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A COMPARISON OF OPTIMAL PORTFOLIO PERFORMANCES OF THREE OPTIMIZATION METHODS

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Abstract:

This study compares performances of three portfolios established based on Markowitz optimization, shrinkage optimization, and Black-Litterman optimization. BIST30 companies are used to test the results. Markowitz optimization is unrestricted, thus generates the highest possible utility. However, portfolio weights display high values of short- selling needs. Shrinkage optimization restricts short selling needs gradually, but it does not block short- selling. On the other hand, Black- Litterman model totally prohibits short- selling. Results show that the lowest utility is originated by Black- Litterman model. Shrinkage model generates average returns and less- than- average risk. Therefore, shrinkage ratio is a strong candidate for future portfolio building. The results also suggest that short selling should be included in portfolio activities to maximize performance. Short- selling improves portfolio performance significantly

Keywords: *Optimization techniques, portfolio choice and investment decisions.*

1. Introduction:

Increasing investments to stock markets has special importance in emerging countries. Finance policies are established in a way to attract more investors and increase financial trust, efficiency, and profits from the stock exchanges. Efficiency and returns can be measured short- term and long- term and it is expected that policies are set to attract more long- term investors. As number of long- term investors increase in a stock market, volatility reduces, trust is built and efficiency increases. Thus, increasing profitability and efficiency has a special importance in stock markets.

This article compares the performances of three optimization models by using stocks from BIST30. The optimization models to compare are Markowitz optimization, shrinkage optimization, and Black- Litterman model of optimization. The results suggest that Markowitz optimization provides the highest return and risk combination for an investor. That optimization is very attractive for risk- lover investor type. Black- Litterman optimization provides very low return and risk combination, which could be a good address for risk- averse investors. However, the return is lower than the short- termed treasury bills. Therefore, Sharpe ratio of Black- Litterman portfolio is negative. On the other hand, shrinkage portfolio presents a higher than medium return, and a medium risk that would be very attractive for a medium risk- averse investor.

The results have some suggestions to investors. Avoiding risk is an advice for every investor, however taking risk is the main way of making profits. Limiting short- selling would benefit highly risk- lover investors in extreme market conditions but on the other hand, it would harm medium risk- lover investors. Shrinkage portfolio in this study, allows short- selling up a medium level. The performance of the portfolio is remarkable. On the other hand, Black Litterman portfolio is suffering from restrictions.

One contribution of this article is that it is the first article that compares the three optimization methods using Turkish stock market to our knowledge. Second, portfolio selection period is three years, which represents the long-term investments. Representing long- term investments is a rare choice, for most of previous studies form portfolios for 3- or 6- months. Third, the performances of the three method are so clear that they suggest a policy: Use more short- selling.

The paper is organized as follows. Section 2 reviews the literature of Markowitz optimization, shrinkage optimization, Black- Litterman optimization and empirical studies that compare optimization methods in Turkey. Section 3 explains the theoretical foundations of the three models. Section 4 introduces data and hypothesis of the study. Section 5 presents empirical results. Section 6 concludes.

2. Literature Review

a. Markowitz Optimization

The first model that explains investment behavior in mathematical terms is Markowitz optimization model based on Markowitz's cornerstone work in 1952 (Markowitz, 1952). Markowitz, detailed his idea in a book (Markowitz, 1959). In his book, he introduces a quadratic utility function that depicts attitude of a risk- averse investor. A risk averse investor likes higher return but is afraid of higher risk: therefore, he holds a tradeoff between risk and return. Markowitz define risk by using several tools, most accepted of which, is standard deviation. By maximizing the utility function, an investor maximizes her return and minimizes risk (Rubinstein, 2002). Asset returns that have negative correlation with each other should be selected for a portfolio so that the total risk of the portfolio will be reduced (Markowitz, 1952).

Benefits of Markowitz (Mean- Variance) optimization can be summarized as follows (Michaud,1989): Optimization provides a framework for investor constraints in the model setup, investor is free to choose a level of risk; exposure to various risk factors, stock universe, and performance benchmarks can be chosen; performance of portfolio is not directly dependent on the performance of individual stocks; due to simplicity in implementation, it allows timely portfolio changes.

Similarly, the two most important problems with the Markowitz model can be summarized as following (Michaud, 1989; Norstad, 2011): First, it is a purely mathematical model, it does not have investment sense and the portfolios may not have investment value. Performance of a portfolio depends on the structure of variance- covariance matrix. Therefore, unintuitive portfolios can come out. Second, the model maximizes estimation errors. Risk and return estimates are subject to estimation errors. The model overweighs securities with large estimated returns, negative correlations, and small variances. It underweights securities with small estimated returns, positive correlations, and large variances. Therefore, estimation can be large. Intuitive constraints should be added to the model to reach meaningful portfolios.

b. Shrinkage Optimization

Due to sensitivity of portfolio weights to mean and variance of portfolios and also due to large estimation errors, classical Markowitz optimization needs to be improved (Elton and Gruber, 1973; Jobson and Korkie, 1981; Jorion, 1985; Jorion, 1986; Bengtsson and Holst, 2002; Chan, Karceski and Lakonishok, 1999; Disatnik and Beninnga, 2007; Laloux, Cizeau, Bouchaud and Potters, 1999; Laloux, Cizeau, Bouchaud and Potters, 2000; Ledoit and Wolf, 2003; Ledoit and Wolf, 2004; Plerou,Gopikrishnan, Rosenow, Amaral and Stanley, 1999; Won, Lim, Kim and Rajaratnam, 2009). A shrinkage method is an improvement which suggests mixing of covariance matrix with a smaller version of itself. A covariance matrix contains the interactions of all asset returns. We call the matrix a pure- diagonal matrix if we demote the covariances between assets to zero, and keep only the variances with the same asset. A pure- diagonal matrix assumes no relationship among assets, and therefore imposes certainty to the problem. We can assign weight to covariance matrix and pure- diagonal matrix in order to create a mixed covariance matrix. In this framework, the covariance matrix represents the Markowitz world; and the pure- diagonal matrix represents the certainty case, in which assets have no covariance. This method partly protects us from the disadvantages of Markowitz optimization. However, this method is not a new method that hedge against the problems caused by Markowitz; it only diminishes the problems. Besides, the choice of the weight for covariance and pure diagonal matrix has no prescription, it usually is a personal decision and depends on experience or solved by trial- and- error method (Pollak, 2011).

c. *Black Litterman Optimization*

Black and Litterman model was first published in 1991 (Black and Litterman 1991a). Black and Litterman (1991b), He and Litterman (1999), and Litterman (2003) are the examples of papers from the authors are the following contributions of the authors that completes the technical details. Several other researchers contributed the model by introducing updates, calibrations, and other applications (Bevan and Winkelmann, 1998; Satchell and Scowcroft, 2000; Drobetz, 2001; Firoozye and Blamont, 2003; Herold, 2003; Idzorek, 2005; Mankert, 2006; Bertsimas, Gupta and Kallus, 2013; Meucci, 2006; Meucci, 2008; Giacommeti, Bertocchi, Rachev and Fabozzi, 2007; Krishnan and Mains, 2005; Beach and Orlov, 2007; Braga and Natale, 2007; Martellini and Ziemann, 2007; Esch and Michaud,2012;Walters, 2014). Among all, one study is more important for this paper. Fusai and Meucci (2003) introduce a non- Bayesian version of the model. Their model is also called the shrinkage model. This model is largely used in applications.

d. *Empirical Studies that Compare Optimization Methods in Turkey*

Caliskan (2010) and Caliskan (2011) form several short termed portfolios from BIST30 companies to compare the performances of Markowitz and Black- Litterman optimization techniques. Zerey and Terzi (2015) form an efficient portfolio from BIST30 companies by employing Finnet Portfolio Advisor Program. The program makes selections based on Markowitz optimization theory. Similarly, Topal and Ilarslan (2009) form a portfolio from BIST30 based on Markowitz optimization by using Excel Solver. Kardiyen (2007) uses mean absolute deviation model, which transforms the portfolio optimization problem in linear programming to form an optimal portfolio in BIST30. Toraman and Yuruk (2014) test Markowitz quadratic base programming model to form efficient portfolios from BIST100 companies. Genel (2004) tests portfolio optimization by genetic algorithms. He finds that genetic algorithms perform at least as good as Markowitz optimization, in many cases they perform much better. Ayan and Akay (2013) develop new measures of performance, and a new optimization technique. Their empirical tests on BIST30 companies show that the new method is more efficient than modern portfolio theory optimization results.

3. Model

The optimization problem for all methods can be defined as following (Beninga, 2008, p. 261-266).

$$Max\ \theta = \frac{E(r_x)-c}{\sigma_x} \tag{1}$$

s.t.

$$\sum_{i=1}^{N} x_i = 1, \qquad x_i = 1, \dots, N \tag{2}$$

where

$$E(r_x) = x^T.R = \sum_{i=1}^{N} X_i \,.\, E(r_i) \tag{3}$$

$$\sigma_p = \sqrt{x^T.\ S_x} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} x_i.x_j.\sigma_{ij}} \tag{4}$$

E(rx) in equation (1) represents the expected return of the portfolio, c is a lower benchmark on returns (such as the risk- free rate), σx represents the volatility of the portfolio. The object of the maximization problem is to maximize the Sharpe ratio. By this means, return at a certain level of risk is maximized. The only restriction of the maximization is the restriction on weights, which is presented in equation (2). X represents the number of assets in the portfolio. The sum of weights should be equal to 1. The last two equations display how expected return (Eqution 3) and volatility (equation 4) of the portfolio are computed. Expected return is the arithmetic average of asset returns, while volatility is the geometric average of asset volatilities of asset returns and their covariance terms. Each optimization type mentioned below uses these terms.

a. Markowitz Optimization

We solve the below optimization problem for implications purposes in Markowitz optimization (Benninga, 2008, p. 357):

$$\begin{bmatrix} Efficient \\ Portfolio \\ Proportions \end{bmatrix} = \begin{bmatrix} Var - Covar \\ Matrix \end{bmatrix}^T * \left\{ \begin{bmatrix} Expected \\ Portfolio \\ Returns \end{bmatrix} - Risk - free\ Rate \right\}$$

b. Shrinkage Method

This model is an enhanced version of Markowitz model. The only difference from the Markowitz optimization is to establish shrinkage variance- covariance matrix. In this paper, I adopt the following formula (Benninga, 2008, p. 308):

$$\begin{bmatrix} \text{Shrinkage} \\ \text{Var} - \text{Covar} \\ \text{Matrix} \end{bmatrix} = \lambda. \begin{bmatrix} Var - Covar \\ Matrix \end{bmatrix} + (1-\lambda). \begin{bmatrix} Pure - Diagonal \\ Matrix \end{bmatrix}$$

According to the formula above, shrunk variance- covariance matrix is a convex combination of sample variance-covariance matrix and pure- diagonal matrix. The shrinkage factor λ is selected to maximize expected accuracy of the formula. This method is especially successful in small samples of assets. The intensity of shrinkage will tend to zero as sample size increases. Shrinkage factor being 0 is equal to Markowitz optimization (Schäfer and Strimmer, 2005).

c. Black- Litterman Model

Black- Litterman model can be perceived as a developed model of shrinkage optimization. Black- Litterman approach solves the problem of shrinkage optimization by weighting the variance- covariance matrix based on market capitalization (Benninga, 2008, p. 355-356). In this paper, we follow Benninga (2008, p. 359) methodology and solve the below equation:

$$\begin{bmatrix} Benchmark \\ Portfolio \\ Returns \end{bmatrix} =$$

$$\begin{bmatrix} Var - Covar \\ Matrix \end{bmatrix} \begin{bmatrix} Benchmark \\ Portfolio \\ Proportions \end{bmatrix} \left[\begin{bmatrix} Benchmark \\ Portfolio \\ Proportions \end{bmatrix}^T \begin{bmatrix} Var - Covar \\ Matrix \end{bmatrix} \begin{bmatrix} Benchmark \\ Portfolio \\ Proportions \end{bmatrix} \right] \begin{pmatrix} Risk - free \\ Rate \end{pmatrix}$$

4. Data and Hypothesis

We use BIST30 stocks to test the hypotheses. We obtain stocks' daily prices for 31.12.2012- 31.12.2014 period from Bloomberg. PGSUS prices were not available for most of the analysis period; therefore, we exclude this stock from analysis. We also obtained daily prices of XU100 index, which is a capitalization-weighted index composed of national market companies in Turkey (Bloomberg, 2016). We use it as a proxy for market return. We use Turkish treasury bills annual compound rate as a proxy for risk- free asset in the market. We obtain these rates from the statistics database of Republic of Turkey Undersecretariat of Treasury website (http://www.treasury.gov.tr/en-US/Mainpage). Market capitalization of companies were obtained from Finnet.

In order to compute daily stock returns, we deploy the following formula:

$$Rt=\ln(Pt+1/Pt) \tag{5}$$

Where Rt is the daily stock return at day t, and Pt is the daily stock price at day t. Then we computed excess returns over the risk- free rate:

$$\text{Excess returns}= Rt- Rf,t \tag{6}$$

The correlation table of excess returns is provided in Table 1. The table shows that some stocks have quite high correlations whereas others have lower or even negative correlations. Having negative correlations among asset returns ensures that there will be efficiency from diversification when we establish a portfolio.

Table 1: Correlation Table for Daily Returns of BIST30 Companies

Correlation Table	ULKER	AKBNK	ARCLK	ENKAI	EREGL	FROTO	GARAN	ISCTR	KCHOL	KRDMD	OTKAR	PETKM	SAHOL	SISE	TCELL	THYAO	TOASO	TUPRS	YKBNK	DOAS	BIMAS	VAKBN	CCOLA	TAVHL	HALKB	TKFEN	TTKOM	KOZAL	EKGYO
ULKER	1	0.37	0.34	0.29	0.29	0.34	0.38	0.41	0.39	0.36	0.34	0.32	0.37	0.38	0.24	0.44	0.40	0.29	0.41	0.35	0.36	0.41	0.30	0.32	0.39	0.32	0.30	0.15	0.05
AKBNK		1	0.53	0.41	0.45	0.50	0.90	0.87	0.68	0.61	0.49	0.54	0.75	0.59	0.49	0.59	0.52	0.55	0.87	0.55	0.49	0.85	0.40	0.34	0.84	0.57	0.54	0.32	0.03
ARCLK			1	0.39	0.36	0.51	0.54	0.53	0.53	0.43	0.42	0.38	0.52	0.49	0.33	0.45	0.50	0.45	0.56	0.47	0.39	0.54	0.36	0.35	0.53	0.41	0.44	0.23	0.06
ENKAI				1	0.32	0.34	0.40	0.43	0.38	0.33	0.29	0.36	0.39	0.38	0.33	0.38	0.32	0.33	0.42	0.36	0.36	0.42	0.25	0.31	0.42	0.34	0.32	0.21	-0.03
EREGL					1	0.33	0.45	0.48	0.43	0.48	0.32	0.42	0.41	0.42	0.34	0.44	0.37	0.32	0.47	0.35	0.32	0.46	0.31	0.36	0.46	0.36	0.35	0.23	-0.01
FROTO						1	0.50	0.50	0.55	0.42	0.43	0.36	0.52	0.48	0.36	0.45	0.56	0.46	0.51	0.48	0.42	0.51	0.36	0.33	0.49	0.44	0.43	0.21	0.02
GARAN							1	0.88	0.65	0.59	0.50	0.56	0.73	0.58	0.47	0.59	0.51	0.51	0.88	0.57	0.46	0.86	0.41	0.34	0.85	0.56	0.52	0.29	0.03
ISCTR								1	0.65	0.63	0.53	0.56	0.74	0.60	0.50	0.61	0.50	0.51	0.88	0.55	0.46	0.86	0.42	0.37	0.85	0.60	0.54	0.30	0.03
KCHOL									1	0.51	0.48	0.45	0.70	0.53	0.42	0.54	0.52	0.57	0.68	0.50	0.49	0.65	0.40	0.35	0.65	0.52	0.49	0.33	0.04
KRDMD										1	0.43	0.53	0.53	0.49	0.38	0.53	0.44	0.43	0.62	0.48	0.37	0.63	0.35	0.33	0.64	0.52	0.41	0.23	0.00
OTKAR											1	0.41	0.46	0.44	0.33	0.47	0.41	0.38	0.53	0.46	0.34	0.55	0.29	0.29	0.50	0.44	0.35	0.22	0.00
PETKM												1	0.52	0.51	0.37	0.48	0.35	0.41	0.58	0.42	0.36	0.58	0.35	0.30	0.59	0.49	0.41	0.19	0.01
SAHOL													1	0.57	0.48	0.57	0.49	0.57	0.75	0.51	0.49	0.73	0.41	0.39	0.71	0.55	0.55	0.28	0.00
SISE														1	0.44	0.55	0.49	0.46	0.61	0.46	0.46	0.58	0.32	0.37	0.59	0.48	0.50	0.24	0.06
TCELL															1	0.46	0.34	0.36	0.50	0.40	0.32	0.50	0.25	0.28	0.48	0.42	0.47	0.18	0.03
THYAO																1	0.45	0.43	0.60	0.51	0.41	0.60	0.38	0.33	0.59	0.55	0.50	0.24	0.05
TOASO																	1	0.42	0.52	0.53	0.38	0.50	0.30	0.34	0.50	0.47	0.45	0.24	0.01
TUPRS																		1	0.54	0.37	0.44	0.51	0.35	0.34	0.51	0.46	0.47	0.27	-0.01
YKBNK																			1	0.56	0.49	0.88	0.43	0.38	0.86	0.59	0.52	0.31	0.03
DOAS																				1	0.34	0.55	0.31	0.26	0.56	0.48	0.39	0.24	0.02
BIMAS																					1	0.46	0.36	0.36	0.45	0.39	0.42	0.23	0.00
VAKBN																						1	0.40	0.35	0.88	0.58	0.54	0.31	0.01
CCOLA																							1	0.25	0.41	0.33	0.36	0.20	0.05
TAVHL																								1	0.37	0.32	0.31	0.23	0.05
HALKB																									1	0.57	0.52	0.31	0.00
TKFEN																										1	0.48	0.18	0.01
TTKOM																											1	0.24	-0.01
KOZAL																												1	-0.01
EKGYO																													1

The sample characteristics of stocks are displayed in Table 2. Average annual returns range within -15% (KOZAL) and 12% (OTKAR). Minimum return is -21,40% (KOZAL) and maximum return is 15,92% (OTKAR). Median values are very close to zero, showing that number of positive and negative return days are almost even. Standard deviations range from 1,75% (TTKOM and BIMAS) to 3,42 (KOZAL). 22 of 29 companies have slightly negative skewness, indicating small left- tail risk. Kurtosis ranges between 5,96 (TOASO) and 0,84 (AKBNK). Beta ranges between 1,39 (HALKB) and 0,59 (ENKAI). Alpha is almost zero for all stocks. Sharpe ratio ranges between -4% (KOZAL) and 5% (OTKAR).

Table 2: Sample Characteristics of the Daily Returns

Summary Statistics	ULKER	AKBNK	ARCLK	ENKAI	EREGL	FROTO	GARAN	ISCTR	KCHOL	KRDMD	OTKAR	PETKM	SAHOL	SISE	TCELL
Average	0.09	-0.03	0.04	0.04	0.07	0.06	-0.03	-0.03	0.03	0.02	0.12	0.07	-0.02	0.05	0.00
Median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard Deviation	2.16	2.26	2.07	1.81	1.87	2.18	2.36	2.14	1.80	2.64	2.65	1.83	2.08	2.15	1.65
Skewness	0.27	0.09	-0.19	-0.36	-0.19	-0.72	-0.01	-0.34	-0.04	0.12	0.71	0.06	-0.26	-0.27	-0.30
Kurtosis	3.96	0.87	1.85	2.08	1.40	5.75	2.73	1.75	2.14	4.29	5.77	2.73	3.04	1.27	1.48
Minimum	-13.80	-9.87	-10.77	-10.54	-8.38	-15.53	-14.15	-11.24	-9.34	-16.03	-11.62	-10.35	-12.88	-10.96	-6.99
Maximum	11.33	9.25	8.85	7.25	7.81	8.53	12.40	7.54	8.29	12.78	15.92	8.87	8.42	7.18	6.18
Beta	0.68	1.32	0.84	0.59	0.67	0.87	1.38	1.24	0.89	1.17	1.00	0.75	1.11	0.96	0.63
Alpha	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R-squared	0.24	0.83	0.40	0.26	0.31	0.38	0.83	0.82	0.60	0.48	0.35	0.41	0.69	0.48	0.35
Sharpe Ratio	0.04	-0.01	0.02	0.02	0.04	0.03	-0.01	-0.01	0.02	0.01	0.05	0.04	-0.01	0.02	0.00

Table 2 (Continued): Sample Characteristics of the Daily Returns

Summary Statistics	THYAO	TOASO	TUPRS	YKBNK	DOAS	BIMAS	VAKBN	CCOLA	TAVHL	HALKB	TKFEN	TTKOM	KOZAL	EKGYO
Average	0.04	0.10	0.05	-0.05	0.07	0.03	-0.02	0.00	0.11	-0.06	-0.07	0.00	-0.15	-0.02
Median	0.15	0.00	0.01	0.00	0.20	0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard Deviation	2.35	2.53	1.94	2.14	2.97	1.75	2.41	2.12	2.33	2.43	2.07	1.75	3.42	2.15
Skewness	-0.34	-0.50	-0.22	-0.32	-0.93	-0.19	-0.36	0.02	0.01	-0.24	-0.86	-0.03	-0.28	-0.41
Kurtosis	4.39	5.96	1.80	3.08	5.94	2.51	2.27	1.12	1.57	3.60	5.51	0.79	4.00	4.92
Minimum	-16.18	-19.59	-9.14	-12.14	-17.26	-8.24	-11.58	-8.66	-8.47	-13.17	-14.82	-6.86	-21.40	-13.30
Maximum	10.34	13.35	6.81	9.66	12.78	7.60	11.14	7.72	9.19	13.64	7.16	6.78	14.38	11.35
Beta	1.09	1.00	0.81	1.24	1.22	0.68	1.38	0.69	0.70	1.39	0.89	0.73	0.80	0.03
Alpha	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R-squared	0.52	0.38	0.42	0.82	0.41	0.37	0.80	0.26	0.22	0.79	0.45	0.42	0.13	0.00
Sharpe Ratio	0.02	0.04	0.03	-0.02	0.02	0.02	-0.01	0.00	0.05	-0.03	-0.03	0.00	-0.04	-0.01

We impose no restrictions on Markowitz optimization model. Therefore, it is expected that Markowitz portfolio performs better than the shrinkage portfolio. Black- Litterman model uses market capitalizations as a weight for the variance- covariance matrix (we can call it a restriction). Therefore, Markowitz portfolio is expected to perform better than Black- Litterman, too. Since the restrictions on Shrinkage portfolio is less than the restrictions on Black-Litterman portfolio, we expect shrinkage portfolio performs better than Black- Litterman portfolio. We use Sharpe ratios as a measure of performance measure. Our hypothesis can be stated as the following:

H0: SRMarkowitz > SRShrinkage >SRBlack- Litterman,
H1: Otherwise.

5. Analysis

This study establishes three portfolios from daily stock returns of BIST30 firms for January 2013- December 2015 period and compares portfolio performances. The first portfolio is optimized by Markowitz model principles; thus we call it Markowitz portfolio. we compute variance- covariance matrix and asset weights based on the method described in Benninga (2008, p. 294-301). The weights obtained from Markowitz optimization is presented in the first column of Table 3. The weights contain many negative values. Negative weights correspond to short selling. In turn, there are weights greater than 1. Investor borrows and sell the assets which have negative weights, and buys the assets which have weights more than 1.

Short- selling is the sale of a security that is not owned by the seller. tHe seller borrows the security to pay back in the future, sells the security in the market. After a while, the seller buys the security from the market and pays back to the lender. Investors apply short- selling when they believe that the price of the security will decline. They aim to make profits from the price difference. The implication is that borrowing costs less than the profits made from capital gains. Short selling contains risks in its stem, but at the same time it can benefit an investor by yielding extra returns. Short- selling reinforces the high- risk, high- return attributes of portfolios. Sobaci, Sensoy and Erturk (2014) shows that short selling increases the performances of investments in BIST.

Table 3: Portfolio Weights based on 3 optimization models

	Markowitz Optimization	Shrinkage Optimization	Black Litterman Optimization
ULKER	0.55	0.53	0.02
AKBNK	0.56	-0.21	0.08
ARCLK	-0.13	0.10	0.03
ENKAI	0.07	0.14	0.05
EREGL	0.94	0.63	0.04
FROTO	0.00	0.24	0.03
GARAN	0.48	-0.17	0.08
ISCTR	0.18	-0.32	0.05
KCHOL	0.81	0.16	0.09
KRDMD	0.13	0.11	0.00
OTKAR	1.25	0.72	0.01
PETKM	1.92	0.76	0.01
SAHOL	-0.94	-0.35	0.05
SISE	0.36	0.25	0.02
TCELL	-0.49	-0.32	0.06
THYAO	0.57	0.24	0.03
TOASO	1.19	0.60	0.03
TUPRS	1.28	0.50	0.05
YKBNK	-3.62	-0.73	0.04
DOAS	0.65	0.33	0.01
BIMAS	-0.17	0.00	0.04
VAKBN	1.55	-0.09	0.03
CCOLA	-0.17	-0.13	0.02
TAVHL	1.27	0.68	0.01
HALKB	-2.35	-0.57	0.03
TKFEN	-2.72	-1.05	0.00
TTKOM	-0.87	-0.38	0.06
KOZAL	-0.94	-0.47	0.01
EKGYO	-0.37	-0.20	0.03
SUM	1.00	1.00	1.00

The second portfolio is shrinkage portfolio. we computed variance covariance matrix, pure diagonal matrix and the shrunk matrix based on the methods defined in Benninga (2008). we determined the shrinkage factor at 0.5. There is no academic research and intuition behind the value of shrinkage factor. It is basically determined by practical factors and risk- averseness of the researcher. A medium value would represent a more or less risk neutral behavior. Benninga (2008) uses the shrinkage factor as 0,3; however, this value is attained by trial- and- error. There is no mathematical intuition has been developed for this value to our knowledge. The weights obtained from shrinkage optimization is presented in the second column of Table 3. There are short- sales, but their number and density is much lower than that of Markowitz optimization. This method allows short sales, but limit them at lower amounts. Therefore, it contains less risk than Markowitz optimization technique.

The last model is Black- Litterman optimization. We followed the methods described in Benninga (2008, p.358) to form the portfolio. The weights are presented in the third column of Table 3. There are no negative weights, and all stocks earn positive but small weights. This method was developed in a way that does not allow short- selling, and thus preferred by risk- averse investors.

Lastly, performances of the portfolios are compared. To do that, I compute the expected daily returns of each portfolio by using the weights. Then, I compute annual average return and annualized standard deviation of each portfolio; and lastly I compute Sharpe ratios of each portfolio. Sharpe ratio is the most applied performance measure in finance. It measures the excess return- risk tradeoff. The higher the Sharpe ratio, the higher the performance. Annual average returns, standard deviations, and Sharpe ratio of each portfolio is presented in Table 4. The Sharpe ratios of the three portfolio are 0,54; 0,49; -0,03. It is clear that the highest performance belongs to Markowitz portfolio. Shrinkage portfolio is a step behind the Markowitz. However, the performance of Black_ Litterman is way behind the other two portfolios. First of all, Sharpe ratio of Black-Litterman portfolio is negative because the annual average return of the portfolio is less than the annual average return of the risk- free rate. Secondly, even though the ranking is expected to be in this order, the very low performance of Black- Litterman is surprising as former studies in the field finds Black- Litterman almost as successful as Markowitz if not more (Caliskan, 2011).

Table 4: Performance of Portfolios

	Markowitz Portfolio	Shrunk Portfolio	Black Litterman Portfolio	Rf
Annual Average Return	0.17	0.09	0.00	0.00
Annual Standard Deviation	0.32	0.17	0.05	0.00
Sharpe Ratio	0.54	0.49	-0.03	

These results suggest that investing by the highly risk-averse instincts would harm investors in the Turkish stock market. Turkey is an emerging market and it is highly risky by the nature. Sharing higher level of risk would benefit the investor in the long- run. It also suggests that prohibiting short- sales mitigates the efficiency of financial portfolios and cause redundant efforts to increase yields.

6. Conclusion:

The results of this article suggest that Markowitz optimization provides the highest return and risk combination for an investor. Risk- seeking investor would love this type of portfolios. Black- Litterman optimization provides very low return and risk combination, which could address highly risk- averse investors. However, the annual average return of Black- Litterman portfolio is lower than the return of the short- termed treasury bills. Therefore, Sharpe ratio of Black- Litterman portfolio is negative. On the other hand, shrinkage portfolio presents a higher than medium return, and a medium risk that would be very attractive for a medium risk- averse investor.

As a conclusion, we can suggest investors not to avoid risk all the times. Limiting short- selling would benefit in negative extreme market conditions but on the other hand, it would harm medium risk- lover investors. Shrinkage portfolio in this study, allows short- selling up a medium level. The performance of the portfolio is remarkable. On the other hand, Black Litterman portfolio is suffering because restrictions on short- selling prohibits the portfolio from making profits. Turkish Stock Exchange does not put short- selling restrictions on stock or derivatives market. The results of this paper, along with Sobaci, Sensoy and Mutahhar (2014) support this policy.

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