İstanbul Üniv. Fen Fak. Mat. Dergisi **61-62** (2002-2003), 105-107

Obituary for Professor Kamil Alnıaçık

Yann Bugeaud

The mathematical contribution of Professor Kamil Alnıaçık concern mainly Diophantine approximation, in particular the study of the classification of real numbers introduced in 1932 by Kurt Mahler. For any real number ξ and any positive integer n, denote by $w_n(\xi)$ the supremum of the real numbers w such that the equation $0 < |P(\xi)| < \mathrm{H}(P)^{-w}$ has infinitely many solutions in integer polynomials $P(X)$ of degree at most n, where $\mathrm{H}(P)$ denotes the maximum of the absolute values of the coefficients of $P(X)$. Mahler classified the transcendental numbers ξ into three classes, according to the behaviour of the sequence $(w_n(\xi))_{n\geq 1}$. For instance, the U-numbers are precisely the real numbers ξ for which $w_n(\xi) = +\infty$ from some integer n onwards. This set is very small, in the sense that it has Lebesgue measure zero and even Hausdorff dimension zero. A celebrated example of a U-number is given by the series $\sum_{n\geq 1} 10^{-n!}$.

In many papers, Professor Kamil Alnıaçık studied the set of U-numbers and its subsets introduced by LeVeque in 1953, who called U_m-number any real number ξ for which $w_m(\xi)$ is infinite, but $w_n(\xi)$ is finite for $n = 1, \ldots, m-1$. Obviously, the set of U-numbers is exactly the disjoint union of the sets of U_m-numbers. Professor Kamil Alnıaçık [1, 2, 3, 8, 9] refined the work of LeVeque and introduced the new notions of semi-strong and irregular semi-strong U_m-numbers. Furthermore, he gave new explicit examples of U_m-numbers and he established [1, 6] the p-adic analogues of his results.

In 1959, Erdős showed that any real number can be written as a sum of two U_1-numbers. Thanks to a very intricate construction, Professor Kamil Alnıaçık [5] was able to show that any real number (except U_1-numbers) can be written as a sum of two U_2-numbers, a result re-established a few years later by Pollington, but with a different proof. I particularly like the very short note [10], where an elegant four-lines proof of Erdős' result (actually, of a more general result) is given. I should also mention the two works [4, 7] on T-numbers, but they unfortunately contain some gaps.

Recently, Professor Kamil Alnıaçık [11, 13] investigated a problem posed by Mahler, who asked which analytic functions f have the property that if ξ is any U_1-number, then so is $f(\xi)$. He kindly sent me his manuscript [13] a few months ago.

During the writing of our joint work [12], I exchanged many e-mails with Professor Kamil Alnıaçık and I enjoyed very much our collaboration. This is with very much sadness that I learnt his death. I deeply regret that I never met him.

References

[1] *On the subclasses U_m in Mahler's classification of the transcendental numbers*, Istanbul Üniv. Fen Fak. Mecm. Ser. A 44 (1979), 39–82.

[2] *On U_m-numbers*, Proc. Amer. Math. Soc. 85 (1982), 499–505.

[3] *On Mahler's U-numbers*, Amer. J. Math. 105 (1983), 1347–1356.

[4] *On T-numbers*, Glasnik Math. 21 (1986), 271–282.

[5] *Representation of real numbers as sums of U_2-numbers*, Acta Arith. 55 (1990), 301–310.

[6] *On p-adic U_m-numbers*, Istanbul Üniv. Fen Fak. Mat. Der. 50 (1991), 1–7.

[7] *On p-adic T-numbers*, Doğa Math. 16 (1992), 119–128. [8] *On semi-strong U-numbers*, Acta Arith. 60 (1992), 349–358.

[9] *A note on some U-numbers*, Indian J. Pure Appl. Math. 25 (1994), 689–691.

[10] (with É. Saias) *Une remarque sur les G_δ-denses*, Arch. Math. 62 (1994), 425–426.

[11] *The points on curves whose coordinates are U-numbers*, Rend. Mat. Roma, Serie VII, 18 (1998), 649–653.

[12] (with Y. Avci and Y. Bugeaud) *On U_m-numbers with small transcendence measures*, Acta. Math. Hungar. To appear.

[13] *The points on algebraic curves whose coordinates are U_2-numbers.* Unpublished text.

Université Louis Pasteur
U. F. R. de mathématiques
7, rue René Descartes
67084 STRASBOURG

e-mail : bugeaud@math.u-strasbg.fr