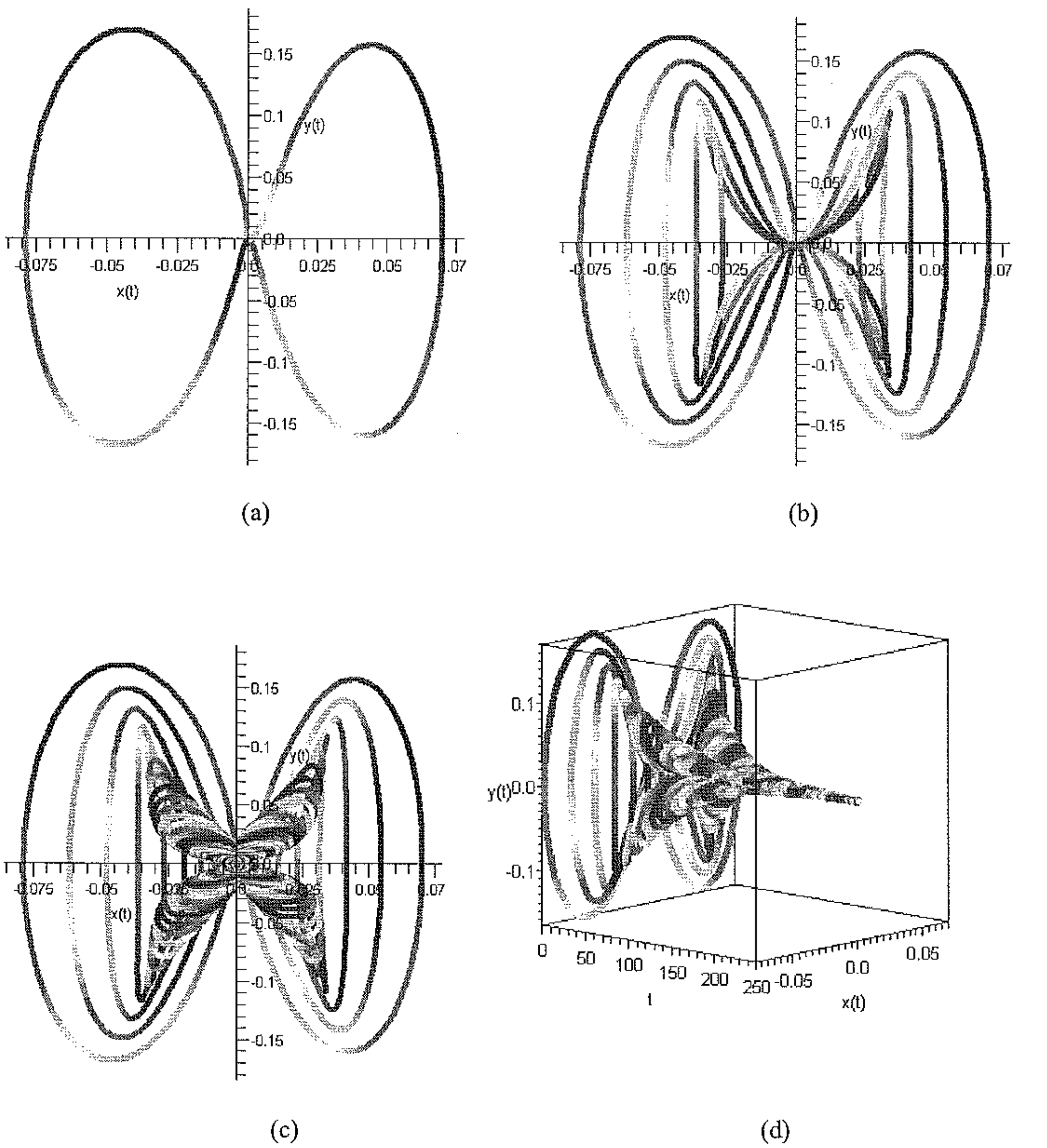
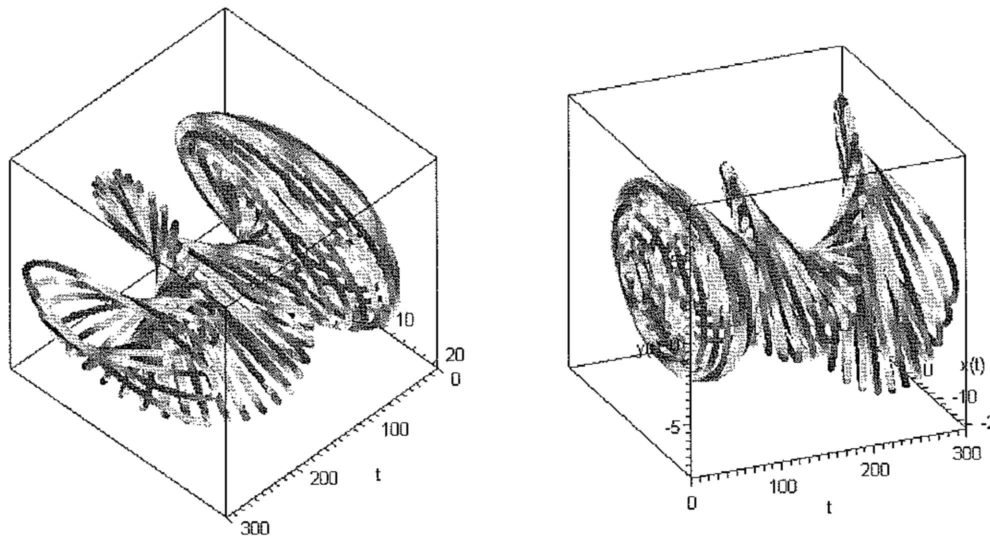


Fig. 3: $\nu = 0.02$, $A = -0.5$, $B = 0$, $C = 0$, $D = 0.9$, $m_1 = m_2$,
(a) $0 \le t \le 6.25$ (b) $0 \le t \le 31.2$ (c) $0 \le t \le 250$ (d) $0 \le t \le 250$

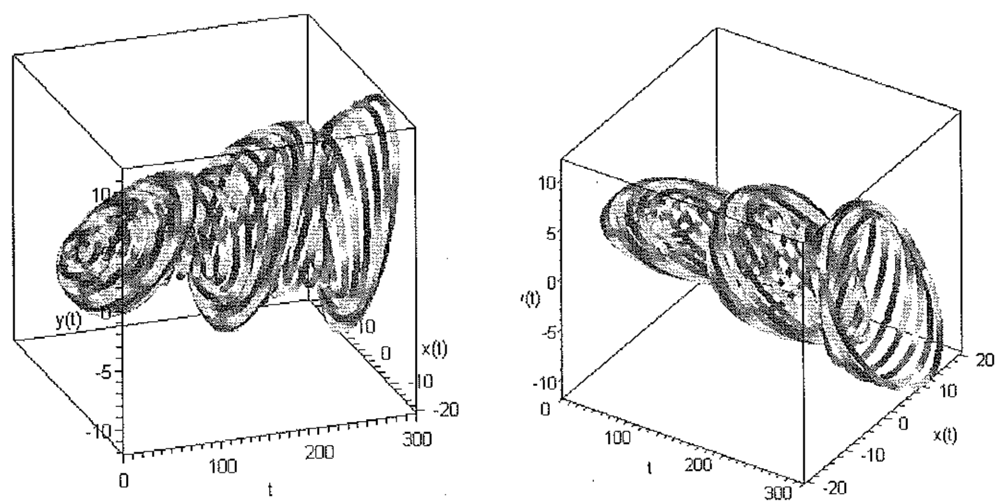
2.

$$\begin{aligned}&\ddot{\varphi}+2\nu\dot{\varphi}-0{,}0074\psi+0{,}898\varphi=e^{-\nu t}\left(A\sin t+B\cos t\right)\\&\ddot{\psi}+2\nu\dot{\psi}-0{,}0061\varphi+0{,}897\psi=e^{-\nu t}\left(C\sin t+D\cos t\right)\\&\varphi(0)=\psi(0)=\dot{\varphi}(0)=\dot{\psi}(0)=0\end{aligned}\qquad(13)$$



(a) (b)

Fig. 4. (a), (b): $\nu=0.000025,\ A=0.9,\ B=-0.5,\ C=0.2,\ D=0.3,\ \ 0\le t\le 300$



(a) (b)

Fig. 5. (a), (b): $\nu=25.10^{-12},\ A=0.9,\ B=0.5,\ C=-0.2,\ D=-0.3,\ \ 0\le t\le 300$

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REFERENCES

1. WIRSCHfNG, P.H., PAEZ, T.L., ORTIZ, K., 1995, *Random Vibrations*; Theory and Practice; John Willey & Sons İnc.
2. BOLOTİN, V.V. 1995, Nonlinear Stability of Structures ; Theory and Computational Techniques, Dynamic Stability of Structures, Springer-Verlag, N.Y., 3-211-82651-3.
3. SEYRANIAN, A P., MAİLYBAEV, A.A., 2004, *Multiparameter Stability Theory with Mechanical Applications,* World Scientific Pub. Co.
4. BUTENİN, N.V., NEIMARK, Yu.L, FUFAEV, N.A., 1987, *İntroduction to* Nonlinear Vibrations Theory, Nauka, Moskova.
5. HIRSCH, M.W., SMALE, S., DEVANEY, R.L., 2004, *Differential Equations,* Dynamical Systems and an İntroduction Chaos, Elsevier Acad. Press, 0-12-349703-5.
6. KUZNETSOV, Y.A., 2004, *Elements of Applied Bifurcation Theory,* Sphıger Verlag, 0-387-21906-4.
7. BOLOTİN, V.V., 1964, *The Dynamic Stability of Elastic Systems,* Holden-Day, İnc, San Francicsco, London.
8. CICEK, C., 2007, *Quasilinear Dynamic Systems and Their Applications to Viscoelastic Vibrations*, PhD. Thesis, I.U. Institute of Science, Istanbul.