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# Sonlu Aralıkta Coulomb Potansiyele Sahip Sturm-Liouville Diferansiyel Denklemlerin Çözümleri İçin Bir Gösterilim

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**Özet:** Bu çalışmada sonlu aralıkta Coulomb potansiyele sahip Sturm-Liouville operatörleri için çevirme operatörü tipinde gösterilimler elde edilmiştir.
**Anahtar kelimeler:** Çevirme operatörü, İntegral Denklemi, Sturm-Liouville operatörü, Coulomb potansiyeli.

# An Integral Representation for Solutions of Sturm-Liouville Differential Equations with Coulomb Potential on a Finite Interval

**Abstract:** In this study, representation with transformation operator has been obtained for Sturm-Liouville operators with Coulomb potential on a finite interval.
**Keywords:** Transformation operator, Integral Equation, Sturm-Liouville operator, Coulomb Potential.

## 1. Giriş

$(a,b)$ sonlu aralığında $\mathbf{l}y := -y''(x) + q(x)\,y(x)$ diferansiyel ifadesinin ürettiği Sturm-Liouville operatörler teorisinde, $q(x)$ fonksiyonu, $q(x) \in L_1(a,b)$ koşulunu sağladığında $\mathbf{l}y$ diferansiyel ifadesi regüler, $(a,b)$ aralığı sonsuz ya da $q(x)$ fonksiyonu verilen aralığın iç noktalarında veya sınırında integrallenemeyen singüleriteye sahip ise singüler diferansiyel ifade olarak adlandırılır.

[6] çalışmasında, $u \in L_2(0,1)$ olmak üzere genelleştirilmiş türev kullanılarak $q(x) = u'(x)$ şeklinde bir potansiyele sahip singüler Sturm-Liouville operatörü tanımlanmıştır.

Ayrıca bu çalışmada $u \in L_2(0,1)$ olmak üzere $q(x) = u'(x)$ potansiyeline sahip $\mathbf{l}y$ diferansiyel ifadesi tarafından diferansiyel operatörlerin self-adjoint genişlemeleri oluşturulmuştur. Genelleştirilmiş fonksiyonlar, [5] 'de kanonik regülarizasyon metodu kullanılarak $|x|^{-a} signx$, $a \neq 2, 4, 6, \ldots$ durumlarında belirtilmiştir. $a < \frac{3}{2}$ durumunda, bu yolla elde edilen genelleştirilmiş fonksiyonlar $L_2$ uzayındaki fonksiyonların genelleştirilmiş türevleri şeklinde gösterilebilir. Böylece $\mathbf{l}y$ diferansiyel ifadesi ve $q(x) = |x|^{-a} signx$ şeklinde bir potansiyele sahip Sturm-Liouville operatörü tanımlanabilir. [1] çalışmasında, $q(x) = Cx^{-a}$ ve $a < \frac{3}{2}$, $C \in R$, $x \in R^+$ olması durumunda, bu tip potansiyele sahip Sturm-Liouville denklemi için sınır-değer probleminde bir regülarizasyon verilmiştir.

[4] çalışmasında ise $q(x) = Cx^{-a}$ ve $a \in [1,2)$ olması durumunda sınır-değer koşullarına göre bu tip potansiyele sahip $\mathbf{l}y$ diferansiyel ifadesi tarafından üretilen operatörlerin tüm self-adjoint genişlemeleri ve dolayısıyla bu tip potansiyele sahip Sturm-Liouville denklemi için sınır-değer probleminin nasıl konulacağı konusu incelenmiştir. Ayrıca [4] ve [6] çalışmalarında ki regülarizasyonlar sadece $a < \frac{3}{2}$ durumu için çakışmaktadır.

$$\mathbf{l}y := -y''(x) + \frac{C}{x^a} y(x) + q(x) y(x) \ , \ 0 < x < p \qquad \text{(I)}$$

diferansiyel ifadesini ele alalım. Burada $C \in R$, $q(x)$- gerçel değerli, sınırlı bir fonksiyondur.

$D_0' = C_0^\infty(0,p)$ kümesinde $L_0' : L_0' y = \mathbf{l}(y)$ operatörünü tanımlayalım. Açıktır ki, $L_2[0,p]$ uzayında $L_0'$ operatörü simetriktir. $L_0$ ın kapanışı olan $L_0'$ operatörü (I) diferansiyel ifadesinin ürettiği minimal operatördür, $L_0$ operatörünün eşleniği olan $L_0^*$ operatörüde (I) diferansiyel ifadesinin ürettiği maksimal operatör olarak adlandırılır.

[4] çalışmasında, tüm maksimal dissipative ve accumulative ve ayrıca $L_0$ operatörünün self-adjoint genişlemeleri; (I) diferansiyel ifadesinin ürettiği maksimal ve minimal operatörlerin sınır koşulları ve tanım kümesine göre çalışılmıştır.

$u(x) = C\dfrac{x^{1-a}}{1-a}$ olmak üzere $(\Gamma_a y)(\mathrm{x}) = y'(x) - u(x)y(x)$ alalım.

[4] de $y(x) \in D(L_0^*)$ ise bu durumda $x \to +0$ iken $(\Gamma_a y)(\mathrm{x})$ fonksiyonu bir limite sahip olduğu gösterilmiştir. Yani

$$\lim_{x \to +0} (\Gamma_a y)(\mathrm{x}) = (\Gamma_a y)(0)$$

dir. Dolayısıyla (I) diferansiyel ifadesinin ürettiği $L_0$ minimal operatörünün $D(L_0)$ tanım kümesi sadece $y(x) \in D(L_0^*)$ fonksiyonlarından oluşur öyle ki, $y(x)$ fonksiyonu $y(0) = y(p) = (\Gamma_a y)(0) = y'(p) = 0$ koşullarını sağlar.

Bu çalışmada $a = 1$ durumu incelenmiştir. Dolayısıyla $u(x) = C \ln x \in L_2[0, p]$ ve $(\Gamma y)(x) = y'(x) - u(x)y(x)$ olarak alınmıştır. Ayrıca bu çalışmada [10] çalışmasında ki

$$y(x,k) = y_0(x,k) + \int_{-x}^{x} K(x,t) e^{ikt} dt$$

çevirme operatörüne benzer bir gösterilim elde edilmiştir.

## 2. İntegral denklemin oluşturulması

$$\mathbf{l}y := -y'' + \left[\frac{C}{x} + q(x)\right] y = l\, y,\ \ l = k^2,\ 0 < x < p \qquad (1)$$

diferansiyel denklemi,

$$y(0) = 0, y(p) = 0 \qquad (2)$$

sınır koşulları ve

$$\begin{cases} y(d+0) = a\, y(d-0) \\ y'(d+0) = a^{-1} y'(d-0) + 2ikb\, y(d-0) \end{cases} \qquad (3)$$

süreksizlik koşullarının ürettiği $L$ problemini ele alalım. Burada $l$ -spektral parametre, $C, a, b \in R, a \neq 1, a > 0$, $d \in \left(\frac{p}{2}, p\right)$, $q(x)$- gerçel değerli, sınırlı ve $q(x) \in L_2(0,p)$ dir.

(1) diferansiyel denkleminde $u(x) = C \ln x \in L_2(0,p)$ olmak üzere $(\Gamma y)(x) = y' - u(x)y$ alınırsa,

$$\mathbf{l}y := -y'' + u'(x)y + q(x)y = -\left[y' - u(x)y\right]' - u(x)y' + q(x)y = k^2 y,$$

$$\mathbf{l}y = -\left[(\Gamma y)\ (x)\right]' - u(x)\ (\Gamma y)\ (x) - u^2(x)y + q(x)y = k^2 y$$

elde edilir. Şimdi $y_1 = y,\ y_2 = (\Gamma y)(x)$ alınırsa,

$$\begin{cases} y_1' - y_2 = u(x)y_1 \\ y_2' + k^2 y_1 = -u(x)y_2 - u^2(x)y_1 + q(x)y_1 \end{cases} \quad (4)$$

$$y_1(0) = 0, y_1(p) = 0 \quad (5)$$

$$\begin{cases} y_1(d+0) = a y_1(d-0) \\ y_2(d+0) = a^{-1} y_1(d-0) + 2ikb y_1(d-0) \end{cases} \quad (6)$$

problemi elde edilir.

(4) sisteminin matris gösterilimi

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} u & 1 \\ -k^2 - u^2 + q & -u \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (7)$$

veya

$A = \begin{pmatrix} u(x) & 1 \\ -k^2 - u^2(x) + q(x) & \text{-}u(x) \end{pmatrix}, y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ olmak üzere $y' = Ay$, matrisinin elemanları integrallenebilir olduğundan [12] de $y' = Ay + f$, $f \in L_1(0,p)$ sistemleri için başlangıç-değer problemin çözümünün varlığı ile ilgili teorem gereği her $x \in [0,p]$, $n = (n_1, n_2)^T \in C^2$ için (7) sisteminin $y_1(x) = n_1$ , $y_2(x) = n_2$ başlangıç koşullarını sağlayan sadece bir tek çözümü vardır. Özel olarak $y_1(0) = 1, y_2(0) = ik$ alınabilir.

**Tanım:** (4) diferansiyel denklemler sisteminin $y_1(x) = y(x) = n_1$, $y_2(x) = (\Gamma y)(x) = n_2$ başlangıç koşullarını sağlayan çözümünün birinci bileşenine, (1) denkleminin aynı koşulları sağlayan çözümü denir.

(4) diferansiyel denklemler siteminin $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(0)=\begin{pmatrix} 1 \\ ik \end{pmatrix}$ başlangıç koşullarını ve (6) süreksizlik koşullarını sağlayan çözümü, $a^{+}=\frac{1}{2}\left(a+a^{-1}\right)$ , $a^{-}=\frac{1}{2}\left(a-a^{-1}\right)$ olmak üzere,

$x<d$ iken,

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x)=\begin{pmatrix} e^{ikx}+\int_0^x u(t)y_1\cos k(x-t)dt-\frac{1}{k}\int_0^x\left[u(t)y_2+u^2(t)y_1-q(t)y_1\right]\sin k(x-t)dt \\ ike^{ikx}-\int_0^x ku(t)y_1\sin k(x-t)dt+\int_0^x\left[u(t)y_2+u^2(t)y_1-q(t)y_1\right]\cos k(x-t)dt \end{pmatrix} \tag{8}$$

$x>d$ iken,

$$\begin{aligned} y_1 &= a^{+}e^{ikx}+a^{-}e^{ik(2d-x)}+b\left[e^{ikx}-e^{ik(2d-x)}\right] \\ &+\int_0^d\left[a^{+}\cos k(x-t)+a^{-}\cos k(x-2d+t)\right]u(t)y_1dt \\ &-\frac{1}{k}\int_0^d\left[u(t)y_2+u^2(t)y_1-q(t)y_1\right]\left[a^{+}\sin k(x-t)-a^{-}\sin k(x-2d+t)\right]dt \\ &+ib\int_o^d\left[\sin k(x-t)+\sin k(x-2d+t)\right]u(t)y_1dt \\ &-\frac{ib}{k}\int_0^d\left[u(t)y_2+u^2(t)y_1-q(t)y_1\right]\left[\cos k(x-2d+t)-\cos k(x-t)\right]dt \\ &+\int_d^x u(t)y_1\cos k(x-t)dt-\frac{1}{k}\int_d^x\left[u(t)y_2+u^2(t)y_1-q(t)y_1\right]\sin k(x-t)dt \end{aligned} \tag{9}$$

$$
\begin{aligned}
y_2 &= ika^{+}e^{ikx} - ika^{-}e^{ik(2d-x)} + ikb\left[e^{ikx} - e^{ik(2d-x)}\right] \\
&\quad + k\int_0^d \left[-a^{+}\sin k(x-t) - a^{-}\sin k(x-2d+t)\right]u(t)y_1dt \\
&\quad + \int_0^d \left[u(t)y_2 + u^2(t)y_1 - q(t)y_1\right]\left[-a^{+}\cos k(x-t) + a^{-}\cos k(x-2d+t)\right]dt \\
&\quad + ikb\int_o^d \left[\cos k(x-t) + \cos k(x-2d+t)\right]u(t)y_1dt \\
&\quad - ib\int_0^d \left[u(t)y_2 + u^2(t)y_1 - q(t)y_1\right]\left[\sin k(x-t) - \sin k(x-2d+t)\right]dt \\
&\quad - k\int_d^x u(t)y_1\sin k(x-t)dt - \int_d^x \left[u(t)y_2 + u^2(t)y_1 - q(t)y_1\right]\cos k(x-t)dt
\end{aligned}
\tag{10}
$$

integral denklemler sistemini elde edilir.

Şimdi (4) diferansiyel denklemler siteminin $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ ik \end{pmatrix}$ başlangıç koşullarını ve (6) süreksizlik koşullarını sağlayan her çözümünün

$x < d$ iken,

$$
\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x) = \begin{pmatrix} e^{ikx} + \int_{-x}^{x} K_{11}(x,t)e^{ikt}dt \\ ike^{ikx} + b(x)e^{ikx} + \int_{-x}^{x} K_{21}(x,t)e^{ikt} + ik\int_{-x}^{x} K_{22}(x,t)e^{ikt} \end{pmatrix}
\tag{11}
$$

$x > d$ iken,

$$
\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x) = \begin{pmatrix} a^{+}e^{ikx} + a^{-}e^{ik(2d-x)} + b\left(e^{ikx} - e^{ik(2d-x)}\right) + \int_{-x}^{x} K_{11}(x,t)e^{ikt}dt \\ ika^{+}e^{ikx} - ika^{-}e^{ik(2d-x)} + ikb\left(e^{ikx} + e^{ik(2d-x)}\right) \\ \quad + b(x)\left[a^{+}e^{ikx} + a^{-}e^{ik(2d-x)} + b\left(e^{ikx} - e^{ik(2d-x)}\right)\right] \\ \quad + \int_{-x}^{x} K_{21}(x,t)e^{ikt} + ik\int_{-x}^{x} K_{22}(x,t)e^{ikt} \end{pmatrix}
\tag{12}
$$

şeklinde bir integral gösterilime sahip olduğunu ispatlayalım. Burada $K_{ij}(x,t), i, j = 1,2$ ve $b(x)$ reel değerli sınırlı fonksiyonlardır. (11) ve (12) ifadeleri, (9) ve (10) çözümünde yerine yazılırsa,

$$\int_{-x}^{x} K_{11}(x,t)e^{ikt}dt = \int_{0}^{d} u(t)\left[a^{+}\cos k(x-t)+a^{-}\cos k(x-2d+t)\right]\left\{e^{ikt}+\int_{-t}^{t} K_{11}(t,s)e^{iks}ds\right\}dt$$

$$-\frac{1}{k}\int_{0}^{d} u(t)\left[a^{+}\sin k(x-t)-a^{-}\sin k(x-2d+t)\right]\left\{ike^{ikt}+b(t)e^{ikt}+\int_{-t}^{t} K_{21}(t,s)e^{iks}ds\right.$$

$$\left.+ik\int_{-t}^{t} K_{22}(t,s)e^{iks}ds\right\}dt$$

$$-\frac{1}{k}\int_{0}^{d}\left(u^{2}(t)-q(t)\right)\left[a^{+}\sin k(x-t)-a^{-}\sin k(x-2d+t)\right]\left\{e^{ikt}+\int_{-t}^{t} K_{11}(t,s)e^{iks}ds\right\}dt$$

$$+ib\int_{o}^{d} u(t)\left[\sin k(x-t)+\sin k(x-2d+t)\right]\left\{e^{ikt}+\int_{-t}^{t} K_{11}(t,s)e^{iks}ds\right\}dt$$

$$\frac{ib}{k}\int_{0}^{d} u(t)\left[\cos k(x-2d+t)-\cos k(x-t)\right]\left\{ike^{ikt}+b(t)e^{ikt}+\int_{-t}^{t} K_{21}(t,s)e^{iks}ds\right.$$

$$\left.+ik\int_{-t}^{t} K_{22}(t,s)e^{iks}ds\right\}dt$$

$$\frac{ib}{k}\int_{0}^{d}\left(u^{2}(t)-q(t)\right)\left[\cos k(x-2d+t)-\cos k(x-t)\right]\left\{e^{ikt}+\int_{-t}^{t} K_{11}(t,s)e^{iks}ds\right\}dt$$

$$+\int_{d}^{x} u(t)\left\{a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+b\left(e^{ikt}-e^{ik(2d-t)}\right)+\int_{-t}^{t} K_{11}(t,s)e^{iks}ds\right\}\cos k(x-t)\,dt$$

$$-\int_{d}^{x} u(t)\left\{ika^{+}e^{ikt}-ika^{-}e^{ik(2d-t)}ik+b\left(e^{ikt}+e^{ik(2d-t)}\right)\right\}\sin k(x-t)\,dt$$

$$-\int_{d}^{x} u(t)b(t)\left[a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+b\left(e^{ikt}-e^{ik(2d-t)}\right)\right]\sin k(x-t)\,dt$$

$$+\int_{d}^{x} u(t)\left\{\int_{-t}^{t} K_{21}(t,s)e^{iks}+ik\int_{-t}^{t} K_{22}(t,s)e^{ikt}\right\}\sin k(x-t)dt$$

$$-\int_{d}^{x}\left(u^{2}(t)-q(t)\right)\left\{a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+b\left(e^{ikt}-e^{ik(2d-t)}\right)+\int_{-t}^{t} K_{11}(t,s)e^{iks}ds\right\}\sin k(x-t)\,dt$$

ve

$$b(x)\left[a^{+}e^{ikx}+a^{-}e^{ik(2d-x)}+b\left(e^{ikx}-e^{ik(2d-x)}\right)\right]+\int_{-x}^{x}K_{21}(x,t)e^{ikt}+ik\int_{-x}^{x}K_{22}(x,t)e^{ikt}=$$

$$k\int_{0}^{d}u(t)\left[-a^{+}\sin k(x-t)-a^{-}\sin k(x-2d+t)\right]\left\{e^{ikt}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds\right\}dt$$

$$+\int_{0}^{d}u(t)\left[-a^{+}\cos k(x-t)+a^{-}\cos k(x-2d+t)\right]\left\{ike^{ikt}+b(t)e^{ikt}+\int_{-t}^{t}K_{21}(t,s)e^{iks}ds\right.$$

$$\left.+ik\int_{-t}^{t}K_{22}(t,s)e^{iks}ds\right\}dt$$

$$+\int_{0}^{d}\left(u^{2}(t)-q(t)\right)\left[-a^{+}\cos k(x-t)+a^{-}\cos k(x-2d+t)\right]\left\{e^{ikt}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds\right\}dt$$

$$+ikb\int_{o}^{d}u(t)\left[\cos k(x-t)+\cos k(x-2d+t)\right]\left\{e^{ikt}+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds\right\}dt$$

$$-ib\int_{0}^{d}u(t)\left[\sin k(x-t)-\sin k(x-2d+t)\right]\left\{ike^{ikt}+b(t)e^{ikt}+\int_{-t}^{t}K_{21}(t,s)e^{iks}ds\right.$$

$$\left.+ik\int_{-t}^{t}K_{22}(t,s)e^{iks}ds\right\}dt$$

$$-k\int_{d}^{x}u(t)\left\{a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+b\left(e^{ikt}-e^{ik(2d-t)}\right)+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds\right\}\sin k(x-t)\,dt$$

$$-\int_{d}^{x}u(t)\left\{ika^{+}e^{ikt}-ika^{-}e^{ik(2d-t)}ikb\left[e^{ikt}+e^{ik(2d-t)}\right]\right\}\cos k(x-t)dt$$

$$-\int_{d}^{x}u(t)b(t)\left[a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+b\left(e^{ikt}-e^{ik(2d-t)}\right)\right]\cos k(x-t)\,dt$$

$$-\int_{d}^{x}u(t)\left\{\int_{-t}^{t}K_{21}(t,s)e^{iks}+ik\int_{-t}^{t}K_{22}(t,s)e^{ikt}\right\}\cos k(x-t)\,dt$$

$$-\int_{d}^{x}\left(u^{2}(t)-q(t)\right)\left\{a^{+}e^{ikt}+a^{-}e^{ik(2d-t)}+b\left(e^{ikt}-e^{ik(2d-t)}\right)+\int_{-t}^{t}K_{11}(t,s)e^{iks}ds\right\}\cos k(x-t)\,dt$$

integral denklemleri elde edilir. Gerekli hesaplamalardan sonra

$$\int_{-x}^{x} K_{11}(x,t)e^{ikt}dt=\frac{a^{+}}{2}\int_{-x}^{x} u\left(\frac{x+t}{2}\right)e^{ikt}dt+\frac{a^{-}}{2}\int_{x-2d}^{x} u\left(d+\frac{t-x}{2}\right)e^{ikt}dt$$

$$-\frac{a^{-}}{2}\int_{x}^{2d-x} u\left(d-\frac{t-x}{2}\right)e^{ikt}dt-\frac{b}{2}\int_{-x}^{2d-x} u\left(\frac{x+t}{2}\right)e^{ikt}dt+\frac{b}{2}\int_{2d-x}^{x} u\left(\frac{x+t}{2}\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{2d-x} u\left(d-\frac{t-x}{2}\right)e^{ikt}dt-\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{0}^{\frac{x+t}{2}}\left[u^{2}(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{x-2d}^{x} u\left(d+\frac{t-x}{2}\right)e^{ikt}dt-\frac{a^{-}}{2}\int_{2d-x}^{x}\left(\int_{d}^{d+\frac{x+t}{2}}\left[u^{2}(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$+\frac{a^{-}}{2}\int_{x-2d}^{x}\left(\int_{d+\frac{t-x}{2}}^{d}\left[u^{2}(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{2d-x}^{x}\left(\int_{d}^{d+\frac{x+t}{2}}\left[u^{2}(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{2d-x}^{x}\left(\int_{0}^{\frac{x+t}{2}}\left[u^{2}(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$+\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x} u(s)K_{11}(s,t+s-x)ds\right)e^{ikt}dt+\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x} u(s)K_{11}(s,t-s+x)ds\right)e^{ikt}dt$$

$$-\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{0}^{d}u(s)\int_{t-x+s}^{t+x-s} K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt-\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x} u(s)K_{22}(s,t+s-x)ds\right)e^{ikt}dt$$

$$+\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x} u(s)K_{22}(s,t-s+x)ds\right)e^{ikt}dt$$

$$-\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{0}^{d}\left(u^{2}(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$+\frac{a^{-}}{2}\int_{-x}^{x}\left(\int_{0}^{d}u(s)\int_{t-x-s+2d}^{t+x+s-2d} K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$+\frac{a^-}{2}\int_{-x}^{x}\left(\int_0^d\left(u^2(s)-q(s)\right)\int_{t-x-s+2d}^{t+x+s-2d}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t+s-x)\,ds\right)e^{ikt}dt\ -\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{22}(s,t-s+x)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-s+x)\,ds\right)e^{ikt}dt\ -\frac{b}{2}\int_{-x}^{x}\left(\int_d^x u(t)\int_{t-x+s}^{t+x-s}K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{x}\left(\int_d^x\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{x-2d}^{x}\left(\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$+\frac{a^-}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{11}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$+\frac{a^-}{2}\int_{x-2d}^{x}\left(\int_{d-\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$+\frac{a^-}{2}\int_{x-2d}^{x}\left(\int_{d-\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)\,ds\right)e^{ikt}dt-\ \frac{a^-}{2}\int_{-x}^{2d-x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t+s-x)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{x-2d}^{x}\left(\int_{d-\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{x-2d}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt\ -\ \frac{b}{2}\int_{-x}^{2d-x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t-s+x)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{x-2d}^{x}\left(\int_{d-\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_2(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{x-2d}^{x}\left(\int_{0}^{d+\frac{t-x}{2}}\left[u^2(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{0}^{\frac{t+x}{2}}\left[u^2(s)+u(s)b(s)-q(s)\right]ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{x-2d}^{x}\left(\int_{0}^{d}u(t)\int_{t+x-2d+s}^{t+x-s}K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt+\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{0}^{d}u(t)\int_{t-x+s}^{t-x+2d-s}K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{x-2d}^{x}\left(\int_{0}^{d}\left[u^2(t)-q(t)\right]\int_{t+x-2d+s}^{t+x-s}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{0}^{d}\left[u^2(t)-q(t)\right]\int_{t-x+s}^{t-x+2d-s}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$+\frac{1}{2}\int_{-x}^{2d-x}\left(\int_{d}^{x}u(t)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt+\frac{1}{2}\int_{2d-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(t)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$+\frac{1}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)\,ds\right)e^{ikt}dt \qquad (13)$$

$$\int_{-x}^{x}K_{21}(x,t)e^{ikt}dt=\frac{a^-}{4}\left(\int_{x-2d}^{x}\left[u^2\left(d+\frac{t-x}{2}\right)+u\left(d+\frac{t-x}{2}\right)b\left(d+\frac{t-x}{2}\right)-q\left(d+\frac{t-x}{2}\right)\right]e^{ikt}dt\right)$$

$$+\frac{a^-}{4}\left(\int_{x}^{2d-x}\left[u^2\left(d-\frac{t-x}{2}\right)+u\left(d-\frac{t-x}{2}\right)b\left(d-\frac{t-x}{2}\right)-q\left(d-\frac{t-x}{2}\right)\right]e^{ikt}dt\right)$$

$$-\frac{a^+}{4}\int_{-x}^{x}\left[u^2\left(\frac{t+x}{2}\right)+u\left(\frac{t+x}{2}\right)b\left(\frac{t+x}{2}\right)-q\left(\frac{t+x}{2}\right)\right]e^{ikt}dt$$

$$-\frac{b}{4}\int_{-x}^{x}\left[u^2\left(\frac{t+x}{2}\right)+u\left(\frac{t+x}{2}\right)b\left(\frac{t+x}{2}\right)-q\left(\frac{t+x}{2}\right)\right]e^{ikt}dt$$

$$-\frac{b}{4}\int_{x-2d}^{x}\left[u^2\left(d+\frac{t-x}{2}\right)+u\left(d+\frac{t-x}{2}\right)b\left(d+\frac{t-x}{2}\right)-q\left(d+\frac{t-x}{2}\right)\right]e^{ikt}dt$$

$$-\frac{b}{4}\int_{2d-x}^{x}\left[u^2\left(d-\frac{t-x}{2}\right)+u\left(d-\frac{t-x}{2}\right)b\left(d-\frac{t-x}{2}\right)-q\left(d-\frac{t-x}{2}\right)\right]e^{ikt}dt$$

$$-\frac{a^-}{2}\int_{x-2d}^{x}\left(\int_{d+\frac{t-x}{2}}^{d}u(s)K_{21}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^-}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{21}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^-}{2}\int_{x-2d}^{x}\left(\int_{d+\frac{t-x}{2}}^{d}\left(u^2(s)-q(s)\right)K_{21}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^-}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}\left(u^2(s)-q(s)\right)K_{21}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{-x}^{x-2d}\left(\int_{d+\frac{t-x}{2}}^{x}u(s)K_{21}(s,t-x+s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{x-2d}^{x}\left(\int_{d}^{x}u(s)K_{21}(s,t-x+s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{-x}^{2d-x}\left(\int_{d}^{x}u(s)K_{21}(s,t+x-s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{2d-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{21}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{-x}^{x-2d}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{x-2d}^{x}\left(\int_{d}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{-xd}^{2d-x}\left(\int_{d}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{2d-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^+}{2}\int_{2d-x}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt\quad-\frac{a^+}{2}\int_{2d-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^+}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x}\left(u^2(s)-q(s)\right)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^{+}}{2}\int_{-x}^{2d-x}\left(\int_{\frac{t+x}{2}}^{x}\left(u^{2}(s)-q(s)\right)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt \tag{14}$$

$$\int_{-x}^{x}K_{22}(x,t)e^{ikt}dt=$$

$$-\frac{a^{+}}{2}\int_{-x}^{x}u\left(\frac{x+t}{2}\right)e^{ikt}dt-\frac{a^{-}}{2}\int_{x-2d}^{x}u\left(d+\frac{t-x}{2}\right)e^{ikt}dt-\frac{a^{-}}{2}\int_{-x}^{2d-x}u\left(d-\frac{t-x}{2}\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{2d-x}u\left(\frac{x+t}{2}\right)e^{ikt}dt-\frac{b}{2}\int_{2d-x}^{x}u\left(\frac{x+t}{2}\right)e^{ikt}dt-\frac{b}{2}\int_{x-2d}^{x}u\left(d+\frac{t-x}{2}\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{2d-x}u\left(d-\frac{t-x}{2}\right)e^{ikt}dt-\frac{b}{2}\int_{-x}^{x}\left(\int_{0}^{d}\left(u^{2}(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$+\frac{a^{+}}{2}\int_{x-2d}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(t)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt-\frac{a^{+}}{2}\int_{-x}^{2d-x}\left(\int_{\frac{x+t}{2}}^{x}u(t)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^{-}}{2}\int_{x-2d}^{x}\left(\int_{d-\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt+\frac{a^{-}}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^{-}}{2}\int_{x-2d}^{x}\left(\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$-\frac{a^{-}}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{11}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{x-2d}^{x}\left(\int_{d-\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{2d-x}\left(\int_{d-\frac{t+x}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)\,ds\right)e^{ikt}dt$$

$$+\frac{1}{2}\int_{-x}^{2d-x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt$$

$$+\frac{1}{2}\int_{x-2d}^{x}\left(\int_{d}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{-x}^{2d-x}\left(\int_{d}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{2d-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{-x}^{x-2d}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t-x+s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{x-2d}^{x}\left(\int_{d}^{x}u(s)K_{22}(s,t-x+s)\,ds\right)e^{ikt}dt-\frac{1}{2}\int_{-x}^{x-2d}\left(\int_{d}^{x}u(s)K_{22}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{1}{2}\int_{2d-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{22}(s,t+x-s)\,ds\right)e^{ikt}dt+\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{0}^{d}u(t)\int_{t-x+s}^{t+x-s}K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt$$

$$-\frac{a^{+}}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt+\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$+\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)\,ds\right)e^{ikt}dt+\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x+t}{2}}^{x}u(s)K_{22}(s,t+x-s)\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{x}\left(\int_{0}^{d}u(t)\int_{t-x+2d-s}^{t+x-2d+s}K_{21}(s,x)\,dx\,ds\right)e^{ikt}dt\quad-\frac{b}{2}\int_{-x}^{x}\left(\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t-x+s)\,ds\right)e^{ikt}dt$$

$$-\frac{b}{2}\int_{-x}^{x}\left(\int_{0}^{d}\left(u^{2}(s)-q(s)\right)\int_{t-x+2d-s}^{t+x-2d+s}K_{11}(s,x)\,dx\,ds\right)e^{ikt}dt \qquad (15)$$

eşitlikleri elde edilir. (13), (14) ve (15) eşitliklerinden $K_{ij}(x,t), i, j=1,2$ fonksiyonları için aşağıdaki integral denklem sistemleri bulunur.

$$K_{11}(x,t)=\frac{a^+}{2}u\left(\frac{t+x}{2}\right)+\frac{a^-}{2}u\left(d+\frac{t-x}{2}\right)-\frac{b}{2}u\left(\frac{t+x}{2}\right)+\frac{b}{2}u\left(d+\frac{t-x}{2}\right)+\frac{b}{2}u\left(d-\frac{t-x}{2}\right)$$

$$-\frac{a^+}{2}\int_0^{\frac{t+x}{2}}\left[u^2(t)+u(t)b(t)-q(t)\right]dt+\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}\left[u^2(t)+u(t)b(t)-q(t)\right]dt$$

$$-\frac{b}{2}\int_0^{d+\frac{t-x}{2}}\left[u^2(t)+u(t)b(t)-q(t)\right]dt+\frac{b}{2}\int_0^{\frac{t+x}{2}}\left[u^2(t)+u(t)b(t)-q(t)\right]dt$$

$$+\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)ds+\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)ds$$

$$-\frac{a^+}{2}\int_0^{d}u(s)\int_{t-x+s}^{t+x-s}K_{21}(s,x)dxds-\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t-x+s)ds$$

$$+\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{22}(s,t+x-s)ds-\frac{a^+}{2}\int_0^{d}\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}(s,x)dxds$$

$$+\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+2d-s)ds+\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}(s,t+x-2d+s)ds$$

$$+\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)ds-\frac{a^-}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t-x+s)ds$$

$$+\frac{a^-}{2}\int_0^{d}u(s)\int_{t-x+2d-s}^{t+x-2d+s}K_{21}(s,x)dxds+\frac{a^-}{2}\int_0^{d}\left(u^2(s)-q(s)\right)\int_{t-x+2d-s}^{t+x-2d+s}K_{11}(s,x)dxds$$

$$+\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)ds+\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)ds$$

$$+\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)ds$$

$$+\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+2d-s)ds+\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}(s,t+x-2d+s)ds$$

$$+\frac{b}{2}\int_0^{d}u(s)\int_{t-x+s}^{t-x+2d-s}K_{21}^{(n-1)}(s,x)dxds+\frac{b}{2}\int_0^{d}\left(u^2(s)-q(s)\right)\int_{t+x-2d+s}^{t+x-s}K_{11}^{(n-1)}(s,x)dxds$$

$$+\frac{b}{2}\int_0^d\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t-x+2d-s}K_{11}(s,x)dxds-\frac{b}{2}\int_0^d u(s)\int_{t+x-2d+s}^{t+x-s}K_{21}(s,x)dxds$$

$$-\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{22}(s,t+x-s)ds$$

$$+\frac{1}{2}\int_d^x u(s)K_{11}(s,t+x-s)ds-\frac{1}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)ds$$

$$+\frac{1}{2}\int_{d-\frac{t+x}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)ds+\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)ds$$

$$-\frac{1}{2}\int_d^x u(s)\int_{t-x+s}^{t+x-s}K_{21}(s,x)dxds-\frac{1}{2}\int_d^x\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}(s,x)dxds$$

$$K_{21}(x,t)=-\frac{a^+}{4}\left[u^2\left(\frac{x+t}{2}\right)+u\left(\frac{x+t}{2}\right)b\left(\frac{x+t}{2}\right)-q\left(\frac{x+t}{2}\right)\right]$$

$$+\frac{a^-}{4}\left[u^2\left(d+\frac{t-x}{2}\right)+u\left(d+\frac{t-x}{2}\right)b\left(d+\frac{t-x}{2}\right)-q\left(d+\frac{t-x}{2}\right)\right]$$

$$-\frac{b}{4}\left[u^2\left(\frac{x+t}{2}\right)+u\left(\frac{x+t}{2}\right)b\left(\frac{x+t}{2}\right)-q\left(\frac{x+t}{2}\right)\right]$$

$$-\frac{b}{4}\left[u^2\left(d+\frac{t-x}{2}\right)+u\left(d+\frac{t-x}{2}\right)b\left(d+\frac{t-x}{2}\right)-q\left(d+\frac{t-x}{2}\right)\right]$$

$$-\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)ds-\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)ds$$

$$-\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}\left(u^2(s)-q(s)\right)K_{11}(s,t-x+s)ds-\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}\left(u^2(s)-q(s)\right)K_{11}(s,t+x-s)ds$$

$$-\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{21}(s,t-x+2d-s)ds-\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{21}(s,t+x-2d+s)ds$$

$$-\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}\left(u^2(s)-q(s)\right)K_{11}(s,t-x+2d-s)\,ds$$

$$-\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}\left(u^2(s)-q(s)\right)K_{11}(s,t+x-2d+s)\,ds$$

$$-\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{21}(s,t-x+s)\,ds\ -\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)\,ds$$

$$+\frac{1}{2}\int_{d}^{x}u(s)K_{21}(s,t-x+s)\,ds+\frac{1}{2}\int_{d}^{x}u(s)K_{21}(s,t+x-s)\,ds$$

$$+\frac{1}{2}\int_{d}^{x}u(s)K_{11}(s,t-x+s)\,ds+\frac{1}{2}\int_{d}^{x}u(s)K_{11}(s,t+x-s)\,ds$$

$$K_{22}(x,t)=-\frac{a^+}{2}u\left(\frac{t+x}{2}\right)-\frac{a^-}{2}u\left(d+\frac{t-x}{2}\right)+\frac{b}{2}u\left(\frac{t+x}{2}\right)-\frac{b}{2}u\left(d+\frac{t-x}{2}\right)$$

$$+\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{21}(s,t-x+s)\,ds\ -\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)\,ds$$

$$-\frac{a^+}{2}\int_{0}^{d}u(s)\int_{t-x+s}^{t+x-s}K_{21}(s,x)\,dx\,ds$$

$$-\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+s)ds-\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}(s,t+x-s)ds$$

$$-\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)ds-\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)ds$$

$$+\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}(s,t+x-s)ds$$

$$-\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}(s,t-x+s)ds-\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}(s,t+x-2d+s)ds$$

$$-\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}(s,t-x+2d-s)ds-\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{22}(s,t+x-2d+s)ds$$

$$+\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{22}(s,t+x-s)ds$$

$$+\frac{b}{2}\int_0^d u(s)\int_{t-x+2d-s}^{t+x-2d+s} K_{21}(s,x)dxds-\frac{b}{2}\int_0^d u(s)\int_{t-x+-s}^{t+x-s} K_{21}(s,x)dxds$$

$$-\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t+x-s} K_{11}(s,x)dxds-\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}\left(u^2(s)-q(s)\right)\int_{t-s-x+2d}^{t+x-2d+s} K_{11}(s,x)dxds$$

$$+\frac{1}{2}\int_{\frac{x-t}{2}}^{x} u(s)K_{11}(s,t-x+s)ds-\frac{1}{2}\int_{\frac{x+t}{2}}^{x} u(s)K_{22}(s,t-x+s)ds$$

$$+\frac{1}{2}\int_d^x u(s)K_{11}(s,t-x+s)ds-\frac{1}{2}\int_d^x u(s)K_{11}(s,t+x-s)ds$$

$$-\frac{1}{2}\int_d^x u(s)K_{22}(s,t-x+s)ds-\frac{1}{2}\int_d^x u(s)K_{22}(s,t+x-s)ds$$

1-) $d<x<2d,\ -x<t<x-2d<2d-x$ bölgesinde $K_{11}(x,t), K_{21}(x,t)$ ve $K_{22}(x,t)$ ifadelerine ardışık yaklaşımlar yöntemi uygulanırsa;

$$K_{11}^{(0)}(x,t)=\frac{a^+}{2}u\left(\frac{t+x}{2}\right)+\frac{a^-}{2}u\left(d+\frac{t-x}{2}\right)-\frac{b}{2}u\left(\frac{t+x}{2}\right)+\frac{b}{2}u\left(d+\frac{t-x}{2}\right)+\frac{b}{2}u\left(d-\frac{t-x}{2}\right)$$

$$-\frac{a^+}{2}\int_0^{\frac{t+x}{2}}\left[u^2(t)+u(t)b(t)-q(t)\right]dt+\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}\left[u^2(t)+u(t)b(t)-q(t)\right]dt$$

$$-\frac{b}{2}\int_0^{d+\frac{t-x}{2}}\left[u^2(t)+u(t)b(t)-q(t)\right]dt+\frac{b}{2}\int_0^{\frac{t+x}{2}}\left[u^2(t)+u(t)b(t)-q(t)\right]dt$$

$$K_{11}^{(n)}(x,t)=\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x} u(s)K_{11}^{(n-1)}(s,t-x+s)ds+\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x} u(s)K_{11}^{(n-1)}(s,t+x-s)ds$$

$$-\frac{a^+}{2}\int_0^d u(s)\int_{t-x+s}^{t+x-s} K_{21}^{(n-1)}(s,x)dxds-\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x} u(s)K_{22}^{(n-1)}(s,t-x+s)ds$$

$$+\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x} u(s)K_{22}^{(n-1)}(s,t+x-s)ds-\frac{a^+}{2}\int_0^d\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t+x-s} K_{11}^{(n-1)}(s,x)dxds$$

$$+\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}^{(n-1)}(s,t-x+2d-s)ds+\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}^{(n-1)}(s,t+x-2d+s)ds$$

$$+\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}^{(n-1)}(s,t-x+2d-s)ds-\frac{a^-}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}^{(n-1)}(s,t-x+s)ds$$

$$+\frac{a^-}{2}\int_{0}^{d}u(s)\int_{t-x+2d-s}^{t+x-2d+s}K_{21}^{(n-1)}(s,x)dxds+\frac{a^-}{2}\int_{0}^{d}\left(u^2(s)-q(s)\right)\int_{t-x+2d-s}^{t+x-2d+s}K_{11}^{(n-1)}(s,x)dxds$$

$$+\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}^{(n-1)}(s,t-x+2d-s)ds+\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{22}^{(n-1)}(s,t+x-2d+s)ds$$

$$+\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}^{(n-1)}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}^{(n-1)}(s,t+x-s)ds$$

$$+\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}^{(n-1)}(s,t-x+2d-s)ds+\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}^{(n-1)}(s,t+x-2d+s)ds$$

$$+\frac{b}{2}\int_{0}^{d}u(s)\int_{t-x+s}^{t-x+2d-s}K_{21}^{(n-1)}(s,x)dxds+\frac{b}{2}\int_{0}^{d}\left(u^2(s)-q(s)\right)\int_{t+x-2d+s}^{t+x-s}K_{11}^{(n-1)}(s,x)dxds$$

$$+\frac{b}{2}\int_{0}^{d}\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t-x+2d-s}K_{11}^{(n-1)}(s,x)dxds-\frac{b}{2}\int_{0}^{d}u(s)\int_{t+x-2d+s}^{t+x-s}K_{21}^{(n-1)}(s,x)dxds$$

$$-\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}^{(n-1)}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{22}^{(n-1)}(s,t+x-s)ds$$

$$+\frac{1}{2}\int_{d}^{x}u(s)K_{11}^{(n-1)}(s,t+x-s)ds-\frac{1}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}^{(n-1)}(s,t-x+2d-s)ds$$

$$+\frac{1}{2}\int_{d-\frac{t+x}{2}}^{d}u(s)K_{22}^{(n-1)}(s,t+x-2d+s)ds+\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}^{(n-1)}(s,t-x+s)ds$$

$$-\frac{1}{2}\int_{d}^{x}u(s)\int_{t-x+s}^{t+x-s}K_{21}^{(n-1)}(s,x)dxds-\frac{1}{2}\int_{d}^{x}\left(u^2(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}^{(n-1)}(s,x)dxds$$

$$K_{21}^{(0)}(x,t)=-\frac{a^+}{4}\left[u^2\left(\frac{x+t}{2}\right)+u\left(\frac{x+t}{2}\right)b\left(\frac{x+t}{2}\right)-q\left(\frac{x+t}{2}\right)\right]$$

$$+\frac{a^-}{4}\left[u^2\left(d+\frac{t-x}{2}\right)+u\left(d+\frac{t-x}{2}\right)b\left(d+\frac{t-x}{2}\right)-q\left(d+\frac{t-x}{2}\right)\right]$$

$$-\frac{b}{4}\left[u^2\left(\frac{x+t}{2}\right)+u\left(\frac{x+t}{2}\right)b\left(\frac{x+t}{2}\right)-q\left(\frac{x+t}{2}\right)\right]$$

$$-\frac{b}{4}\left[u^2\left(d+\frac{t-x}{2}\right)+u\left(d+\frac{t-x}{2}\right)b\left(d+\frac{t-x}{2}\right)-q\left(d+\frac{t-x}{2}\right)\right]$$

$$K_{21}{}^{(n)}(x,t)=-\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t-x+s)\,ds-\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t+x-s)\,ds$$

$$-\frac{a^+}{2}\int_{\frac{x-t}{2}}^{x}\left(u^2(s)-q(s)\right)K_{11}{}^{(n-1)}(s,t-x+s)\,ds$$

$$-\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{21}{}^{(n-1)}(s,t+x-2d+s)\,ds$$

$$-\frac{a^+}{2}\int_{\frac{x+t}{2}}^{x}\left(u^2(s)-q(s)\right)K_{11}{}^{(n-1)}(s,t+x-s)\,ds$$

$$-\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{21}{}^{(n-1)}(s,t-x+2d-s)\,ds$$

$$-\frac{a^-}{2}\int_{d+\frac{t-x}{2}}^{d}\left(u^2(s)-q(s)\right)K_{11}{}^{(n-1)}(s,t-x+2d-s)\,ds$$

$$-\frac{a^-}{2}\int_{d-\frac{x+t}{2}}^{d}\left(u^2(s)-q(s)\right)K_{11}{}^{(n-1)}(s,t+x-2d+s)\,ds$$

$$-\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{21}{}^{(n-1)}(s,t-x+s)ds-\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t-x+s)ds$$

$$+\frac{1}{2}\int_{d}^{x}u(s)K_{21}{}^{(n-1)}(s,t-x+s)ds+\frac{1}{2}\int_{d}^{x}u(s)K_{21}{}^{(n-1)}(s,t+x-s)ds$$

$$+\frac{1}{2}\int_{d}^{x}u(s)K_{11}{}^{(n-1)}(s,t-x+s)ds+\frac{1}{2}\int_{d}^{x}u(s)K_{11}{}^{(n-1)}(s,t+x-s)ds$$

$$K_{22}{}^{(0)}(x,t)=-\frac{a^{+}}{2}u\left(\frac{t+x}{2}\right)-\frac{a^{-}}{2}u\left(d+\frac{t-x}{2}\right)+\frac{b}{2}u\left(\frac{t+x}{2}\right)-\frac{b}{2}u\left(d+\frac{t-x}{2}\right)$$

$$\begin{aligned}
K_{22}{}^{(n)}(x,t)=&\frac{a^{+}}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{21}{}^{(n-1)}(s,t-x+s)ds-\frac{a^{+}}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t+x-s)ds\\
&-\frac{a^{+}}{2}\int_{0}^{d}u(s)\int_{t-x+s}^{t+x-s}K_{21}{}^{(n-1)}(s,x)dxds-\frac{a^{-}}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{22}{}^{(n-1)}(s,t+x-2d+s)ds\\
&-\frac{a^{-}}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}{}^{(n-1)}(s,t-x+s)ds-\frac{a^{-}}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}{}^{(n-1)}(s,t+x-s)ds\\
&-\frac{a^{-}}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}{}^{(n-1)}(s,t-x+2d-s)ds-\frac{b}{2}\int_{0}^{d}u(s)\int_{t-x+-s}^{t+x-s}K_{21}{}^{(n-1)}(s,x)dxds\\
&+\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t+x-s)ds\\
&-\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{11}{}^{(n-1)}(s,t-x+s)ds-\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{11}{}^{(n-1)}(s,t+x-2d+s)ds\\
&-\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}u(s)K_{22}{}^{(n-1)}(s,t-x+2d-s)ds-\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}u(s)K_{22}{}^{(n-1)}(s,t+x-2d+s)ds\\
&+\frac{b}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{22}{}^{(n-1)}(s,t-x+s)ds-\frac{b}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{22}{}^{(n-1)}(s,t+x-s)ds\\
&+\frac{b}{2}\int_{0}^{d}u(s)\int_{t-x+2d-s}^{t+x-2d+s}K_{21}{}^{(n-1)}(s,x)dxds-\frac{b}{2}\int_{d-\frac{x+t}{2}}^{d}\left(u^{2}(s)-q(s)\right)\int_{t-s-x+2d}^{t+x-2d+s}K_{11}{}^{(n-1)}(s,x)dxds\\
&-\frac{b}{2}\int_{d+\frac{t-x}{2}}^{d}\left(u^{2}(s)-q(s)\right)\int_{t-x+s}^{t+x-s}K_{11}{}^{(n-1)}(s,x)dxds-\frac{1}{2}\int_{d}^{x}u(s)K_{22}{}^{(n-1)}(s,t+x-s)ds\\
&+\frac{1}{2}\int_{\frac{x-t}{2}}^{x}u(s)K_{11}{}^{(n-1)}(s,t-x+s)ds-\frac{1}{2}\int_{\frac{x+t}{2}}^{x}u(s)K_{22}{}^{(n-1)}(s,t-x+s)ds
\end{aligned}$$

$$+\frac{1}{2}\int_d^x u(s)K_{11}^{(n-1)}(s,t-x+s)ds-\frac{1}{2}\int_d^x u(s)K_{11}^{(n-1)}(s,t+x-s)ds$$

$$-\frac{1}{2}\int_d^x u(s)K_{22}^{(n-1)}(s,t-x+s)ds$$

integral denklemleri elde edilir. O halde

$$\int_{-x}^{x}\left|K_{11}^{(0)}(x,t)\right|dt = a^+\int_0^x|u(t)|dt+\left|a^-\right|\int_{d-x}^{d}|u(t)|dt+|b|\int_0^x|u(t)|dt+|b|\int_{d-x}^{d}|u(t)|dt$$
$$+|b|\int_d^{d+x}|u(t)|dt+\ a^+\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$
$$+\left|a^-\right|\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$
$$+2|b|\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$
$$+\ |b|\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$

$$\int_{-x}^{x}\left|K_{21}^{(0)}(x,t)\right|dt=\frac{a^+}{2}\int_0^x\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt+\frac{\left|a^-\right|}{2}\int_{d-x}^{d}\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$
$$+\frac{|b|}{2}\int_{d-x}^{d}\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt+\frac{|b|}{2}\int_0^x\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$

$$\int_{-x}^{x}\left|K_{22}^{(0)}(x,t)\right|dt=a^+\int_0^x|u(t)|dt+\left|a^-\right|\int_{d-x}^{d}|u(t)|dt+|b|\int_0^x|u(t)|dt+|b|\int_{d-x}^{d}|u(t)|dt$$

dolayısıyla,

$$\int_{-x}^{x}\left|K_{11}^{(0)}(x,t)\right|dt\le\left[2a^++2\left|a^-\right|+6|b|\right]\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$

$$\int_{-x}^{x}\left|K_{21}^{(0)}(x,t)\right|dt\le\left[\frac{a^+}{2}+\frac{\left|a^-\right|}{2}+|b|\right]\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$

$$\int_{-x}^{x}\left|K_{22}^{(0)}(x,t)\right|dt\le\left[a^++\left|a^-\right|+2|b|\right]\int_0^x(x-t)\left[|u(t)|^2+|u(t)||b(t)|+|q(t)|\right]dt$$

eşitsizlikleri bulunur.

$$c_1=\max\left\{\left[2a^{+}+2\left|a^{-}\right|+6\left|b\right|\right],\left[\frac{a^{+}}{2}+\frac{\left|a^{-}\right|}{2}+\left|b\right|\right],\left[a^{+}+\left|a^{-}\right|+2\left|b\right|\right]\right\}$$ ve

$s(x)=\int_0^x (x-t)\left[\left|u(t)\right|^2+\left|u(t)\right|\left|b(t)\right|+\left|q(t)\right|\right]dt$ olarak alınırsa, her $i,j=1,2$ için

$\int_{-x}^{x}\left|K_{ij}^{(0)}(x,t)\right|dt\le c_1 s(x)$ olur. Ayrıca

$$\begin{aligned}\int_{-x}^{x}\left|K_{11}^{(n)}(x,t)\right|dt &\le \frac{a^{+}}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds+\frac{a^{+}}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds\\
&+a^{+}p\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{21}^{(n-1)}(s,x)\right|dxds+\frac{a^{+}}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{22}^{(n-1)}(s,x)\right|dxds\\
&+\frac{a^{+}}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{22}^{(n-1)}(s,x)\right|dxds+a^{+}p\int_0^x\left[\left|u(s)\right|^2+\left|q(s)\right|\right]\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds\\
&+\left|a^{-}\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds+\left|a^{-}\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds\\
&+\left|a^{-}\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{22}^{(n-1)}(s,x)\right|dxds+\frac{\left|a^{-}\right|}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{22}^{(n-1)}(s,x)\right|dxds\\
&+\left|a^{-}\right|p\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{21}^{(n-1)}(s,x)\right|dx\,ds\\
&+\left|a^{-}\right|p\int_0^x\left[\left|u(s)\right|^2+\left|q(s)\right|\right]\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dx\,ds\\
&+\left|b\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{22}^{(n-1)}(s,x)\right|dxds+\left|b\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{22}^{(n-1)}(s,x)\right|dxds\\
&+\frac{\left|b\right|}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds+\frac{\left|b\right|}{2}\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds\\
&+\left|b\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds+\left|b\right|\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{11}^{(n-1)}(s,x)\right|dxds\\
&+\left|b\right|p\int_0^x\left|u(s)\right|\int_{-s}^{s}\left|K_{21}^{(n-1)}(s,x)\right|dx\,ds\end{aligned}$$

$$+|b|p\int_0^x |u(s)|\int_{-s}^{s}\left|K_{21}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+|b|p\int_0^x\left[|u(s)|^2+|q(s)|\right]\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\frac{|b|}{2}\int_0^x |u(s)|\int_{-s}^{s}\left|K_{22}{}^{(n-1)}(s,x)\right|dx\,ds+\frac{|b|}{2}\int_0^x |u(s)|\int_{-s}^{s}\left|K_{22}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+2\int_0^x |u(s)|\int_{-s}^{s}\left|K_{22}{}^{(n-1)}(s,x)\right|dx\,ds+p\int_0^x |u(s)|\int_{-s}^{s}\left|K_{21}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+p\int_0^x\left[|u(s)|^2+|q(s)|\right]\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds+\int_0^x |u(s)|\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$K_{21}{}^{(n)}(x,t)=\frac{a^+}{2}\int_0^x\left|u(s)\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds+\frac{a^+}{2}\int_0^x\right|u(s)\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\frac{a^+}{2}\int_0^x\left[|u(s)|^2+|q(s)|\right]\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\frac{a^+}{2}\int_0^x\left[|u(s)|^2+|q(s)|\right]\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\left|a^-\right|\int_0^x\left[|u(s)|^2+|q(s)|\right]\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\left|a^-\right|\int_0^x\left[|u(s)|^2+|q(s)|\right]\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\left|a^-\right|\int_0^x |u(s)|\int_{-s}^{s}\left|K_{21}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\left|a^-\right|\int_0^x |u(s)|\int_{-s}^{s}\left|K_{21}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\frac{1}{2}\int_0^x |u(s)|\int_{-s}^{s}\left|K_{21}{}^{(n-1)}(s,x)\right|dx\,ds+\frac{1}{2}\int_0^x |u(s)|\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\int_0^x |u(s)|\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds+\int_0^x |u(s)|\int_{-s}^{s}\left|K_{11}{}^{(n-1)}(s,x)\right|dx\,ds$$

$$+\int_0^x |u(s)| \int_{-s}^{s} \left|K_{21}^{(n-1)}(s,x)\right| dx\, ds + \int_0^x |u(s)| \int_{-s}^{s} \left|K_{21}^{(n-1)}(s,x)\right| dx\, ds$$

$$\int_{-x}^{x} \left|K_{22}^{(n)}(x,t)\right| dt = \frac{a^+}{2}\int_0^x |u(s)| \int_{-s}^{s} \left|K_{21}^{(n-1)}(s,x)\right| dxds + \frac{a^+}{2}\int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds$$

$$+ a^+ p\int_0^x |u(s)| \int_{-s}^{s} \left|K_{21}^{(n-1)}(s,x)\right| dxds + \left|a^-\right|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds$$

$$+ \left|a^-\right|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds + \left|a^-\right|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{22}^{(n-1)}(s,x)\right| dxds$$

$$+ \left|a^-\right|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{22}^{(n-1)}(s,x)\right| dxds + |b|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds$$

$$+ 2|b|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds + 2|b|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{22}^{(n-1)}(s,x)\right| dxds$$

$$+ |b|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{22}^{(n-1)}(s,x)\right| dxds + 2p|b|\int_0^x |u(s)| \int_{-s}^{s} \left|K_{21}^{(n-1)}(s,x)\right| dxds$$

$$+ |b| p\int_0^x \left[|u(s)|^2 + |q(s)|\right] \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dx\, ds$$

$$+ |b| p\int_0^x \left[|u(s)|^2 + |q(s)|\right] \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dx\, ds$$

$$+ \frac{1}{2}\int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds + \frac{1}{2}\int_0^x |u(s)| \int_{-s}^{s} \left|K_{22}^{(n-1)}(s,x)\right| dxds$$

$$+ \int_0^x |u(s)| \int_{-s}^{s} \left|K_{11}^{(n-1)}(s,x)\right| dxds + \int_0^x |u(s)| \int_{-s}^{s} \left|K_{22}^{(n-1)}(s,x)\right| dxds$$

yukarıdaki eşitsizlikler kullanılırsa, $n = 1$ için;

$$\int_{-x}^{x} \left|K_{11}^{(1)}(x,t)\right| dt \le \left[2a^+ + \frac{7}{2}\left|a^-\right| + 6|b| + 3 + 2p\left(a^+ + \left|a^-\right| + |b| + 1\right)\right] c_1 \frac{s^2(x)}{2!}$$

$$\int_{-x}^{x} \left|K_{21}^{(1)}(x,t)\right| dt \le \left[2a^+ + 4\left|a^-\right| + 5\right] c_1 \frac{s^2(x)}{2!}$$

$$\int_{-x}^{x} \left|K_{22}^{(1)}(x,t)\right| dt \le \left[a^+ + 4\left|a^-\right| + 6|b| + 3 + p\left(a^+ + 4|b|\right)\right] c_1 \frac{s^2(x)}{2!}$$

eşitsizlikleri elde edilir.

$$c=\max\left\{\left[2a^{+}+\frac{7}{2}\left|a^{-}\right|+6\left|b\right|+3+2p\left(a^{+}+\left|a^{-}\right|+\left|b\right|+1\right)\right],\ \left[2a^{+}+4\left|a^{-}\right|+5\right],\right.$$
$$\left.\left[a^{+}+4\left|a^{-}\right|+6\left|b\right|+3+p\left(a^{+}+4\left|b\right|\right)\right],\ c_1\right\}$$

olarak alınırsa; her $i,j=1,2$ için $\int_{-x}^{x}\left|K_{ij}^{(1)}(x,t)\right|dt\le c^2\frac{s^2(x)}{2!}$ eşitsizlikleri elde edilir.

Ayrıca $n=2$ için $i,j=1,2$ olmak üzere $\int_{-x}^{x}\left|K_{ij}^{(2)}(x,t)\right|dt\le c^3\frac{s^3(x)}{3!}$ elde edilir.

Tümevarım yöntemi uygulanırsa, her $i,j=1,2$ için $\int_{-x}^{x}\left|K_{ij}^{(n)}(x,t)\right|dt\le c^{n+1}\frac{s^{n+1}(x)}{(n+1)!}$ eşitsizliği geçerli olur. Benzer işlemler diğer

2-) $2d<x,\ -x<t<2d-x$, 3-) $d<x<2d,\ x-2d<t<2d-x$, 4-) $2d<x,\ -x<t<x-2d$

5-) $2d<x,\ 2d-x<t<x$, 6-) $d<x<2d,\ x-2d<t<x$

bölgelerinde de yapılabilir.

Dolayısıyla $s(x)=\int_0^x(x-t)\left[\left|u(t)\right|^2+\left|u(t)\right|\left|b(t)\right|+\left|q(t)\right|\right]dt$ olmak üzere her $i,j=1,2$

için

$$\int_{-x}^{x}\left|K_{ij}(x,t)\right|dt\le e^{cs(x)}-1$$

eşitsizliği sağlanır.

Bu durumda aşağıdaki teorem ispatlanmış olur.

**Teorem:** $L$ probleminin $\begin{pmatrix}y_1\\y_2\end{pmatrix}(0)=\begin{pmatrix}1\\ik\end{pmatrix}$ başlangıç koşullarını sağlayan çözümü için

$x<d$ iken,

$$\begin{pmatrix}y_1\\y_2\end{pmatrix}(x)=\begin{pmatrix}e^{ikx}+\int_{-x}^{x}K_{11}(x,t)e^{ikt}dt\\ ike^{ikx}+b(x)e^{ikx}+\int_{-x}^{x}K_{21}(x,t)e^{ikt}+ik\int_{-x}^{x}K_{22}(x,t)e^{ikt}\end{pmatrix}$$

$x>d$ iken,

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x)=\begin{pmatrix} a^+e^{ikx}+a^-e^{ik(2d-x)}+b\left[e^{ikx}-e^{ik(2d-x)}\right]+\int_{-x}^{x}K_{11}(x,t)e^{ikt}dt \\ ika^+e^{ikx}-ika^-e^{ik(2d-x)}+ikb\left[e^{ikx}+e^{ik(2d-x)}\right] \\ +b(x)\left[a^+e^{ikx}+a^-e^{ik(2d-x)}+b\left[e^{ikx}-e^{ik(2d-x)}\right]\right] \\ +\int_{-x}^{x}K_{21}(x,t)e^{ikt}+ik\int_{-x}^{x}K_{22}(x,t)e^{ikt} \end{pmatrix}$$

gösterilimleri mevcuttur.

Burada $b(x)=-\frac{1}{2}\int_0^x\left[u^2(s)-q(s)\right]e^{-\frac{1}{2}\int_s^x u(t)dt}ds$ ve

$$K_{11}(x,x)=\frac{\left(a^++b\right)}{2}\left[u(x)+b(x)\right],\ K_{22}(x,x)=-\frac{\left(a^++b\right)}{2}\left[u(x)+2b(x)\right]$$

$$K_{21}(x,x)=b'(x)-\frac{1}{2}\int_0^x\left[u^2(s)-q(s)\right]K_{11}(\mathrm{s},\mathrm{s})ds-\frac{1}{2}\int_0^x \mathrm{u}(\mathrm{s})K_{21}(s,s)ds$$

$$\frac{\partial K_{i,j}(x,.)}{\partial x},\ \frac{\partial K_{i,j}(x,.)}{\partial t}\in L_2[0,p],\ (i,j=1,2)$$

şeklindedir.

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