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Singüler Potansiyelli Dirac Diferensiyel Denklemlerin Çözümleri İçin İntegral Gösterilimleri

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Özet: Bu çalışmada, [4]'de incelenen ve self-adjoint genişlemeleri yazılan singüler katsayılı Dirac operatörleri için çevirme operatörü tipinde gösterilimler elde edilmiştir.
Anahtar kelimeler: Çevirme operatörü, İntegral denklemi, Dirac operatörü

Integral Representations for Solutions of Dirac Differential Equations With Singular Potential

Abstract: In this study, representations with transformation operator have been obtained for Dirac operators with singular coefficients which have been written self-adjoint extensions and have been considered in [4].
Key Words: Transformation operator, Integral equation, Dirac operator

1. Giriş

İntegrallenebilen potansiyellere sahip Dirac diferansiyel operatörlerin spektral teorisi, [1,2] de verilmiştir. Singüler potansiyellere sahip bazı Dirac operatörleri [3, sayfa 486-500]' çalışılmıştır.

$$\mathbf{l}_1[y]=\begin{cases} y_2'+a(x)y_1 \\ -y_1'+b(x)y_2 \end{cases} \qquad \mathbf{l}_2[y]=\begin{cases} y_2'+p(x)y_1+q(x)y_2 \\ -y_1'+q(x)y_1+p(x)y_2 \end{cases}$$

diferansiyel ifadeleri tarafından sonlu bir aralıkta üretilen Dirac operatörleri [4] de tanımlanmıştır. Burada $a(x)$ birinci mertebeden singüler fonksiyondur öyleki $u(x)\equiv\int a(x)dx\in L_2$, $\nu^{\pm 1}(x)\equiv\exp\left\{\pm\int q(x)dx\right\}\in L_2$ ve $b(x)$ ve $p(x)$ sınırlı ölçülebilir fonksiyonlardır. $\mathbf{l}_1[\cdot]$ ve $\mathbf{l}_2[\cdot]$ ifadeleri ile minimal ve maksimal operatörler belirlenmiş ve minimal operatörlerin self-adjoint genişlemeleri tanımlanmıştır.

Singüler katsayılara sahip Sturm-Liouville operatörleri [3,5] de geniş olarak çalışılmıştır.

Bu çalışmada, [4]'de incelenen ve self-adjoint genişlemeleri yazılan singüler katsayılı Dirac operatörleri için çevirme operatörü tipinde gösterilimler elde edilmiştir.

2. İntegral Denkleminin Oluşturulması

Sonlu aralıkta

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' + \begin{pmatrix} a(x) & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad 0<x<\pi \tag{1}$$

Dirac diferansiyel denklemler sistemi ele alınsın. Burada $a(x)$ birinci mertebeden singüler fonksiyondur öyleki $u(x)\equiv\int a(x)dx\in L_2[0,\pi]$ ve $\lambda\in C$ dir.

$\tilde{y}_2=y_2+u(x)y_1$ dönüşümü ile

$$\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix}' = \begin{pmatrix} \lambda u(x) & -\lambda \\ \lambda(u^2(x)+1) & -\lambda u(x) \end{pmatrix}\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix} \tag{2}$$

sistemi elde edilir. Bu bağıntının sağ tarafındaki matrisin elemanları $L_1(0,\pi)$ uzayına aittir. Böylece $\forall x\in[0,\pi]$ için $c_1,c_2\in C$ olmak üzere (1) sisteminin $y_1(x)=c_1$ ve $\tilde{y}_2(x)=c_2$ başlangıç koşullarını sağlayan bir tek $y(x)=\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix}(x)$ çözümü vardır ve $y(x)\in AC[0,\pi]$ dir.

İlk olarak (2) sisteminin çözümü elde edilsin. Bunun için (2) sisteminin homojen kısmının çözümü;

$y_1(x,\lambda)=e^{i\lambda x}$, $\quad \tilde{y}_2(x,\lambda)=-ie^{i\lambda x}$

şeklinde alınır. Dolayısıyla,

$$y_1(x,\, l\,) = c_1 \cos l\, x + c_2 \sin\, l\, x$$

$$\tilde{y}_2(x,\, l\,) = c_1 \sin\, l\, x - c_2 \cos\, l\, x$$

şeklinde lineer bağımsız reel çözümler ailesi elde edilir.

Şimdi ise homojen olmayan (2) sisteminin çözümü yazılacak olursa,

$$y_1(x,\, l\,) = c_1 \cos\, l\, x + c_2 \sin\, l\, x + l \int_0^x u^2(x) y_1 \sin\, l\,(t-x)dt - l \int_0^x u(x) \tilde{y}_2 \sin\, l\,(t-x)dt$$

$$+\, l \int_0^x u(x) y_1 \cos\, l\,(t-x)dt$$

$$\tilde{y}_2(x,\, l\,) = c_1 \sin\, l\, x - c_2 \cos\, l\, x + l \int_0^x u^2(x) y_1 \cos\, l\,(t-x)dt - l \int_0^x u(x) \tilde{y}_2 \cos\, l\,(t-x)dt$$

$$-\, l \int_0^x u(x) y_1 \sin\, l\,(t-x)dt$$

şeklinde olur.

(2) sistemi ve $\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ koşulunun oluşturduğu problem L ile gösterilirse bu L probleminin çözümü,

$$\begin{cases} y_1(x,\, l\,) = e^{i l x} + l \int_0^x u^2(x) y_1 \sin\, l\,(t-x)dt - l \int_0^x u(x) \tilde{y}_2 \sin\, l\,(t-x)dt \\ \qquad + l \int_0^x u(x) y_1 \cos\, l\,(t-x)dt \\ \tilde{y}_2(x,\, l\,) = -ie^{i l x} + l \int_0^x u^2(x) y_1 \cos\, l\,(t-x)dt - l \int_0^x u(x) \tilde{y}_2 \cos\, l\,(t-x)dt \\ \qquad - l \int_0^x u(x) y_1 \sin\, l\,(t-x)dt \end{cases} \tag{3}$$

olur. Diğer taraftan (2) sisteminin $\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ başlangıç koşullarını sağlayan $\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix}(x,\, l\,)$ çözümünün

$$\begin{pmatrix} y_1 \\ \tilde{y}_2 \end{pmatrix}(x,\, l\,) = \begin{pmatrix} 1 + ib(x) \\ -i + b(x) \end{pmatrix} e^{i l x} + \int_{-x}^{x} K(x,t) \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{i l t} dt \tag{4}$$

şeklinde bir gösterime sahip olduğu gösterilsin. Burada $K(x,t)=\begin{pmatrix} K_{11}(x,t) & K_{12}(x,t) \\ K_{21}(x,t) & K_{22}(x,t) \end{pmatrix}$ ve $b(x)=b_1(x)+ib_2(x)$ biçimindedir.

(4) ifadesi koordinatları ile yazılırsa aşağıdaki ifadeler alınır:

$$y_1(x,l)=e^{ilx}-b_2(x)e^{ilx}+ib_1(x)e^{ilx}+\int_{-x}^{x}K_{11}(x,t)e^{ilt}dt-i\int_{-x}^{x}K_{12}(x,t)e^{ilt}dt\,, \qquad (5)$$

$$y_2(x,l)=-ie^{ilx}+b_1(x)e^{ilx}+ib_2(x)e^{ilx}+\int_{-x}^{x}K_{21}(x,t)e^{ilt}dt-i\int_{-x}^{x}K_{22}(x,t)e^{ilt}dt\,, \qquad (6)$$

(4) şeklinde verilen $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x,l)$ çözümünün (3) denklemini sağlaması için,((5), (6) eşitlikleri (3) eşitliğinde yerlerine yazılırsa)

$$-b_2(x)e^{ilx}+ib_1(x)e^{ilx}+\int_{-x}^{x}K_{11}(x,t)e^{ilt}dt-i\int_{-x}^{x}K_{12}(x,t)e^{ilt}dt=l\int_0^x u^2(t)\sin l(t-x)e^{ilt}dt$$

$$-l\int_0^x u^2(t)\sin l(t-x)b_2(t)e^{ilt}dt+li\int_0^x u^2(t)\sin l(t-x)b_1(t)e^{ilt}dt$$

$$+l\int_0^x u^2(t)\sin l(t-x)\int_{-t}^{t}K_{11}(t,s)e^{ils}dsdt-li\int_0^x u^2(t)\sin l(t-x)\int_{-t}^{t}K_{12}(t,s)e^{ils}dsdt$$

$$+li\int_0^x u(t)\sin l(t-x)e^{ilt}dt-l\int_0^x u(t)\sin l(t-x)b_1(t)e^{ilt}dt-li\int_0^x u(t)\sin l(t-x)b_2(t)e^{ilt}dt$$

$$-l\int_0^x u(t)\sin l(t-x)\int_{-t}^{t}K_{21}(t,s)e^{ils}dsdt+il\int_0^x u(t)\sin l(t-x)\int_{-t}^{t}K_{22}(t,s)e^{ils}dsdt$$

$$+l\int_0^x u(t)\cos l(t-x)e^{ilt}dt-l\int_0^x u(t)\cos l(t-x)b_2(t)e^{ilt}dt+li\int_0^x u(t)\cos l(t-x)b_1(t)e^{ils}dt$$

$$+l\int_0^x u(t)\cos l(t-x)\int_{-t}^{t}K_{11}(t,s)e^{ils}dsdt-li\int_0^x u(t)\cos l(t-x)\int_{-t}^{t}K_{12}(t,s)e^{ils}dsdt \qquad (7)$$

ve

$$b_1(x)e^{ilx}+ib_2(x)e^{ilx}+\int_{-x}^{x}K_{21}(x,t)e^{ilt}dt-i\int_{-x}^{x}K_{22}(x,t)e^{ilt}dt=l\int_0^x u^2(t)\cos l(t-x)e^{ilt}dt$$

$$-l\int_0^x u^2(t)\cos l(t-x)b_2(t)e^{ilt}dt+li\int_0^x u^2(t)\cos l(t-x)b_1(t)e^{ilt}dt$$

$$+ l\int_0^x u^2(t)\cos l(t-x)\int_{-t}^{t} K_{11}(t,s)e^{ils}dsdt - li\int_0^x u^2(t)\cos l(t-x)\int_{-t}^{t} K_{12}(t,s)e^{ils}dsdt$$

$$+ li\int_0^x u(t)\cos l(t-x)e^{ilt}dt - l\int_0^x u(t)\cos l(t-x)b_1(t)e^{ilt}dt - l\int_0^x u(t)\cos l(t-x)b_2(t)e^{ilt}dt$$

$$- l\int_0^x u(t)\cos l(t-x)\int_{-t}^{t} K_{21}(t,s)e^{ils}dsdt + il\int_0^x u(t)\cos l(t-x)\int_{-t}^{t} K_{22}(t,s)e^{ils}dsdt$$

$$- l\int_0^x u(t)\sin l(t-x)e^{ilt}dt + l\int_0^x u(t)\sin l(t-x)b_2(t)e^{ilt}dt - li\int_0^x u(t)\sin l(t-x)b_1(t)e^{ils}dt$$

$$- l\int_0^x u(t)\sin l(t-x)\int_{-t}^{t} K_{11}(t,s)e^{ils}dsdt - li\int_0^x u(t)\sin l(t-x)\int_{-t}^{t} K_{12}(t,s)e^{ils}dsdt \qquad (8)$$

eşitliklerinin sağlanması gerekir.

Tersine $K(x,t)$ matris fonksiyonu (7) ve (8) eşitliklerini sağlıyorsa, $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x,l)$ fonksiyonu (4) eşitliğini sağlamalıdır.

(7) eşitliğinden,

$$l\int_0^x u^2(t)\sin l(t-x)e^{ilt}dt = -\frac{il}{4}\int_{-x}^{x} u^2\left(\frac{x+t}{2}\right)e^{ilt}dt + \frac{il}{2}e^{ilx}\int_0^x u^2(t)dt$$

$$- l\int_0^x u^2(t)\sin l(t-x)b_2(t)e^{ilt}dt = \frac{il}{4}\int_{-x}^{x} u^2\left(\frac{x+t}{2}\right)b_2\left(\frac{x+t}{2}\right)e^{ilt}dt - \frac{il}{2}e^{ilx}\int_0^x u^2(t)b_2(t)dt$$

$$li\int_0^x u^2(t)\sin l(t-x)b_1(t)e^{ilt}dt = -\frac{l}{2}e^{ilx}\int_0^x u^2(t)b_1(t)dt + \frac{l}{4}\int_{-x}^{x} u^2\left(\frac{x+t}{2}\right)b_1\left(\frac{x+t}{2}\right)e^{ilt}dt$$

$$l\int_0^x u^2(t)\sin l(t-x)\int_{-t}^{t} K_{11}(t,s)e^{ils}dsdt = -\frac{l}{2i}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x} u^2(V)K_{11}(V,t+V-x)dV\right)e^{ilt}dt$$

$$+\frac{l}{2i}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x} u^2(V)K_{11}(V,x+t-V)dV\right)e^{ilt}dt$$

$$- li\int_0^x u^2(t)\sin l(t-x)\int_{-t}^{t} K_{12}(t,s)e^{ils}dsdt = \frac{l}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x} u^2(V)K_{12}(V,t+V-x)dV\right)e^{ilt}dt$$

$$-\frac{l}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x} u^2(V)K_{12}(V,x+t-V)dV\right)e^{ilt}dt$$

$$l\,i\int_0^x u(t)\sin l\,(t-x)e^{ilt}dt=\frac{l}{4}\int_{-x}^x u(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}e^{ilx}\int_0^x u(t)dt$$

$$l\int_0^x u(t)\sin l\,(t-x)b_1(t)e^{ilt}dt=\frac{l}{4i}\int_{-x}^x u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2i}e^{ilx}\int_0^x u(t)b_1(t)dt$$

$$l\,i\int_0^x u(t)\sin l\,(t-x)b_2(t)e^{ilt}dt=-\frac{l}{2}e^{ilx}\int_0^x u(t)b_2(t)dt+\frac{l}{4}\int_{-x}^x u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt$$

$$l\int_0^x u(t)\sin l\,(t-x)\int_{-t}^t K_{21}(t,s)e^{ils}dsdt=-\frac{l}{2i}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u(V)K_{21}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{l}{2i}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u(V)K_{21}(V,x+t-V)dV)e^{ilt}dt$$

$$l\,i\int_0^x u(t)\sin l\,(t-x)\int_{-t}^t K_{22}(t,s)e^{ils}dsdt=-\frac{l}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u(V)K_{22}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u(V)K_{22}(V,x+t-V)dV)e^{ilt}dt$$

$$l\int_0^x u(t)\cos l\,(t-x)e^{ilt}dt=\frac{l}{4}\int_{-x}^x u(\frac{x+t}{2})e^{ilt}dt+\frac{l}{2}e^{ilx}\int_0^x u(t)dt$$

$$-\,l\int_0^x u(t)\cos l\,(t-x)b_2(t)e^{ilt}dt=-\frac{l}{4}\int_{-x}^x u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}e^{ilx}\int_0^x u(t)b_2(t)dt$$

$$l\,i\int_0^x u(t)\cos l\,(t-x)b_1(t)e^{ilt}dt=\frac{l\,i}{2}e^{ilx}\int_0^x u(t)b_1(t)dt+\frac{l\,i}{4}\int_{-x}^x u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt$$

$$l\int_0^x u(t)\cos l\,(t-x)\int_{-t}^t K_{11}(t,s)e^{ils}dsdt=\frac{l}{2}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u(V)K_{11}(V,t+x-V)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u(V)K_{11}(V,t+V-x)dV)e^{ilt}dt$$

$$-\,l\,i\int_0^x u(t)\cos l\,(t-x)\int_{-t}^t K_{12}(t,s)e^{ils}dsdt=-\frac{l\,i}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u(V)K_{12}(V,t+V-x)dV)e^{ilt}dt$$

$$-\frac{li}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u(V)K_{12}(V,t+x-V)dV)e^{ilt}dt$$

ifadeleri elde edilir. Bu ifadeler (7) de yerlerine yazılırsa,

$$-b_2(x)e^{ilx}+ib_1(x)e^{ilx}+\int_{-x}^{x}K_{11}(x,t)e^{ilt}dt-i\int_{-x}^{x}K_{12}(x,t)e^{ilt}dt=-\frac{il}{4}\int_{-x}^{x}u^2(\frac{x+t}{2})e^{ilt}dt$$

$$+\frac{il}{2}e^{ilx}\int_{0}^{x}u^2(t)dt+\frac{il}{4}\int_{-x}^{x}u^2(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{il}{2}e^{ilx}\int_{0}^{x}u^2(t)b_2(t)dt$$

$$-\frac{l}{2}e^{ilx}\int_{0}^{x}u^2(t)b_1(t)dt+\frac{l}{4}\int_{-x}^{x}u^2(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt$$

$$+\frac{li}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u^2(V)K_{11}(V,t+V-x)dV)e^{ilt}dt$$

$$-\frac{li}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u^2(V)K_{11}(V,x+t-V)dV)e^{ilt}dt+\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u^2(V)K_{12}(V,t+V-x)dV)e^{ilt}dt$$

$$-\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u^2(V)K_{12}(V,x+t-V)dV)e^{ilt}dt+\frac{l}{4}\int_{-x}^{x}u(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}e^{ilx}\int_{0}^{x}u(t)dt$$

$$\frac{li}{4}\int_{-x}^{x}u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt-\frac{li}{2}e^{ilx}\int_{0}^{x}u(t)b_1(t)dt+\frac{l}{2}e^{ilx}\int_{0}^{x}u(t)b_2(t)dt$$

$$-\frac{l}{4}\int_{-x}^{x}u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{li}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u(V)K_{21}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{li}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u(V)K_{21}(V,x+t-V)dV)e^{ilt}dt-\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u(V)K_{22}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u(V)K_{22}(V,x+t-V)dV)e^{ilt}dt+\frac{l}{4}\int_{-x}^{x}u(\frac{x+t}{2})e^{ilt}dt+\frac{l}{2}e^{ilx}\int_{0}^{x}u(t)dt$$

$$-\frac{l}{4}\int_{-x}^{x}u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}e^{ilx}\int_{0}^{x}u(t)b_2(t)dt+\frac{li}{2}e^{ilx}\int_{0}^{x}u(t)b_1(t)dt$$

$$+\frac{li}{4}\int_{-x}^{x}u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt+\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u(V)K_{11}(V,t+x-V)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^{x}\Big(\int_{\frac{x-t}{2}}^{x}u(V)K_{11}(V,t+V-x)dV\Big)e^{ilt}dt-\frac{li}{2}\int_{-x}^{x}\Big(\int_{\frac{x-t}{2}}^{x}u(V)K_{12}(V,t+V-x)dV\Big)e^{ilt}dt$$

$$-\frac{li}{2}\int_{-x}^{x}\Big(\int_{\frac{x+t}{2}}^{x}u(V)K_{12}(V,t+x-V)dV\Big)e^{ilt}dt \qquad (7')$$

eşitliği alınır. Buradan

$$b_1(x)=\frac{l}{2}\int_0^x u^2(t)dt-\frac{l}{2}\int_0^x u^2(t)b_2(t)dt\ ,\ b_2(x)=\frac{l}{2}\int_0^x u^2(t)b_1(t)dt$$

$$K_{11}(x,t)=\frac{l}{4}u^2\Big(\frac{x+t}{2}\Big)b_1\Big(\frac{x+t}{2}\Big)-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u^2(V)K_{12}(V,t+V-x)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u^2(V)K_{12}(V,t+x-V)dV+\frac{l}{2}u\Big(\frac{x+t}{2}\Big)-\frac{l}{2}u\Big(\frac{x+t}{2}\Big)b_2\Big(\frac{x+t}{2}\Big)$$

$$-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{22}(V,t+V-x)dV+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{11}(V,t+V-x)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{22}(V,t+x-V)dV+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{11}(V,t+x-V)dV$$

$$K_{12}(x,t)=-\frac{l}{4}u^2\Big(\frac{x+t}{2}\Big)+\frac{l}{4}u^2\Big(\frac{x+t}{2}\Big)b_2\Big(\frac{x+t}{2}\Big)+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u^2(V)K_{11}(V,t+V-x)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u^2(V)K_{11}(V,t+x-V)dV+\frac{l}{2}u\Big(\frac{x+t}{2}\Big)b_1\Big(\frac{x+t}{2}\Big)-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{21}(V,t+V-x)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{21}(V,t+x-V)dV+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{12}(V,t+V-x)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{12}(V,t+x-V)dV$$

eşitlikleri elde edilir.

(8) eşitliğinden,

$$l\int_0^x u^2(t)\cos l(t-x)e^{ilt}dt=\frac{l}{4}\int_{-x}^{x}u^2\Big(\frac{x+t}{2}\Big)e^{ilt}dt+\frac{l}{2}e^{ilx}\int_0^x u^2(t)dt$$

$$-l\int_0^x u^2(t)\cos l(t-x)b_2(t)e^{ilt}dt=-\frac{l}{4}\int_{-x}^x u^2(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}e^{ilx}\int_0^x u^2(t)b_2(t)dt$$

$$li\int_0^x u^2(t)\cos l(t-x)b_1(t)e^{ilt}dt=\frac{li}{2}e^{ilx}\int_0^x u^2(t)b_1(t)dt+\frac{li}{4}\int_{-x}^x u^2(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt$$

$$l\int_0^x u^2(t)\cos l(t-x)\int_{-t}^t K_{11}(t,s)e^{ils}dsdt=\frac{l}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u^2(V)K_{11}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u^2(V)K_{11}(V,x+t-V)dV)e^{ilt}dt$$

$$-li\int_0^x u^2(t)\cos l(t-x)\int_{-t}^t K_{12}(t,s)e^{ils}dsdt=-\frac{il}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u^2(V)K_{12}(V,t+V-x)dV)e^{ilt}dt$$

$$-\frac{li}{2}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u^2(V)K_{12}(V,x+t-V)dV)e^{ilt}dt$$

$$li\int_0^x u(t)\cos l(t-x)e^{ilt}dt=\frac{li}{4}\int_{-x}^x u(\frac{x+t}{2})e^{ilt}dt+\frac{li}{2}e^{ilx}\int_0^x u(t)dt$$

$$l\int_0^x u(t)\cos l(t-x)b_1(t)e^{ilt}dt=\frac{l}{4}\int_{-x}^x u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt+\frac{l}{2}e^{ilx}\int_0^x u(t)b_1(t)dt$$

$$li\int_0^x u(t)\cos l(t-x)b_2(t)e^{ilt}dt=\frac{li}{2}e^{ilx}\int_0^x u(t)b_2(t)dt+\frac{li}{4}\int_{-x}^x u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt$$

$$l\int_0^x u(t)\cos l(t-x)\int_{-t}^t K_{21}(t,s)e^{ils}dsdt=\frac{l}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u(V)K_{21}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u(V)K_{21}(V,x+t-V)dV)e^{ilt}dt$$

$$li\int_0^x u(t)\cos l(t-x)\int_{-t}^t K_{22}(t,s)e^{ils}dsdt=\frac{li}{2}\int_{-x}^x(\int_{\frac{x-t}{2}}^x u(V)K_{22}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{li}{2}\int_{-x}^x(\int_{\frac{x+t}{2}}^x u(V)K_{22}(V,x+t-V)dV)e^{ilt}dt$$

$$l\int_0^x u(t)\sin l(t-x)e^{ilt}dt=\frac{l}{4i}\int_{-x}^{x}u(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2i}e^{ilx}\int_0^x u(t)dt$$

$$-l\int_0^x u(t)\sin l(t-x)b_2(t)e^{ilt}dt=-\frac{l}{4i}\int_{-x}^{x}u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt+\frac{l}{2i}e^{ilx}\int_0^x u(t)b_2(t)dt$$

$$li\int_0^x u(t)\sin l(t-x)b_1(t)e^{ilt}dt=-\frac{l}{2}e^{ilx}\int_0^x u(t)b_1(t)dt+\frac{l}{4}\int_{-x}^{x}u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt$$

$$l\int_0^x u(t)\sin l(t-x)\int_{-t}^{t}K_{11}(t,s)e^{ils}dsdt=\frac{l}{2i}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u(V)K_{11}(V,t+x-V)dV)e^{ilt}dt$$

$$-\frac{l}{2i}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u(V)K_{11}(V,t+V-x)dV)e^{ilt}dt$$

$$li\int_0^x u(t)\sin l(t-x)\int_{-t}^{t}K_{12}(t,s)e^{ils}dsdt=-\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u(V)K_{12}(V,t+V-x)dV)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u(V)K_{12}(V,t+x-V)dV)e^{ilt}dt$$

ifadeleri elde edilir. Bu ifadeler (8) de yerlerine yazılırsa,

$$b_1(x)e^{ilx}+ib_2(x)e^{ilx}+\int_{-x}^{x}K_{21}(x,t)e^{ilt}dt-i\int_{-x}^{x}K_{22}(x,t)e^{ilt}dt=\frac{l}{4}\int_{-x}^{x}u^2(\frac{x+t}{2})e^{ilt}dt$$

$$+\frac{l}{2}e^{ilx}\int_0^x u^2(t)dt-\frac{l}{4}\int_{-x}^{x}u^2(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}e^{ilx}\int_0^x u^2(t)b_2(t)dt$$

$$+\frac{il}{2}e^{ilx}\int_0^x u^2(t)b_1(t)dt+\frac{il}{4}\int_{-x}^{x}u^2(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u^2(V)K_{11}(V,t+V-x)dV)e^{ilt}dt+\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u^2(V)K_{11}(V,x+t-V)dV)e^{ilt}dt$$

$$-\frac{il}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u^2(V)K_{12}(V,t+V-x)dV)e^{ilt}dt-\frac{il}{2}\int_{-x}^{x}(\int_{\frac{x+t}{2}}^{x}u^2(V)K_{12}(V,x+t-V)dV)e^{ilt}dt$$

$$+\frac{il}{4}\int_{-x}^{x}u(\frac{x+t}{2})e^{ilt}dt+\frac{il}{2}e^{ilx}\int_0^x u(t)dt-\frac{l}{2}\int_{-x}^{x}u(\frac{x+t}{2})b_1(\frac{x+t}{2})e^{ilt}dt-\frac{il}{2}e^{ilx}\int_0^x u(t)b_2(t)dt$$

$$-\frac{il}{2}\int_{-x}^{x}u(\frac{x+t}{2})b_2(\frac{x+t}{2})e^{ilt}dt-\frac{l}{2}\int_{-x}^{x}(\int_{\frac{x-t}{2}}^{x}u(V)K_{21}(V,t+V-x)dV)e^{ilt}dt$$

$$-\frac{l}{2}\int_{-x}^{x}\Big(\int_{\frac{x+t}{2}}^{x}u(V)K_{21}(V,x+t-V)dV\Big)e^{ilt}dt+\frac{il}{2}\int_{-x}^{x}\Big(\int_{\frac{x-t}{2}}^{x}u(V)K_{22}(V,t+V-x)dV\Big)e^{ilt}dt$$

$$+\frac{il}{2}\int_{-x}^{x}\Big(\int_{\frac{x+t}{2}}^{x}u(V)K_{22}(V,x+t-V)dV\Big)e^{ilt}dt+\frac{il}{4}\int_{-x}^{x}u\Big(\frac{x+t}{2}\Big)e^{ilt}dt-\frac{il}{2}e^{ilx}\int_{0}^{x}u(t)dt$$

$$+\frac{il}{2}e^{ilx}\int_{0}^{x}u(t)b_2(t)dt+\frac{il}{2}\int_{-x}^{x}\Big(\int_{\frac{x+t}{2}}^{x}u(V)K_{11}(V,t+x-V)dV\Big)e^{ilt}dt$$

$$-\frac{il}{2}\int_{-x}^{x}\Big(\int_{\frac{x-t}{2}}^{x}u(V)K_{11}(V,t+V-x)dV\Big)e^{ilt}dt-\frac{l}{2}\int_{-x}^{x}\Big(\int_{\frac{x-t}{2}}^{x}u(V)K_{12}(V,t+V-x)dV\Big)e^{ilt}dt$$

$$+\frac{l}{2}\int_{-x}^{x}\Big(\int_{\frac{x+t}{2}}^{x}u(V)K_{12}(V,t+x-V)dV\Big)e^{ilt}dt \qquad (8')$$

eşitliği alınır. Buradan

$$b_1(x)=\frac{l}{2}\int_0^x u^2(t)dt-\frac{l}{2}\int_0^x u^2(t)b_2(t)dt\ ,\ b_2(x)=\frac{l}{2}\int_0^x u^2(t)b_1(t)dt$$

$$K_{21}(x,t)=\frac{l}{4}u^2\Big(\frac{x+t}{2}\Big)-\frac{l}{4}u^2\Big(\frac{x+t}{2}\Big)b_2\Big(\frac{x+t}{2}\Big)+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u^2(V)K_{11}(V,t+V-x)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u^2(V)K_{11}(V,t+x-V)dV-\frac{l}{2}u\Big(\frac{x+t}{2}\Big)b_1\Big(\frac{x+t}{2}\Big)-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{21}(V,t+V-x)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{21}(V,t+x-V)dV-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{12}(V,t+x-V)dV$$

$$+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{12}(V,t+V-x)dV$$

$$K_{22}(x,t)=-\frac{l}{2}u\Big(\frac{x+t}{2}\Big)-\frac{l}{4}u^2\Big(\frac{x+t}{2}\Big)b_1\Big(\frac{x+t}{2}\Big)-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u^2(V)K_{12}(V,t+V-x)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u^2(V)K_{12}(V,t+x-V)dV+\frac{l}{2}u\Big(\frac{x+t}{2}\Big)b_2\Big(\frac{x+t}{2}\Big)-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{22}(V,t+V-x)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x} u(V)K_{22}(V,t+x-V)dV+\frac{l}{2}\int_{\frac{x-t}{2}}^{x} u(V)K_{11}(V,t+V-x)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x} u(V)K_{11}(V,t+x-V)dV$$

eşitlikleri elde edilir.

Ayrıca

$$\begin{pmatrix} b_1(x) \\ b_2(x) \end{pmatrix}' = \begin{pmatrix} 0 & -\frac{l}{2}u^2 \\ \frac{l}{2}u^2 & 0 \end{pmatrix} \begin{pmatrix} b_1(x) \\ b_2(x) \end{pmatrix} + \begin{pmatrix} \frac{l}{2}u^2 \\ 0 \end{pmatrix}$$ şeklinde sistem yazılır. Bu bağıntının sağ tarafındaki matrisin elemanları $L_1(0,p)$ uzayına aittir. Böylece $\forall x \in [0,p]$ için $c_1, c_2 \in C$ olmak üzere sisteminin $y_1(x) = c_1$ ve $y_2(x) = c_2$ başlangıç koşullarını sağlayan bir tek $b(x) = \begin{pmatrix} b_1(x) \\ b_2(x) \end{pmatrix}$ çözümü vardır ve $b_1(x), b_2(x) \in AC[0,p]$ dir.

3. İntegral Denklemler Sisteminin Çözümünün Varlığı

İntegral denklemler sisteminin çözümünün varlığı için ardışık yaklaşımlar metodu uygulansın. Bunun için

$$K_{11}^{(0)}(x,t) = \frac{l}{4}u^2(\frac{x+t}{2})b_1(\frac{x+t}{2}) + \frac{l}{2}u(\frac{x+t}{2}) - \frac{l}{2}u(\frac{x+t}{2})b_2(\frac{x+t}{2})$$

$$K_{11}^{(n)}(x,t) = -\frac{l}{2}\int_{\frac{x-t}{2}}^{x} u^2(V)K_{12}^{(n-1)}(V,t+V-x)dV + \frac{l}{2}\int_{\frac{x+t}{2}}^{x} u^2(V)K_{12}^{(n-1)}(V,x+t-V)dV$$

$$-\frac{l}{2}\int_{\frac{x-t}{2}}^{x} u(V)K_{22}^{(n-1)}(V,t+V-x)dV + \frac{l}{2}\int_{\frac{x+t}{2}}^{x} u(V)K_{22}^{(n-1)}(V,t+x-V)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x} u(V)K_{11}^{(n-1)}(V,t+x-V)dV + \frac{l}{2}\int_{\frac{x-t}{2}}^{x} u(V)K_{11}^{(n-1)}(V,t+V-x)dV$$

$$K_{12}^{(0)}(x,t) = -\frac{l}{4}u^2(\frac{x+t}{2}) + \frac{l}{4}u^2(\frac{x+t}{2})b_2(\frac{x+t}{2}) + \frac{l}{2}u(\frac{x+t}{2})b_1(\frac{x+t}{2})$$

$$K_{12}^{(n)}(x,t) = \frac{l}{2}\int_{\frac{x-t}{2}}^{x} u^2(V)K_{11}^{(n-1)}(V,t+V-x)dV - \frac{l}{2}\int_{\frac{x+t}{2}}^{x} u^2(V)K_{11}^{(n-1)}(V,x+t-V)dV$$

$$-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{21}^{(n-1)}(V,t+V-x)dV+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{21}^{(n-1)}(V,t+x-V)dV$$

$$+\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{12}^{(n-1)}(V,t+x-V)dV+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{12}^{(n-1)}(V,t+V-x)dV$$

$$K_{21}^{(0)}(x,t)=\frac{l}{4}u^2(\frac{x+t}{2})-\frac{l}{4}u^2(\frac{x+t}{2})b_2(\frac{x+t}{2})-\frac{l}{2}u(\frac{x+t}{2})b_1(\frac{x+t}{2})$$

$$K_{21}^{(n)}(x,t)=-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u^2(V)K_{12}^{(n-1)}(V,t+V-x)dV-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u^2(V)K_{12}^{(n-1)}(V,x+t-V)dV$$

$$-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{22}^{(n-1)}(V,t+V-x)dV-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{22}^{(n-1)}(V,t+x-V)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{11}^{(n-1)}(V,t+x-V)dV+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{11}^{(n-1)}(V,t+V-x)dV$$

$$K_{22}^{(0)}(x,t)=-\frac{l}{4}u^2(\frac{x+t}{2})b_1(\frac{x+t}{2})-\frac{l}{2}u(\frac{x+t}{2})+\frac{l}{2}u(\frac{x+t}{2})b_2(\frac{x+t}{2})$$

$$K_{22}^{(n)}(x,t)=-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u^2(V)K_{12}^{(n-1)}(V,t+V-x)dV-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u^2(V)K_{12}^{(n-1)}(V,x+t-V)dV$$

$$-\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{22}^{(n-1)}(V,t+V-x)dV-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{22}^{(n-1)}(V,t+x-V)dV$$

$$-\frac{l}{2}\int_{\frac{x+t}{2}}^{x}u(V)K_{11}^{(n-1)}(V,t+x-V)dV+\frac{l}{2}\int_{\frac{x-t}{2}}^{x}u(V)K_{11}^{(n-1)}(V,t+V-x)dV$$

$$s_1(x)=\frac{l}{2}\int_0^x\left|u^2(V)b_1(V)\right|dV+l\int_0^x\left|u(V)\right|dV+l\int_0^x\left|u(V)b_2(V)\right|dV$$

$$s_2(x)=\frac{l}{2}\int_0^x\left|u^2(V)\right|dV+\frac{l}{2}\int_0^x\left|u^2(V)b_2(V)\right|dV+l\int_0^x\left|u(V)b_1(V)\right|dV$$

ve $\max\{s_1(x),s_2(x)\}=s(x)$ olarak alınırsa,

$K_{11}^{(0)}(x,t),K_{12}^{(0)}(x,t),K_{21}^{(0)}(x,t),K_{22}^{(0)}(x,t)$ fonksiyonlarının ifadelerinden aşağıdaki eşitsizlikler elde edilir:

$$\int_{-x}^{x}\left|K_{11}^{(0)}(x,t)\right|dt \le \frac{l}{2}\int_{0}^{x}\left|u^2(V)b_1(V)\right|dV + l\int_{0}^{x}\left|u(V)\right|dV + l\int_{0}^{x}\left|u(V)b_2(V)\right|dV = s_1(x) \le s(x)$$

$$\int_{-x}^{x}\left|K_{12}^{(0)}(x,t)\right|dt \le \frac{l}{2}\int_{0}^{x}\left|u^2(V)\right|dV + \frac{l}{2}\int_{0}^{x}\left|u^2(V)b_2(V)\right|dV + l\int_{0}^{x}\left|u(V)b_1(V)\right|dV = s_2(x) \le s(x)$$

$$\int_{-x}^{x}\left|K_{21}^{(0)}(x,t)\right|dt \le \frac{l}{2}\int_{0}^{x}\left|u^2(V)\right|dV + \frac{l}{2}\int_{0}^{x}\left|u^2(V)b_2(V)\right|dV + l\int_{0}^{x}\left|u(V)b_1(V)\right|dV = s_2(x) \le s(x)$$

$$\int_{-x}^{x}\left|K_{22}^{(0)}(x,t)\right|dt \le \frac{l}{2}\int_{0}^{x}\left|u^2(V)b_1(V)\right|dV + l\int_{0}^{x}\left|u(V)\right|dV + l\int_{0}^{x}\left|u(V)b_2(V)\right|dV = s_1(x) \le s(x)$$

$$\int_{-x}^{x}\left|K_{11}^{(1)}(x,t)\right|dt \le \frac{l}{2}\int_{0}^{x}\left|u^2(V)b_1(V)\right|dV + l\int_{0}^{x}\left|u(V)\right|dV + l\int_{0}^{x}\left|u(V)b_2(V)\right|dV = s_1(x) \le s(x)$$

$K_{11}^{(1)}(x,t)$ fonksiyonu için benzer değerlendirmeler alınacak olunursa,

$$\int_{-x}^{x}\left|K_{11}^{(1)}(x,t)\right|dt \le \frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u^2(V)\right|\left|K_{12}^{(0)}(V,t+V-x)\right|dV + \frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u^2(V)\right|\left|K_{12}^{(0)}(V,x+t-V)\right|dV$$

$$+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u(V)\right|\left|K_{22}^{(0)}(V,t+V-x)\right|dV + \frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u(V)\right|\left|K_{22}^{(0)}(V,t+x-V)\right|dV$$

$$+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u(V)\right|\left|K_{11}^{(0)}(V,t+x-V)\right|dV + \frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u(V)\right|\left|K_{11}^{(0)}(V,t+V-x)\right|dV$$

eşitsizliği yazılır. Bu eşitsizliğin sağ tarafındaki integraller değerlendirilirse;

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u^2(V)\right|\left|K_{12}^{(0)}(V,t+V-x)\right|dV = \frac{l}{2}\int_{0}^{x}\left|u^2(V)\right|dV\int_{-V}^{V}\left|K_{12}^{(0)}(V,s)\right|ds \le \frac{s^2(x)}{2!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u^2(V)\right|\left|K_{12}^{(0)}(V,x+t-V)\right|dV = \frac{l}{2}\int_{0}^{x}\left|u^2(V)\right|dV\int_{-V}^{V}\left|K_{12}^{(0)}(V,s)\right|ds \le \frac{s^2(x)}{2!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u(V)\right|\left|K_{22}^{(0)}(V,t+V-x)\right|dV = \frac{l}{2}\int_{0}^{x}\left|u(V)\right|dV\int_{-V}^{V}\left|K_{22}^{(0)}(V,s)\right|ds \le \frac{s^2(x)}{2!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u(V)\right|\left|K_{22}^{(0)}(V,t+x-V)\right|dV = \frac{l}{2}\int_{0}^{x}\left|u(V)\right|dV\int_{-V}^{V}\left|K_{22}^{(0)}(V,s)\right|ds \le \frac{s^2(x)}{2!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u(V)\right|\left|K_{11}^{(0)}(V,t+x-V)\right|dV = \frac{l}{2}\int_{0}^{x}\left|u(V)\right|dV\int_{-V}^{V}\left|K_{11}^{(0)}(V,s)\right|ds \le \frac{s^2(x)}{2!}$$

$$+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}|u(V)|\left|K_{11}^{(0)}(V,t+V-x)\right|dV=\frac{l}{2}\int_{0}^{x}|u(V)|dV\int_{-V}^{V}\left|K_{11}^{(0)}(V,s)\right|ds\le\frac{s^{2}(x)}{2!}$$

elde edilir. Dolayısıyla

$$\int_{-x}^{x}\left|K_{11}^{(1)}(x,t)\right|dt\le\frac{6s^{2}(x)}{2!}\le\frac{[cs(x)]^{2}}{2!}$$

olur. Burada $c\ge 6$ dır.

Benzer şekilde

$$\int_{-x}^{x}\left|K_{12}^{(1)}(x,t)\right|dt\le\frac{6s^{2}(x)}{2!}\le\frac{[cs(x)]^{2}}{2!}$$

$$\int_{-x}^{x}\left|K_{21}^{(1)}(x,t)\right|dt\le\frac{6s^{2}(x)}{2!}\le\frac{[cs(x)]^{2}}{2!}$$

$$\int_{-x}^{x}\left|K_{22}^{(1)}(x,t)\right|dt\le\frac{6s^{2}(x)}{2!}\le\frac{[cs(x)]^{2}}{2!}$$

elde edilir.

Böylece $\int_{-x}^{x}\left|K_{ij}^{(0)}(x,t)\right|dt\le s(x)$ olmak üzere

$$\int_{-x}^{x}\left|K_{ij}^{(m)}(x,t)\right|dt\le\frac{[cs(x)]^{m+1}}{(m+1)!},\quad m=0,1,\ldots \tag{9}$$

eşitsizliğin doğruluğu gösterilebilir. Bunun için tümevarım yöntemi kullanılırsa, $m=0,1$ için yukarıdaki eşitsizliğin doğruluğu açıktır. $(m-1)$ için doğru olduğu kabul edilsin ve m için doğru olduğu gösterilsin. $K_{11}(x,t)$ için,

$$\int_{-x}^{x}\left|K_{11}^{(m)}(x,t)\right|dt\le\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u^{2}(V)\right|\left|K_{12}^{(m-1)}(V,t+V-x)\right|dV$$

$$+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u^{2}(V)\right|\left|K_{12}^{(m-1)}(V,x+t-V)\right|dV+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}|u(V)|\left|K_{22}^{(m-1)}(V,t+V-x\right|dV$$

$$+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}|u(V)|\left|K_{22}^{(m-1)}(V,t+x-V)\right|dV+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}|u(V)|\left|K_{11}^{(m-1)}(V,t+x-V)\right|dV$$

$$+\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}|u(V)|\left|K_{11}^{(m-1)}(V,t+V-x)\right|dV$$

eşitsizliği elde edilir. Burada

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u^2(V)\right|\left|K_{12}^{(m-1)}(V,t+V-x)\right|dV=\frac{l}{2}\int_{0}^{x}\left|u^2(V)\right|dV\int_{-V}^{V}\left|K_{12}^{(m-1)}(V,s)\right|ds$$

$$\leq\frac{l}{2}\int_{0}^{x}\left|u^2(V)\right|\frac{[cs(V)]^m}{m!}dV\leq\frac{c^m}{m!}\int_{0}^{x}s^m(V)ds(V)\leq\frac{c^m s^{m+1}(x)}{(m+1)!}$$

olur.

Benzer şekilde,

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u^2(V)\right|\left|K_{12}^{(m-1)}(V,x+t-V)\right|dV\leq\frac{c^m s^{m+1}(x)}{(m+1)!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u(V)\right|\left|K_{22}^{(m-1)}(V,t+V-x)\right|dV\leq\frac{c^m s^{m+1}(x)}{(m+1)!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u(V)\right|\left|K_{22}^{(m-1)}(V,t+x-V)\right|dV\leq\frac{c^m s^{m+1}(x)}{(m+1)!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x+t}{2}}^{x}\left|u(V)\right|\left|K_{11}^{(m-1)}(V,t+x-V)\right|dV\leq\frac{c^m s^{m+1}(x)}{(m+1)!}$$

$$\frac{l}{2}\int_{-x}^{x}dt\int_{\frac{x-t}{2}}^{x}\left|u(V)\right|\left|K_{11}^{(m-1)}(V,t+V-x)\right|dV\leq\frac{c^m s^{m+1}(x)}{(m+1)!}$$

elde edilir. O halde

$$\int_{-x}^{x}\left|K_{11}^{(m)}(x,t)\right|dt\leq\frac{6c^m s^{m+1}(x)}{(m+1)!}\leq\frac{c^{m+1}s^{m+1}(x)}{(m+1)!}=\frac{[cs(x)]^{m+1}}{(m+1)!}$$

eşitsizliği sağlanır.

(9) eşitsizliklerinden $\sum_{m=0}^{\infty}K_{ij}^{(m)}(x,t)$ serisinin $L_1[0,p]$ uzayında düzgün yakınsak olduğu açıktır ve serinin toplamı olan $K_{ij}(x,\cdot)\in L_1[0,p]$ fonksiyonları aşağıdaki eşitsizliği sağlar:

$$\int_{-x}^{x}\left|K_{ij}(x,t)\right|dt\leq e^{cs(x)}-1.$$

Böylece aşağıdaki teorem ispatlanmış olur.

Teorem. $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ başlangıç koşullarını sağlayan L probleminin her bir çözümü

için $\int_{-x}^{x} \left| K_{ij}(x,t) \right| dt \le e^{cs(x)} - 1$ olmak üzere,

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x, l) = \begin{pmatrix} 1 + ib(x) \\ -i + b(x) \end{pmatrix} e^{ilx} + \int_{-x}^{x} K(x,t) \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{ilt} dt$$

eşitsizliği sağlanır.

Burada $b(x) = b_1(x) + ib_2(x)$ ve

$b_2(x) = \frac{l}{2} \int_0^x u^2(t) b_1(t) dt \quad b_1(x) = \frac{l}{2} \int_0^x u^2(t) dt - \frac{l}{2} \int_0^x u^2(t) b_2(t) dt$ gerçel değerli mutlak

sürekli fonksiyonlar , $K(x,t) = \begin{pmatrix} K_{11}(x,t) & K_{12}(x,t) \\ K_{21}(x,t) & K_{22}(x,t) \end{pmatrix}$ ve c pozitif sabittir.

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