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GIYASEDDIN JEMSHID EL-KASHI AND STALACTITES

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INTRODUCTION

Stalactite is one of the elements that act as a common denominator between various architectural styles in the medieval Islamic world. It has always been a curious and interesting subject for architectural historians. Ever changing light and shade effects, a sense of infinity and deeper symbolic meaning created by stalactite ceilings leave a magical impression on the viewer. Its apparently complex geometry is a challenge for researchers. But Giyaseddin Jemshid el-Kashi, in his book *Miftah el-Hisab* (Key for Arithmetic), had a different approach to stalactites [1] :

> the length at the base of the largest side is called the scale of the stalactite... All sides... are equal to each other and equal to the scale [2].

Such a simplified concept of a modular system may serve as a key to resolve stalactites. If what he said is true and stalactites were indeed designed according to a rational modular geometry, we will be able not only to solve the 'Sufi secrets', but also to reach medieval builders' knowledge in geometry [3].

But before reaching any conclusions, there is a lot to be studied. El-Kashi and his interest in stalactites deserve more attention in order to acquire a thorough understanding of the section on stalactites in his book. Only then, his account can safely be tested on existing examples of stalactites.

EL-KASHI'S LIFE AND HIS WORKS

El-Kashi was born and educated in Kashan, a town in central Persia. He made observations about a lunar eclipse on 2 June, 1406 in Kashan where he wrote the following treatises:

1. This book has an important place in the history of science. Since it was completed and presented to Uluğ Bey in 1427, numerous manuscript copies and several publications have been made. We have recorded 20 manuscripts (10 of them are in İstanbul) in various libraries of different countries and 3 publications. The complete list is presented in the Appendix A.

2. See Note 7 (page 37-38).

3. In el-Hayat newspaper, 22 February, 1992, Muhammed el-Esad claims that the geometry of stalactites was regarded as 'Sufi secrets' which passed down through generations by those people who belonged to the Sufi sect.

Sullem el-Sema (1 March, 1407, Arabic) on the sizes and distances of the celestial bodies;

Hakani Zic (1413/4, Persian) on the improvements of *Ilhani Zic*, written by Nasreddin el-Tusi;

An untitled treatise (January, 1416, Persian) on astronomical instruments mentioned in the *Almagest* and by earlier astronomers;

Nuzhet el-Hedayik (10 February, 1416, Arabic), on the description of the equatorium he invented (Kennedy, 1960, 1-2; Vernet, 1974, 703).

From several references in his books we learn that el-Kashi spent most of his time in Kashan during this period and visited several towns in central Persia for his astronomical works, where he lived in poverty (Kennedy, 1960, 1). He dedicated his first treatise to a vezir named Kemaleddin, the second either to Şah Ruh or to Uluğ Bey of the Timurid dynasty, and the third to Sultan İskender of the Black Sheep dynasty (Kennedy, 1960, 2; Barthold, 1963, 130). By dedicating his treatises to leaders of rival dynasties, el-Kashi was apparently seeking a royal patronage but cautiously avoiding to show any preferences.

For about five years after 1416, we have no information about el- Kashi's works. During this period, Semerkand has already become a major center of scientific activities, thanks to Uluğ Bey. He was acting almost as an independent ruler in Transoxania (*Maveraünnehir*) and he was a scientist himself. In those years there were more than one hundred scientists in Semerkand gathered around Uluğ Bey. The most prominent among them was his former tutor, Kadızade-i Rumi [4]. They held regular scientific meetings and discussed mainly mathematical and astronomical topics (Sayılı, 1960, 13- 15). Uluğ Bey was also a great patron in architecture. His Medrese in Buhara was completed in 1417 (Knobloch, 1972, 164). The construction of his Medrese in Semerkand was started in the same year and finished in 1420 (Barthold, 1963, 119). Scientific activities in Semerkand naturally shifted to this medrese after this date. Probably during these meetings, Uluğ Bey noticed the deficiencies in the astronomical tables of earlier times and decided to build an observatory in Semerkand to set up new observations (Sayılı, 1960, 43). On Kadızade's advice, Uluğ Bey invited el-Kashi to Semerkand to be involved in the construction of the observatory (Barthold, 1963, 130).

4. His real name is Musa Paşa b. Muhammed b. Mahmud. He was born in Bursa and completed his education there. He moved to Semerkand in order to advance his studies. There, he was known by the name 'Rumi' because of his Anatolian connection (Adıvar, 1970, 14- 15).

El-Kashi arrived in Semerkand towards the end of 1420 or 1421 (Sayılı, 1960, 11-12). In the letter he wrote to his father about a year later the mentioned that he was in charge of the construction of the observatory and on his persistence a huge meridian arch (as el-Kashi called it, a 'geometrical *minber*' with the name *Suds-i Fahri*) was built [5] (Sayılı, 1960, 99, 51). Construction was finished in a short time, around 1423 (Sayılı, 1960, 11). In July 1424, el-Kashi completed his *el-Risale el-Muhitiye* in which he determined the *pi* value with an unprecedented precision (Kennedy, 1960, 5). Closeness of the dates cannot be a coincidence. As reflected in his letter, he took the construction of this meridian arch as a challenge and, true to his nature, probably determined its curvature with utmost precision and wrote a treatise about it afterwards. This assumption also suggests a close relation between his current activities and his scientific works. In June 1426, he finished the second version of *Nuzhet el-Hedayik* and on 2 March, 1427, he completed his major work, *Miftah el-Hisab*, and presented it to Uluğ Bey (Kennedy, 1960, 6). In this work, which has an outstanding place in the history of mathematics and which concerns us most, el-Kashi explained arithmetical operations, taught the method of extracting roots by the system which today is called after Ruffini-Horner, calculated the Tartiglian triangle, computed the sum of series up to the fourth power of natural numbers, developed the sexagimal

5. Nothing is left from Uluğ Bey's observatory in Semerkand. As a result of the excavations by Russians in 1908, only a huge meridian arc was unearthed. According to descriptions of contemporary writers, the observatory was composed of three stories, its height was equal to that of St.Sophia in Istanbul and it contained pictures of heavenly bodies, mountains, seas, deserts, etc. (Barthold, 1963, 132).

system which was used since ancient Babylonians by astronomers, invented decimal fractions which were not known in Europe before 1585, and dealt with regular and semi-regular (the five Platonic and to Archimedean) bodies (Vernet, 1974, 703; Schirmer, 1936, 518). He also described arches, domes, stalactites by their types and calculated their areas.

In his last work, *Risale el-veter v'el jaib*, el-Kashi illustrated an advanced iterative method of computing the sine of one degree to any required accuracy. But unfortunately before finishing his last work, he died on the morning of Wednesday 22 June, 1429 at the observatory outside Semerkand (Kennedy, 1960, 6-7).

In addition to the above mentioned works, el-Kashi wrote six more short treatises. One of these concerns us since it is about orientation of the *Kible* direction by astronomical observations. On 14 August, 1589, el-Kashi's great-great-great-grandson el- Rezzak had proudly completed a manuscript copy of *Miftah el-Hisab* (British Library, Add. 7470).

EL-KASHI'S PERSONALITY

El-Kashi was praised by his contemporaries as 'the second Ptolemy' and the next generation was calling a mathematician of their own time 'the second Giyaseddin Jemshid' (Kennedy, 1960, 9). He certainly was a scientist of a high calibre, a master of mathematics with extraordinary abilities, a competent observer, an ingenious inventor, and a prolific writer. But we have to know more about him in order to make a scrupulous evaluation of his account on stalactites. The letter to his father gives us some clues to partially assess his personal character.

General impression of the letter is self-praise and self-declared superiority over his colleagues. Judging merely by the number of works produced, he was most probably right. Looking at the sheer number of works produced, he was the most accomplished. The majority of his works was original and in some topics he was unique or ahead of his time, whereas others were usually working on commentaries. But anecdotes mentioned in the letter were always biased and were told only to prove his superiority. He was so obsessed with self-esteem that at times he even contradicted himself. When he wanted to emphasize his eminence as a scientist, he explained that the greatest authorities in all sciences were gathered together in Semerkand; to exemplify his superiority, however, he repeatedly illustrated their deficiencies and even characterized them as novices (Sayılı, 1960, 44-48). Our author mentioned only Kadızade's name among all these scientists. Evidently he was the only one who gave el-Kashi any competition at all. In speaking of Kadızade, he contradicted himself in the next sentence. After mentioning that Kadızade possessed the theoretical knowledge contained in the *Almagest*, he added that Kadızade was only a beginner in theoretical astronomy (Sayılı, 1960, 107).

El-Kashi was not only careless enough to contradict himself in the letter to his father, but also did not hesitate to misinform Uluğ Bey about Meragha Observatory. He was in favor of employing large astronomical instruments and he attached great importance to the special type of meridian arch, *Suds-i Fahri*. In order to convince Uluğ Bey for its construction, he unscrupulously stated that a geometrical *minber*, called *Suds-i Fahri*, was constructed in Meragha Observatory (Sayılı, 1960, 98). There was no such instrument in that observatory. According to Wilber (1969, 10), the height of the meridian was recorded on the pavement by the rays of the sun passing through the slit in the large dome.

6. There is no direct information on the Hashimi 'gez' used in Semerkand during this period. According to our unpublished study on measuring units in Islamic countries, we can assert by circumstantial evidences that it was probably a unit of approximately 71 cm. A brief summary of the research leading to this conclusion is given below in Appendix B.

The following passage exhibits various aspects of el-Kashi. He was a quick-witted, resourceful mathematician; he was an ill mannered man, who did not hesitate to insult his colleagues in front of a large audience; he was a person with varied interests, who was involved in architecture later in his life:

> Another day, the ground had been leveled at the site of the observatory for the purpose of finding the meridian line. This had been performed by renowned masons, and the ground had become dry... We wished to check first to see whether the surface was level or not. His Majesty... and all people of high rank and notables, as well as the scientists, were present around the leveling instrument the masons had prepared for this work. For this purpose a triangle had been constructed, each side of which measured four Hashimi *gez* **[6]**.
>
> The son of the architect who is the head of the masons said that, as a precautionary measure, one should first check to see whether both sides of the triangle were equal or not. This servant said that even if they were not equal the leveling could be done with it. Kadızade and other masters who were conversant with these matters objected at once. They said, 'How can it be? This is impossible!' This servant said, 'Now the weather is still cool and the sun has not risen high. Let us first check the leveling. Then I shall explain why this is possible.'
>
> When the work was done, they came back to their question and asked me to prove my claim. We all set down, and I began to explain. This servant said, 'Suppose that in this triangle one of the sides, which according to your claim should be equal, is shorter than the other one by one *gez*.' I drew such a triangular figure and brought geometrical proofs bearing upon the question. For one sidereal hour I gave preliminary explanations and proofs of various kinds for it, until they throroughly understood it and gave their consent. Some who were more learned understood it in a shorter time and some others in a longer time, but they all gave their consent.
>
> This servant minced no words, for nearly five hundred of the distinguished personalities were present. I said, 'You can comprehend such an easy problem, on which I gave all these proofs, in two sidereal hours. God knows that with me such problems are self-evident. Indeed, in this case, the question was clear in my mind as soon as Master Ismail said we should check and see whether both sides are equal or not. What need was there for all this talk and argument.' (Sayılı, 1960, 101-102).

The surveying technology had not changed much in the fifteenth century since ancient times. Ancient Egyptian and Roman masons were using identical instruments for leveling: an A-shaped frame with a plumb-bob suspended from its vertex (Clarke and Engelbach, 1930, Fig.264; Neuberger, 1930, Fig.536). When the base is horizontal, the plumb-bob string is perpendicular to the horizontal bar at its mid-point **[Figure 1a]**. In medieval Europe, similar instruments were used by masons to check the leveling of wall courses. They are basically a straight-edge board with a raised part on top having a plumb-bob suspended from the center (Shelby, 1961, 129). In one example, the raised part is a semicircle **[Figure 1b1]**, in the other one it is a triangle **[Figure 1b2]**. These Egyptian, Roman and medieval leveling instruments were all designed according to the same geometric principle: the altitude of an isosceles triangle divides it into two equal right-angled triangles. There is no reason to doubt that the triangular leveling instrument prepared by Master Ismail was also designed according to the same principle, with a plumb-bob hanging from its vertex and a mark at the middle of its base **[Figure 1c]**. In modern Iran, masons are still working with similar instruments (Wulff, 1966, 111). In contrast to other examples, Master Ismail's level was huge in size, about 285 cm laterally, because it was meant to be used as a surveying instrument, not as a masons' tool.

When el-Kashi reacted to Master Ismail's suggestion, he was quick to realize the geometrical principle involved: as long as the surface is horizontal, the plumb-bob line is always perpendicular to the base, even if the triangle is not an isosceles one. He was also resourceful enough to apply his knowledge to an immediate practical problem. As el-Kashi did, let us suppose that one of the legs is shorter than the other one by one *gez*. Leveling can still be done by applying the triangle twice over the same location, but reversing it diametrically. If the ground is horizontal, the plumb-bob will not align itself with the middle point of the base but will mark another point which will be the same in both cases. This new point is the correct position **[Figure 1d]**. If the ground is not horizontal, the plumb- bob will mark two different points; the correct position can be set by taking the middle point between these two **[Figure 1e]**. Indeed, one contemporary author commented on el-Kashi's strength by saying that Uluğ Bey was obliged to put up with his boorish manners because he could not dispense with his assistance (Kennedy, 1960, 8).

EL-KASHI'S INTEREST IN STALACTITES

In the letter, el-Kashi was assuring his father:

> As to the advice you had given to the effect that as I am busy with the affairs of the auspicious observatory I should not occupy myself with any other science, especially prosody and the like, and I am obedient and submissive to the orders given (Sayılı, 1960, 93).

If he has occupied himself with poetry in Kashan, what could stop him to get involved with something else in Semerkand? Apparently his father knew our author and his tendencies well enough to warn him against any such involvements

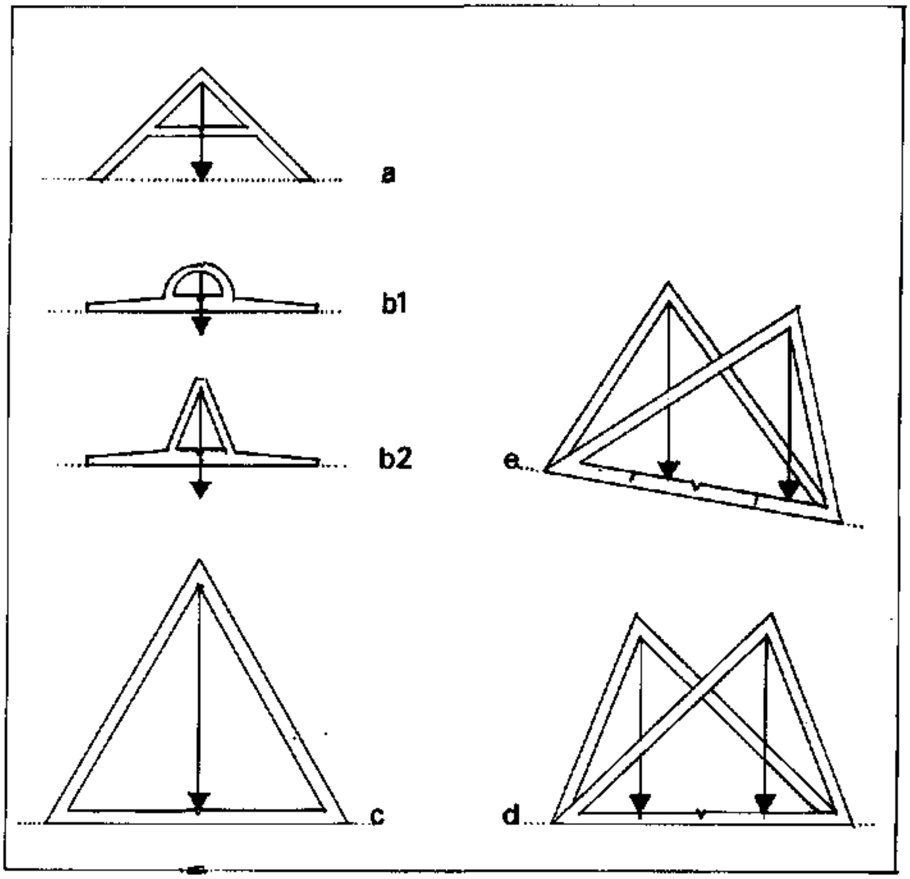

Figure 1. Leveling instruments in history; the problem solved by el-Kashi.

outside his task. But in Semerkand, el-Kashi's current task was to build the observatory and inevitably he had to get involved with architecture. In this context, stalactites and their seemingly complex spatial geometry would have been an academic challenge for his inquisitive mind. We also know that he is the only Muslim mathematician who dealt with Platonic and Archimedean three-dimensional bodies (Schirmer, 1936, 518).

El-Kashi's interest in stalactites was certainly not merely academical. In the section on architectural elements in *Miftah el-Hisab*, his main objective was to establish some rules to calculate their surface areas. He was not a surveyor himself. But it may be assumed that some surveyors were working at the construction of the observatory and/or other constructions in Semerkand. According to a list of the general accounts of commencement of building operations in Timurid literary sources, the word *muhendis* appears in all cases and O'Kane (1987, 38) suggests that it would more closely approximate 'surveyor' than 'architect'. What would they do after the construction had started? It is natural to expect that they would continue performing as building surveyors during the construction process. The word *muhendis* appears only once in the extensive published list of the fifteenth and pre-fifteenth century craftsmen in the Iranian world (O'Kane, 1987, 38, 371-82). This may be explained by the fact that these surveyors were not considered as craftsmen but were operating only as agents of the building supervisors, estimating each amount of completed work for payments. Most probably, El-Kashi's aim was to set some guidelines of area measurements for these building surveyors.

It is not unusual for mathematicians to use architectural elements as practical applications of geometry. Heron of Alexandria was one of them. He lived in the second half of the first century AD (Neugebauer, 1957, 178) and is generally accepted as being one of the main sources for medieval European and Islamic geometry. In his existing books, various architectural elements were used as demonstrating examples (Bruins, 1964). He also wrote a treatise on vaults (not extant) and he was known as *mechanicus* which can be described as 'one who applies geometry to solid matter' (Kidson, 1956, 250). Our learned author, el-Kashi, evidently had studied Heron's works directly, since he is the only one in the Islamic world who knows the method of immersion of Archimedes as described by Heron (Schirmer, 1936, 518).

It is not known whether there were any stalactites in Uluğ Bey Observatory. Since stalactites were fashionable in that period, it may be expected that there were. Even if it was not the case, however, during el-Kashi's stay, Semerkand was busy with various construction activities and some of them certainly included stalactites. Around Uluğ Bey Medrese, a *hankah* (residence for *Sufis*), two mosques, a caravansaray, and a complex of baths were under construction (Knobloch, 1972, 133; Barthold, 1963, 123-5). There is another reference which may imply our author's involvement with architecture. Barthold (1963, 123-125) convincingly argues that Mesjid-i Mukatta which was seen by Babur at the beginning of the sixteenth century, can be identified as the cathedral mosque built by Alike-Kukeltash during Uluğ Bey's reign. While describing the richly decorated monument, Babur mentioned that there was a considerable difference between the *kıble* directions of the medrese and of the mosque and added that the *kıble* of the mosque was more reliable because its orientation had been determined by the observation of stars (Barthold, 1963, 122) . We know that el-Kashi had written a book on orientation of *kıble* directions. It would not be far-fetched to assume that he is the one who determined the correct orientation of Kukeltash's mosque and wrote his book afterwards. Let us turn our main point and affirm that there were numerous buildings under construction at the time in Semerkand for el-Kashi to observe and analyse the stalactites.

MIFTAH EL-HISAB
(Fourth Article, Ninth Chapter, Third Section) [7]

ON THE SURFACE AREA OF STALACTITES (*MUKARNAS*)

It is a stepped ceiling with sides and a [horizontal] plane. Every edge of this plane intersects with the adjacent one forming a certain angle. This angle can be a right-angle, a half right-angle, a one and a half right-angle, or other angles **[8] [Figure 2]**.

Two adjacent sides rise vertically on an imaginary horizontal plane. Ceilings that lie over these two are composed of one or two inclined surfaces which can be either uniform or curved. Each one of the two vertical sides with their ceiling is called a 'home' (*beyt*). All adjacent homes that have bases lying on the same imaginary horizontal plane are called a 'course' (*tabaka*).

As has been related, the length at the base of the largest side is called the 'scale of the stalactite' (*mikyas el-mukarnas*).

We have observed four [types of stalactites]:

'simple stalactite' (*mukarnas el-sazij*), which is also called by builders as 'atop-minber' (*brominber*);
'mudded stalactite' (*mukarnas el-matiin*);
'arched stalactite' (*mukarnas el-mukavas*);
'Shirazi stalactite' (*mukarnas el-Shirazi*).

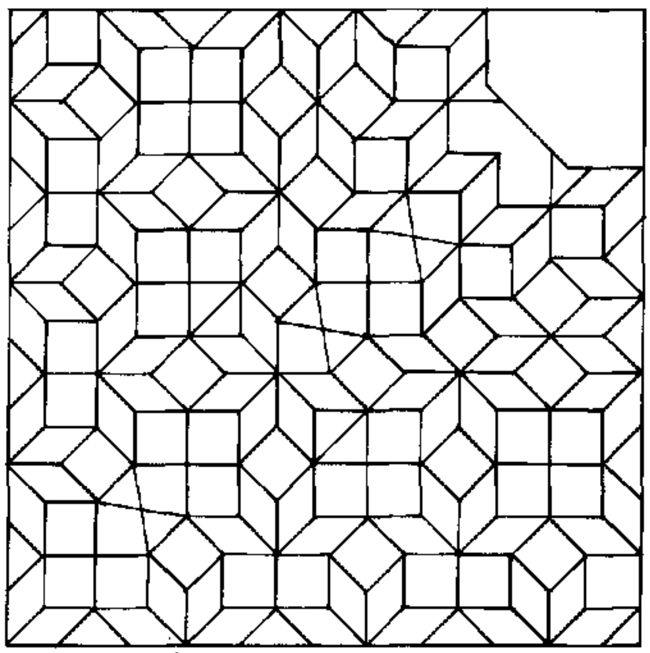

Figure 2. The stalactite plan drawing, based on the plaster slab which was dicovered at Taht-ı Süleyman (Harb, 1978, Pl.1).

7. We want to express our deepest gratitude to Dr. Halid Asfour and to Mr. Taner Avcı for their invaluable help in the translation of the following text from Arabic.

The translation is based primarily on the microfilm copy (Archive No.2461) of Miftah el-Hisab in Nur-i Osmaniye Library (1451, No.2967). Two Arabic publications and other copies are consulted in ambiguous cases. Arabic or Persian words corresponding to the key terms in the text are given in parenthesis. For the transliteration of these words, Turkish pronunciations are adopted, except 'ch', 'sh', 'gh'. The sexagimal numerals as given by el-Kashi are also quoted in parenthesis following their decimal equivalents.

In the Arabic text, el-Kashi generally uses the first person plural. For the sake of fluency in the English translation, this style has been transformed into the passive form. However in the final part where the text relates to builders, third person plural is retained as in the original. Furthermore, square brackets are used to insert words or phrases so as to complement or clarify the original text.

8. Basically this is the description of a geometrical network of horizontal, vertical and 45° diagonal lines. A stalactite plan drawn on a plaster slab, which was discovered during the excavations of the palace of Abaka Han at Taht-i Suleyman, reveals the same system (Harb, 1978, pl.1). The palace is dated to c. 1275 by Wilber (1962, 112). Additional evidence can be found in Morocco where stalactite making still survives as a traditional craft. There, similar plans were drawn on paper (Paccard, 1983, 303-310). Whoever had drawn these plans, apparently, used a practical method of drawing horizontal, vertical, and diagonal lines on plaster or on paper.

ON THE SIMPLE STALACTITE

In this [type, horizontal] edges of sides of homes compose nothing but [the following shapes] **[9]**:

'rhombi' (*muayyen*);
'rhombus-likes' (*shabihet bil muayyen*);
'rectangles' (*mustatil*).

Surfaces over these sides, i.e. their ceilings, are composed of [the following shapes]:

'squares' (*murabba*) **[Figure 3a]**;
rhombi **[Figure 3b and 3c]**;
'almonds' (*loze*) **[Figure 3f]**;
'half-squares' **[Figure 3d and 3d']**;
'half-rhombi' **[Figure 3e and 3e']**;
'two-leggeds' (*zevat er-rijleyn*) **[Figure 3g]**, which are also called 'almond-complement' (*tamam el-loze*);
'barleycorns' (*jaudenha*) **[Figure 3j]**, which are few **[10]**.

All sides of the squares and the rhombi; longer sides of the almonds and the two-leggeds, legs of the half-squares and half-rhombi; shorter sides of the barleycorns are equal to each other and equal to the scale. The barleycorns are located only at the uppermost course **[11]**.

In order to survey the area of the simple stalactite, the operation is carried first by the aid of the scale [of the stalactite]. If it is needed to convert the area into another scale, such as the 'cubit' (*zira*) or a different scale, [necessary calculations] are carried out later.

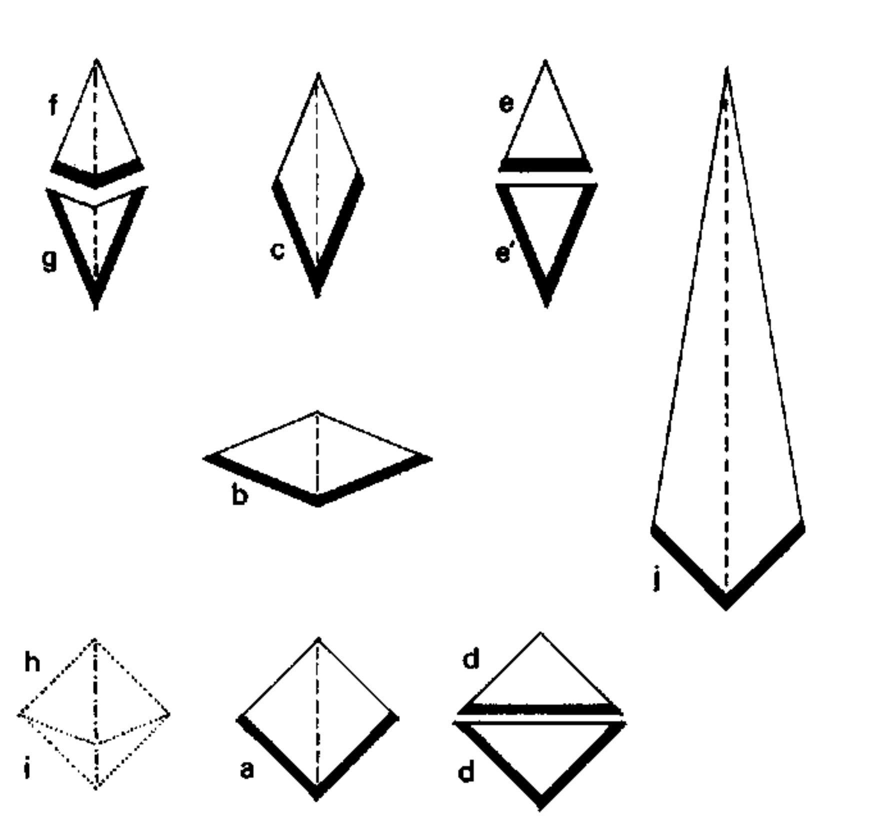

Figure 3. Ceiling elements of the simple stalactite (illustrated by el-Kashi).

9. This passage seems rather confusing. We assume that he is referring to horizontal edges, since vertical edges can form nothing but squares or rectangles. It is also not clear which shape or shapes he classifies under the term 'rhombus-like'.

10. Except the half-square and the half-rhombi, five of these shapes are illustrated by el-Kashi. The term 'almond-complement' clearly suggests that the two-legged and the almond are complementary shapes. We deduce, from area calculations which he further discusses, that these two shapes form a rhombus when coupled together. It is difficult to understand, however, why el-Kashi does not mention similar components of a square (Figure 3 h, i).

11. The barleycorns are one of the characteristic features of stalactites in Transoxania (Maveraünnehir), Horasan and Persia. In these regions, stalactites fill the whole space within an arch. In order to fit the outline of the stalactite into the profile of the arch, some adjustments are required, especially at the top portion where the arch profile has the least slope. Short edges of barley-corn bases fit the overall geometry of the stalactite, but their lengths vary according to the arch profile used.

In northern Azerbeyjan and Anatolia, however, stalactite designs generally exhibit a different approach (Wilber, 1969, 90). In those examples, stalactite outlines are independent of the arch profile, if there is any, and there is no need for adjustments or any barley-corns.

The survey can be done by counting the number of edges above or below [vertical surfaces] that corresponds to one of the [following]:

side of the square or its equivalent;
shorter side of the almond and the almond-complement;
the base of the half-rhombus.

Then, to each group of sides [following numerical values] are allocated:

the side of the square or the rhombus, 1;	
the shorter side of the almond or the almond-complement,	0,414214 (0^o 24 51 10 8);
the base of the half-rhombus,	0,765367 (0^o 45 55 19 15).

[Each side is multiplied by its allocated value and] all these are added together. Then the sum is multiplied by thickness of the course, *i.e.* height of the vertical sides, which is usually equal to the scale. The result is the area of wall surfaces in each course, in terms of the scale of the stalactite.

[To survey the area of ceilings in each course, following numerical values] are then allocated to: [12]

the square,	1;
the rhombus,	0,707107 (0^o 42 25 35 04);
the almond,	0,414214 (0^o 24 51 10 08);
the half-rhombus,	0,353553 (0^o 21 12 47 32);
the almond-complement,	0,292093 (0^o 17 34 24 56); [13]
the half-square,	0,5.

When [the number of each shape is multiplied by its allocated value and] all are added together, the sum is equal to the area of surfaces of ceilings in each course, in terms of the scale of the stalactite.

Then the areas of all courses are surveyed [following the same procedure] and the total sum is the area of the stalactite. [In fact,] if the area of [horizontal] plane on which the stalactite is built is surveyed, the area of the whole stalactite ceiling is obtained.

If this is to be converted into cubits [scale], it is divided by the square of one cubit which can be expressed in terms of the [above mentioned] scale and its parts [14]. The quotient is the required result.

ON THE MUDDED STALACTITE

We have seen this [type] in the old buildings of Isfahan [15]. Mostly it resembles the simple stalactite, except for the fact that the thicknesses of its courses are not uniform. In some cases it may be composed of two or three courses and its ceilings do not have any [horizontal] edges. Its area is analogous to the one of the simple stalactite [16].

ON THE ARCHED STALACTITE

This [type] is similar to the simple stalactite but ceilings of homes are bent and there are bent surfaces penetrating [upwards] between ceilings of adjacent homes. These ['penetrating' (*yatakhalel*)elements] are in the form of either the triangles or the two-leggeds, which are composed of two triangles [17]. The bent almonds and the barleycorns may take place in some ceilings which are composed by the above mentioned triangles. [Horizontal] edges of these surfaces can have only one of the [following] four values:

12. The following area calculations are based on the assumption that ceilings are horizontal. According to el-Kashi's previous definition, however, ceilings are always inclined. Besides, there is not a single existing stalactite example with flat ceilings. In order to obtain the correct results, these areas should have been divided by the cosine of the inclination angle. It is difficult to explain el-Kashi's negligence. Possibly, the slopes of ceilings in most cases were low and, for practical purposes of surveyors, the discrepancy might have been considered insignificant.

13. This is a mistake of carelessness; the correct value should be 0,292893. Apart from this mistake, el-Kashi's calculations are remarkably accurate.

14. This is a crucial information pointing to the direct relation between the scale of the stalactite and the cubit which was used in the construction of the building. The scale is certainly a part of the cubit, but not necessarily subdivision of it, such as a 'palm' (kabza), a 'span' (shibr), or a 'foot' (kadem).

15. This the clue for el-Kashi's visit to Isfahan. He is possibly referring to the stalactites over the 'eyvan's of Mesjid-i Juma which were unique and well known by the colossal sizes of their elements. Unfortunately, he does not give more detailed information which would have been very useful to determine the original forms of these stalactites.

16. Judging by the existing examples, the mudded stalactite is a rare type. The stalactite in Veramin Mesjid-i Jami, with its gradually decreasing course heights, fits el-Kashi's description.

17. 'Penetrating elements' are not unique for the arched stalactite. The triangles and the two-leggeds of the simple stalactite are in fact penetrating elements, but el-Kashi chooses to give their definition at a later stage of his account.

In general, these penetrating elements are the complementary forms of home elements. Their directions and roles are reversed in a stalactite composition. Home elements are collected together towards the apex, whereas penetrating elements spread out to form a console. In a simplified arrangement, there is a corresponding penetrating element underneath every home element. This duality in basic stalactite elements is acknowledged by some scholars: Harb (1978, 28-42) specifies these as 'finished components' (fertigteil) and 'penetrating members' (zwischenglieder); Erdmann (1972) classifies as 'squinches' and 'pendentives'; Wilber (1969, 72) describes as 'concave cells' and 'convex brackets'.

18. This curious way of defining the side of an octagon, of which the radius of the inscribed circle is equal to the scale, is not unique for el-Kashi. Heron of Alexandria defined an octagon drawn on a square with almost same words (Bruins, 1964, 64-65). It cannot be a mere coincidence but is another evidence to indicate that el-Kashi had really studied Heron's works.

19. There is a small discrepancy between this figure and the correct value, 1,726059 (0° 43 33 48 40). For the source of the discrepancy, see Note 25. When el-Kashi further explains how he calculates this coefficient it becomes clear that the assumes a specific curvature and slope for home elements. It is difficult to accept that this profile is true for every stalactite.

20. El-Kashi apparently bases the following calculations on the assumption that the three dimensional curved profile, which corresponds to the scale on the horizontal plane, forms the hypotenuse of each triangle and he determines the lengths of their perpendiculars by using the Pythagorean theorem. He then multiplies these perpendiculars by the corresponding horizontal sides to find the areas.

the scale of the stalactite **[Figure 4a, b]**;
one half of the diagonal of the [scale] square **[Figure 4d]**;
remaining part of the diagonal of the [scale] square laid over its side **[Figure 4f]; [18]**
side of an octagon of which one half of the longer diagonal [*i.e.* radius of the circumscribed circle] is equal to the scale **(Figure 4e, h)**.

In order to survey the area of these surfaces, each edge that corresponds to one of the four [above mentioned] options are counted. [Following numerical values] are allocated to:

the first option,	1;
the second option,	0,707107 (0° 42 25 35 04);
the third option,	0,414214 (0° 24 51 10 08);
the fourth option,	0,765367 (0° 45 55 19 15).

Then [the number of each edge is multiplied by its corresponding value and] all of them are added together. When the total is multiplied by [the number] 1,726045 (1° 43 33 45 41), the product is the area of surfaces of all homes, in terms of the scale of the stalactite. This number is called 'the conversion coefficient' (*tadil*) **[19]**.

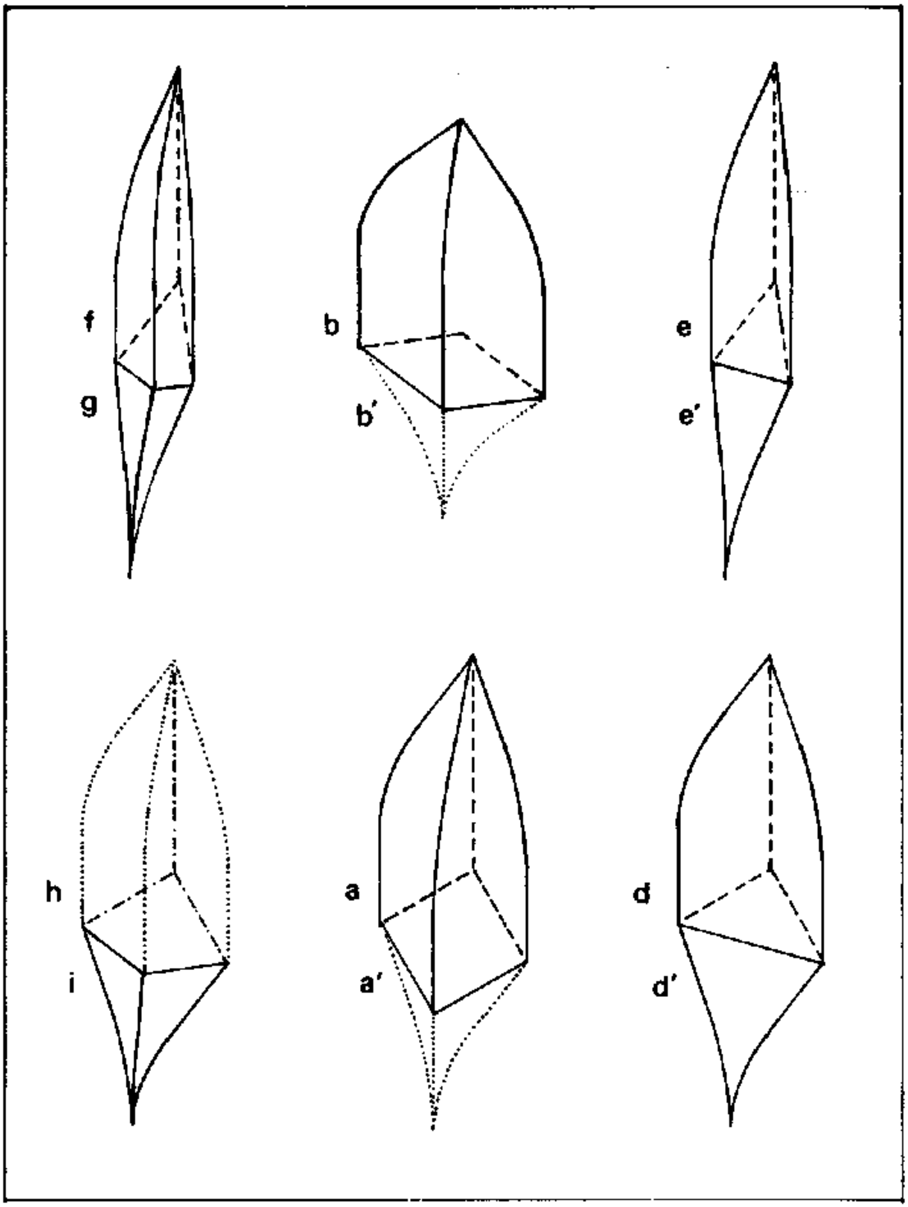

Figure 4. Three dimensional elements of the arched stalactite.

21. Discrepancies between el-Kashi's and our calculations are negligible, except the last one. Most probably it is due to a small mistake in his sexagimal multiplication since there is only one integer difference in seconds.

El-Kashi specifically identifies two different types of two- leggeds. The two-short-legged is the one which is called almond- complement. He does not mention the two-long-legged in the previous section (see note 10). Similarly, it complements another type of almond (Figure 4h). In the same context, he does not consider the two-leggeds which complement the square and the almond (Figure 4a', b'), probably because these do not have any ceilings.

22. The name leaves the impression that this stalactite type was originated in Shiraz, a town in southern Persia. But his impression can not be substantiated by existing monuments. We think it is more plausible to attribute the name to Kevameddin Shirazi, the famous court architect of Şah Ruh, who lived and worked in Horasan from 1410 to around 1440 (O'Kane, 1987, 373, 376). He had a very distinct personal style and was the author of several unique architectural innovations, such as a structural system composed of interlacing transverse arches forming quarter domelets filled in with stalactites. He was also a skilled astrologer and earliest surviving prototypes of the Shirazi stalactite can be detected in buildings designed by him.

Later, especially Safavid examples of this stalactite exhibit a more complex geometrical network composed of concentric rings of various star forms. A paper roll in Istanbul (T.S.M.K., H.1956) which contains 81 stalactite plan drawings and three pages of stalactite plan drawings from Uzbekistan (Pugachenkova, 1962, 209) certainly belong to this type and possibly date from the late fifteenth or sixteenth centuries.

It must be noted that, as a result of earlier and seemingly unrelated development, some of the stone stalactites constructed in the second half of the thirteenth century in Anatolia (such as Konya Sahipata, Sivas Gök Medrese, Sivas Çifte Minareli, Erzurum Çifte Minareli) show similar characteristics to the Shirazi stalactite.

23. In this passage, el-Kashi is particularly vague. It is not clear which two are to be added together and 0,765290 is used both as part of the calculation process and as the conversion coefficient. Our interpretation brings some sense into this confusion; but by this way corresponding areas of the almonds are calculated, not of the two-leggeds. Maybe this vagueness can be explained by his unfamiliarity with the Shirazi stalactite, which supposed to be recently introduced while he was writing his book.

In order to survey the area of surfaces of the bent triangles and the two-leggeds that penetrate between ceilings, [following numerical values] are allocated to: [20].

the triangle [*i.e.*half rhombus], **[Figure 4e']**
0,567129 (0°_2 34 01 38 55)
$[((1{,}530578)^2 - (0{,}765367/2)^2)^{1/2}\,(0{,}765367/2) = 0{,}567125]$;

the two-short-legged, **[Figure 4g]**
0,610328 (0°_2 36 37 10 56)
$[((1{,}530578)^2 - (0{,}414214)^2)^{1/2}\,(0{,}414214) = 0{,}610329]$;

the two-long-legged, **[Figure 4i]**
1,014473 (1°_2 00 52 06 59)
$[((1{,}530578)^2 - (0{,}765367)^2)^{1/2}\,(0{,}765367) = 1{,}014474]$;

the bent almond, **[Figure 4f]**
0,633709 (0° 38 01 21 03)
[((1,530578) (0,414214) = 0,633987 (0° 38 02 21 03)] **[21]**.

[Then the number of each penetrating element is multiplied by its corresponding value and all of them are added together to obtain the area in terms of the scale].

If the barleycorns take place at the uppermost course, [in order to obtain its area] half of its shorter diagonal is multiplied by its longer diagonal, which consists of a number of the scales. Then this product is multiplied by the number of the barleycorns. When the result is added to the area of surfaces of homes and the area of [elements] that penetrate between ceilings of home (such as the triangles, the two-leggeds and the almonds) the total is the area of the stalactite.

ON THE SHIRAZI STALACTITE [22]

This [type] is similar to the arched stalactite, except for the fact that the bases of its arched homes [are longer but] do not exceed four times the ones that were previously mentioned. The Shirazi stalactite cannot be surveyed by counting. Apart from bent ceilings of homes and the penetrating triangles and two-leggeds, its ceilings contain triangles, squares, 'pentagons' (*muhammes*), 'hexagons' (*museddes*), 'hanging pieces' (*shurfa*) and other forms which are either plane or curved. In some cases a side without a ceiling, which has a *mihrab* drawn on it, may take place in a course.

In order to survey the area of the Shirazi stalactite, firstly, a ruler is prepared. This ruler has the same length as the scale of the stalactite and it is subdivided into:

sixty, if sexagimal numerals are used;
ten, if Indian numerals are used.

The bases of the sides of all homes in all courses, excluding the sides without a ceiling, are measured with this ruler [and all of them are added together]. When the total is multiplied by the conversion coefficient, 1,726045, the product will be equal to the area of all surfaces of homes.

Then each exterior perpendicular is taken away from one of its longer sides [the remaining part is equal to 0,765290]. Both [*i.e.* two equal shorter sides] are added together. The sum is multiplied by 0,765290 (0° 45 55 02 27) in order to obtain the area of the two-leggeds **[23]**.

The triangles, the squares, the pentagons, the hexagons, the sides without ceilings, and other forms that take place in its ceilings, excluding surfaces of homes and the two leggeds, are also surveyed by the aid of the ruler as mentioned above. The result is then added to the areas of surfaces of homes and the two-leggeds in order to obtain the total area of the stalactite.

HOW TO PREPARE THE PATTERNS (*tezniib*)

T
For builders [*sic*]

Builders firstly draw a rectangle which has a width that is equal to the scale of the stalactite and a length that is equal to twice of the width, such as ABCD **[Figure 5]**.

Then they draw the line AH that makes an angle of one-third of a right angle with the side AB. They divide the line AH into five parts. From point H, they take two parts on the line AH to mark the point R and on the line HC to mark the point E. HR is equal to HE. From points R and E, they draw two arcs with a radius of ER which intersect at point T inside the rectangle. When they draw the arch RE from the center T, [the length of the arc] is certainly equal to one-sixth of the circumference [of the circle]. Then they extend the lines DA and DC by a small amount to points M and L respectively. They draw MK parallel to AB and LK parallel to BC.

Afterwards, builders produce many gypsum boards corresponding to the surface KMARECL, in which RE is an arc. Then they construct each home by surrounding it with two boards in such a way that EC is always vertical **[24]**.

24. El-Kashi does not explain rest of the construction process. From studies in Iraq and Morocco, some information can be gathered about a common technique (Wilber, 1969, 73). A gypsum or wooden form strips are prepared corresponding to the outline of each course and it is horizontally embedded within the growing system as spaces between them are filled with stalactite elements. We believe the same technique was employed in Transoxania, Horasan and Persia. In fact, during the restoration work in Medrese el-Giyasiye, similar gypsum strips are revealed (O'Kane, 1987, Pl.22.6).

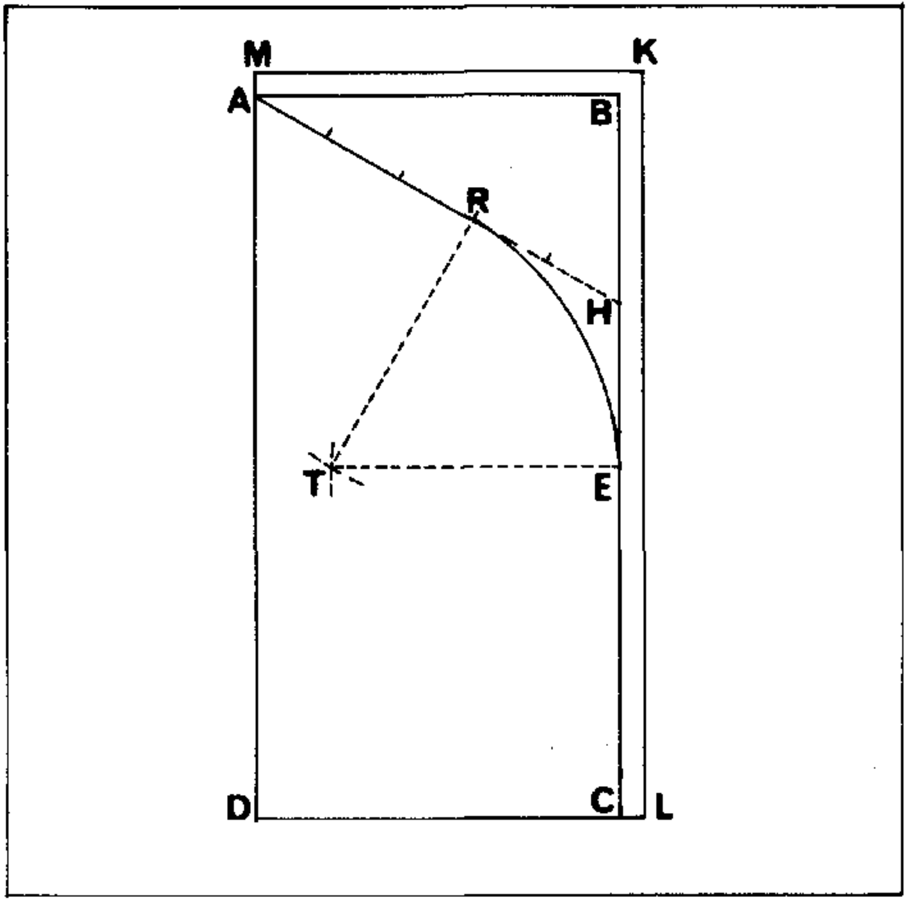

Figure 5. Pattern drawing for the arched and Shirazi stalactites, (illustrated by el-Kashi).

Assuming AB is equal to 1, [following] values are calculated:

AR = 0,692820 (0° 41 34 09 11),
RE = 0,837758 (0° 50 15 55 44),
EC = 0,960756 (0° 57 38 43 14), [0,960770 (0° 57 38 46 14)] **[25]**
AREC = 2,491334 (2° 29 28 48 09), [2,491348 (2° 29 28 51 09)]
ARE = 1,530578 (1° 31 50 04 54),
1/2 ARE = 0,765289 (0° 45 55 02 27),
CE+1/2 ARE = 1,726045 (1° 43 33 45 41). [1,726059 (1° 43 33 48 41)]

[The number] 1,726045 is called 'the conversion coefficient' and it is employed in surveying the areas.

In some cases, where a home is positioned behind the arch, the vertical leg of the board, *i.e.* EC, is reduced [or increased]. Builders need this solution for adjustment **[26]**. In order to measure the area in those cases, the conversion coefficient should be reduced or increased corresponding to the change done on the leg of the board. Numerical values used [in this section] are summarized as:

0,414214 0° 24 51 10 08
If the scale is taken as one, it is the length of one of the shorter sides of the almond; it is the area of the almond when the square of the scale is one.

0,765367 0° 45 55 19 15
It is the length of the shorter diagonal of the rhombus; it is the side of an octagon of which half of its longer diagonal is equal to the scale.

0,707107 0° 42 25 35 04
If the scale is taken as one, it is half of the diagonal of one scale square; it is the area of the rhombus when the square is one.

0,353553 0° 21 12 47 32
It is the area of the half-rhombus.

0,292093 0° 17 34 24 56
[0,292893]
It is the area of the almond-complement.

1,726045 1° 43 33 45 41
[1,726059 1° 43 33 48 41]
The conversion coefficient. To obtain the area, the base of each home in the arched stalactite, including the Shirazi stalactite, is multiplied by it.

0,765290 0° 45 55 02 27
To obtain the area of the two legged, the exterior perpendicular is multiplied by it **[27]**.

0,567129 0° 34 01 38 55
It is the area of the triangle in the arched stalactite.

0,610328 0° 36 37 10 56
It is the area of the two-short-legged which is composed of two bent triangles.

1,014473 1° 00 52 06 59
It is the area of the two-long-legged which is composed of two bent triangles.

0,633709 0° 38 01 21 03
[0,633987 0° 38 02 21 03]
It is the area of the almond-like which is composed of two bent triangles. (El-Kashi, 1451, f.94-97).

25. El-Kashi makes a calculation error. The correct value should be 0,960770. This error repeats itself in the following calculations wherever EC is invoved.

26. When stalactites fill the whole space within a structural arch, adjustments are required where stalactite elements meet the arch at an awkward angle (See Note 11).

27. This explanation adds to the confusion (See Note 23).

CONCLUSION

El-Kashi's section on stalactites is a real contribution to the history of architecture in the Islamic world. It is the only literary source, so far, which gives more or less a detailed account of elements, geometry and construction of stalactites. Being a prolific writer in astronomy and mathematics, el-Kashi's approach to the subject is quite systematic. Although repetitive at times, it is free from unnecessary details. His calculations not only clarify the topic that the discusses, but also serve as clues for the deduction of certain details that he misses.

However, one has to be aware of the fact that el-Kashi's account has some limitations. He seems to be concerned mainly with the survey of areas. In order to realize his objective, he logically decomposes the stalactites into basic elements and deals with them separately. This approach serves his purpose and helps us see the stalactites in a more simplified manner; but unfortunately the composition and design of the stalactites are totally neglected.

His account is also not comprehensive enough to cover all stalactites, even in Semerkand. Although his style gives an impression of authority on the subject, he was not a builder and did not have sufficient information on various details. We know enough about el-Kashi to say that he was not a man to accept and admit his deficiencies. His information was probably based on several stalactite examples under construction while he was in Semerkand. Evidently, these were not enough to set some general rules for all stalactites. For example, he totally neglects geometrical systems other than octagonal ones; the arched profile that he illustrates cannot be applied to every stalactite, especially in other regions. It remains to be seen how generalized his account is, especially his statement about the scale of the stalactite, when it is applied to specific existing examples.

APPENDIX A

COPIES AND PUBLICATIONS OF *MIFTAH EL-HISAB*

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MANUSCRIPTS:
(1430) Topkapı S. M. K., A. 3479.
(1451) Süleymaniye K., Nur-i Osmaniye, 2967.
(1458) Süleymaniye K., H. Hüsnü Paşa, 1268.
(1468) Topkapı S. M.K., A. 3143
(1473) Süleymaniye K., Yeni Cami, 804.
(1589) British Library, Add, 7470. Rich.
(1645) Süleymaniye K., Şehid Ali paşa, 1997.
(1780) Princeton University Library, Yahudi Ms.1189.
(1807) S. O. A. S. Library, 444486, London.
(XIXth.C.) Topkapı S. M. K., 1607 H.606.
Süleymaniye K., Esad Ef. 3175.
Süleymaniye K., Hamidiye 883.
Süleymaniye K., Fatih 5421/2.
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(1956) *Klyuch Arifmetiki, trans.* B.A. Rosenfald, eds. V. S. Segal and A.P. Yushkevich, Moscow.
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APPENDIX B

THE HASHIMI *GEZ*

In Arabic and Persian literary sources, earliest available references to the Hashimi 'cubit' (Arabic, *zira*; Persian, *gez*; Turkish, *arşın*), which is also commonly called 'royal cubit', are from the tenth century (Sauvaire, 1986). The authors have diverse opinions about the origin of this cubit (Abbasids, Umayyads, Omer or ancient Persians) but they all agreed that it is the one used for surveying and construction. Especially in the early sources, the relation between the royal and the common cubits were generally given as 4/3; but in the tenth century, ibn Havkal and el-Mukaddesi specifically mentioned that this relation is 3/2 in Persia (Sauvaire, 1886, 485, 490). A Persian traveller, Nasir-i Khusrau, who visited Jerusalem in 1047 recorded the existence of an inscription which stated the dimensions of Harem-i Sherif in terms of royal cubit (*gez-i melik*) and added that it was the same as the one which is known in Horasan as *gez-i Shaigan* and was equivalent to 3/2 of the common cubit (*arish*), or a fraction less (Le Strange, 1893, 29). This common cubit which was known by various names in different countries was always used for legal (*sheri*) matters. Its origin dates back to early Islamic period. Numerous metrological compilations in the nineteenth and twentieth centuries recorded this cubit in almost every Muslim country as a distinct unit. Variations in their lengths were quite small and they ranged between 47,5-49 cm. The ones in Iran had an average value of 48 cm and Querry (1871, I, 369) specifically mentioned that the legal cubit in Persia was equal to 48 cm. Considering the fact that the length of the legal cubit had remained practically unaltered all through its history, it can be estimated that the Hashimi *gez* in Persia was equal to 70-72 cm. between the tenth and twelfth centuries.

In Egypt and Syria, starting from the twelfth century, some of the sources attributed a longer length and sometimes a different name to the Hashimi cubit. The unknown author of *Guide du Kateb* defined the royal cubit as being equivalent to 3/2 hand or common cubits (Sauvaire, 1886, 499). Ibn Mammati (d. 1209) and ibn el- Atir (d. 1233) called the same cubit as 'carpenters' cubit'; el- Kalkashandi (d.1418) and el-Makrizi (d. 1442) used the name 'work cubit' and stated specifically that it is the Hashimi cubit (Sauvaire, 1886, 500, 518). Moreover, Mujireddin (d. 1522) made a rope and measured the dimensions of the Harem-i Sherif, Jerusalem, in terms of work cubit (Sauvaire, 1876, 120). M. van Berchem (1927, 97) compared his figures with the actual dimensions and concluded that the work cubit used in Memluk lands was equal to 70-71 cm. This unit can serve as an indirect evidence to assume the continuity of the almost identical unit in Persia into the sixteenth century.

We have more direct evidences from later periods. Gmelins (1774, 140) stated that the 'shortened *gez*' of Persia was equal to the Russian *arshin*. Paucton (1780), in his extensive metrological compilation, recorded in Persia a unit called 'royal *gez*' of 71,6 cm. and in Russia vaious *arshin* units ranging between 71,2-71,8 cm. Apparently, *arşın* was a common word in Persian and Russian languages and in Turkish it is the word for cubit (however, the same unit was used by Ottomans under a different name, *Halebi*). The origin of this word is not certain, but many scholars believe that it comes from Turkish (Geiger, 1935, 119). Turkish speaking peoples of Horasan and Transoxania probably acted as the common link between Persia and Russia in this respect. That was where el-Kashi lived in the fifteenth century.

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GIYASEDDİN CEMŞİD EL-KAŞİ VE MUKARNASLAR

Alındı: 24.6.1992
Anahtar Sözcükler: Mimarlık Tarihi, Gıyaseddin Cemşid el-Kaşi, Mukarnaslar, Matematik Tarihi, Yer Ölçme Tarihi, Ölçü Birimleri Tarihi.

ÖZET

Giyaseddin Cemşid el-Kaşi'nin *Miftah el-Hisab* adlı kitabında sözünü ettiği mukarnas ölçeği mukarnasların karmaşık görünen geometrisini çözümlemekte anahtar görevi görebilir. Kitaptaki konuyla ilgili bölümün çevrisini ve analizini yapmadan önce el-Kaşi'yi daha derinlemesine incelemeyi gerekli gördük.

Kaşan'da doğan el-Kaşi aynı kentte 1407-1416 yılları arasında astronomi ile ilgili dört risale yazar. Bu sıralarda Semerkand Uluğ Bey'in önderliğinde yoğun bir bilim etkinliğine sahne olmaktadır. Uluğ Bey 1420 yılında gözlemevinin yapımını başlatır ve bu amaçla el-Kaşi'yi Semerkand'a davet eder. Gelişinden hemen sonra gözlemevinin bilimsel ve yapımsal sorumluluğunu yüklenen el-Kaşi kısa sürede yapımı bitirir ve ardından matematik ve astronomi üzerine dört çalışmasını daha tamamlar. Bunlardan 1427 yılında yazdığı *Miftah el-Hisab*, matematik konusunda ileri seviyedeki katkıları yanısıra ondalık kesirleri tanıtmasıyla dikkat çeker. Aynı kitabın bir bölümü kemer, kubbe ve mukarnasları içerir. Bunlardan başka tarihi belirsiz altı risale daha yazan el-Kaşi 1429 yılında gözlemevinde çalışırken ölür.

Bilim adamı olarak üstün değerlere sahip el-Kaşi'nin kişiliği hakkında bazı ipuçlarını babasına yazdığı mektuptan çıkartabiliyoruz. Sürekli kendisini övme merakı yüzünden yer yer kendisiyle çelişkiye düştüğünü ve gözlemevini büyük boyutlu araçlarla donatmak amacıyla Ulug Bey'i yanıltmaktan çekinmediğini görüyoruz. Mektubundaki bir pasaj kişiliğini çok iyi sergilediği gibi mimarlık

teknolojisi tarihi açısından bizi yakından ilgilendiriyor. Yer tesviyesi için kullanılan üçgen biçimindeki düzecin ikizkenar olması gerekmediğini savunan el-Kaşi bunu hemen anlayamadıkları için diğer matematikçileri azarlıyor. Böylece mimariyle yakın ilişki içinde bulunduğunu da gözlemiş oluyoruz. Çağdaşı bir yazar Uluğ Bey'in el-Kaşi'nin kaba tutumlarından hoşlanmadığını ancak bilgisi yüzünden katlanmak zorunda kaldığını söylüyor.

Aynı mektuptan el-Kaşi'nin işi dışında aruz gibi farklı konularla da ilgilenmiş olduğunu öğreniyoruz. Bu tür çok yönlü merakları olan yazarımızın gözlemevi yapımı nedeniyle mimarlıkla, dolayısıyla mukarnaslarla yakınlık kurmuş olması çok doğal. Ancak kitabında bazı mimari elemanlara yer vermesinin nedeninin yalnızca bir akademik merak olmaması gerek. Genel olarak elemanların alanlarını ölçme konusunda bazı kurallar saptamaya çalışan yazarımızın temel amacının yapım sırasında bina ölçümüyle uğraşan kişiler için bir el kitabı hazırlamak olduğu düşünülebilir. Nitekim İskenderiye'li Heron gibi bazı ünlü matematikçilerin de kitaplarında bu amaçla bazı mimari elemanlara yer verdiğini biliyoruz. Uluğ Bey Medresesinde mukarnaslar kullanılmamış olabilir. Ancak bu dönemde yoğun bir yapım etkinliğine sahne olan Semerkand'da yazarımızın incelemek için mukarnas bulmakta güçlük çekmiş olduğunu sanmıyoruz.

Miftah el-Hisab'taki Mukarnaslar Bölümünün Özet Çevirisi:

Mukarnaslar düzlem ve kenarlarıyla kırılarak yükselen bir tavandır. Her kenar yanındakiyle dik, yarım dik veya birbuçuk dik açı yaparak kesişir. İki düzlem ve tavanının oluşturduğu birime 'yuva' adı verilir. Yatay düzlemdeki en uzun kenar mukarnas ölçeği olarak kabul edilir. Mukarnaslar dörde ayrılır: basit, çamurdan, kavisli ve Şirazi.

Basit mukarnaslarda yuvalar yalnızca baklava-benzeri ve dikdörtgenlerden oluşur. Tavanlarında ise şu şekiller bulunur: kare, baklava, badem, yarım-kare, yarım-baklava, iki-bacaklı (badem-tamamlayan da denir) ve arpa-tanesi. Kare, baklava, iki-bacaklının uzun kenarı, yarım-kare ve yarım-baklavanın bacağı, ve arpa-tanesinin kısa kenarı birbirlerine ve hepsi mukarnas ölçeğine eşittir. Karenin kenarı = 1 ise; bademin veya tamamlayanın kısa kenarı = 0,414214; yarım baklava = 0,765367. Tavanlardaki şekillerin alan değerleri ise şöyledir: kare = 1; baklava = 0,707107; badem = 414214; yarım-baklava = 0,353553; badem-tamamlayan = 292893; yarım-kare = 0,5.

Çamurdan mukarnası İsfahan'daki eski yapılarda gördük. Esas olarak basit mukarnasa benzer, ancak sıra yükseklikleri eşit değildir.

Kavisli mukarnas da basit mukarnasa benzer, fakat tavanları kavislidir ve tavanların arasına üçgen veya iki-bacaklılar girer. yuva yüzeylerinin taban kenarları yalnızca şu değerlerden birine sahip olabilir: 1; 0,707107; 0,414214; 0,765367. Bütün kenarların toplamı düzeltme katsayısıyla (1,726045) çarpıldığında yuvaların alanı bulunmuş olur. Araya giren elemanların alan değerleri şöyledir: Üçgen = 0,567129; kısa-iki-bacaklı = 0,610328; uzun-iki- bacaklı = 1,014473; kavisli badem = 0,633709. Eğer mukarnasın en üst sırasında arpa-taneleri bulunuyorsa, bunların alanını bulmak için uzun köşegeni kısa köşegenin yarısıyla çarparız.

Şirazi mukarnas kavisli mukarnasa benzer, fakat yuva tabanları daha derindir ve tavanlarında üçgen, dörtgen, beşgen, altıgen, sarkıt veya başka şekiller bulunur. Bu mukarnas türünde alan ölçümü ancak bir cetvel aracılığıyla yapılabilir. Bulunan yuva kenarlarının değeri yine düzeltme katsayısıyla çarpılır.

> Yapı ustaları önce her yuvanın profilini veren alçıdan bir örnek hazırlar, sonra bunu çoğaltır ve yuvaları bunlarla inşa ederler. Ancak kemerin arkasına rastlayan elemanlarda düzeltme yapmak gerekebilir.

Sonuç olarak, el-Kaşi'nin mukarnaslar konusunda anlattıklarının mimarlık tarihi araştırmaları açısından önemli katkıları olacağını söyleyebiliriz. El-Kaşi sistematik bir yaklaşımla mukarnasları temel elemanlara indirgeyerek açıklamakta, ancak tasarım ve kompozisyon konularını ihmal etmektedir. Ayrıca, verdiği bilgileri farklı yörelerdeki mukarnaslar için genel kurallar olarak kabul etmek yanıltıcı olabilir. Bu konuda mevcut örnekler üzerinde araştırmalar yapmayı yararlı görmekteyiz.