COMBINED Lp (L_1, L_2 and L_∞) STATE ESTIMATORS IN POWER SYSTEMS

Nurten ERMİŞ

Sakarya Üniv., Sakarya MYO, Elektrik Programı, Adapazarı. Tel: 264 2774001(172),
nermis@sakarya.edu.tr,

ABSTRACT

In this study, the objective is to compare the behavior of the Lp (p = 1, 2 and ∞) state estimators and their combinations when applied to a three phase balanced power system under steady state conditions. The general approach in state estimation is to minimize a residual of measurement equations. The estimate is dependent on the norm used to measure this residual. The performance of the Lp estimators is compared under both normal and bad data conditions. The results are illustrated using the IEEE 6, 30 and 57-bus test systems. The comparison is conducted based on accuracy and computational time. The main conclusion of this study is that the computational time and the number of iterations required for L1 and L∞ estimators to converge substantially reduced using a combination of the L2 estimator with these estimators. The use of a suitable filtering technique and weighting factors may enhance the estimator output.

Keywords - Power system state estimation, least absolute value (LAV), least squares (LS), weighted least squares (WLS), maximum absolute deviation estimators.

GÜÇ SİSTEMLERİNDE BİRLEŞTİRİLMİŞ L_p (L_1, L_2 and L_∞) DURUM KESTİRİMCİLERİ

ÖZET

Bu çalışmanın amacı, sürekli durumda ve üç fazlı dengeli güç sistemlerine uygulandığında, Lp (p = 1, 2 ve ∞) durum kestirimcilerinin ve onların kombinasyonlarının davranışlarını karşılaştırmaktır. Durum kestiriminde genel yaklaşım, ölçüm denklemlerindeki farkı minimum yapmaktır. Kestirim, bu farkı ölçmek için kullanılan norma bağlıdır. Lp kestirimcilerinin performansları, hem normal hem de kötü veri koşullarında karşılaştırılmıştır. Sonuçlar, IEEE 6, 30 ve 57 baralı sistemler kullanılarak elde edilmiştir. Karşılaştırma, doğruluk ve hesaplama süresine göre yapılmıştır. Bu çalışmanın ana sonucu, L_1 ve L∞ kestirimcilerinin yakınsaması için gerekli süre ve iterasyon sayısı, bu kestirimcilerin L_2 kestirimcisi ile kombine edildikleri zaman azalmaktadır. Uygun filtreleme ve ağırlıklı faktörler kullanıldığında, kestirim sonuçları daha da iyileştirilebilir.

Anahtar kelimeler - Güç sistemlerinde durum kestirimi, en küçük mutlak değer, en küçük kareler, ağırlıklı en küçük kareler, maksimum mutlak değer kestirimcileri.

1. INTRODUCTION

State estimation, based on mathematical relation between system state variables and actual measurements, is an essential function in energy management system for system security monitoring and the control of power systems [1]. In electric power systems, efficient control and analysis of a complex power system requires accurate data information from energy control center. The incorporation of computers into these processes has increased with the implementation of data acquisition systems. Rapid data collection and processing are provided by modern telemetry systems, although physical measurements are never entirely free of noise and random errors. Unfortunately, due to financial constraints it is not practical to measure all state variables for the complete determination of the system state. Therefore, the information obtained from telemetry must be supplemented by the less accurate predictions of consumptions at the nodes in the network. These predictions are frequently referred to as pseudomeasurements. Measurements and pseudomeasurements are used to calculate system parameters (bus voltage and its angle) in the network through the use of a state estimator which provide a means between the mathematical model of the system and the input data [2].

With the increasing complexity of modern power systems there is a need for efficient state estimators which will form a basis for the implementation of real time control of these systems. Among the potential algorithms and techniques for state estimation Lp based estimators are of great interest. Many techniques have been developed for estimating power system situations. Most of these techniques are based on results obtained from L_2 estimators [3, 4]. Recently, a new focus arises on L_1 and $L\infty$ estimators [5, 6]. This paper proposes a combination of Lp (p = 1, 2 and ∞) estimators for estimating bus voltages and their angles. The proposed methods are tested on IEEE 6-bus, 30-bus and 57-bus test systems.

2. STATE ESTIMATION IN POWER SYSTEMS

In electric power systems, different types of analog and/or digital measuring devices are used to measure active power, reactive power, voltage, and currents. These continuous or analog quantities are monitored using current and potential transformers. The analog quantities pass through transducers and analog-to-digital converters, and the digital output is then telemetered to control centers, over various communication links. The data received at the energy control centers are analyzed and processed by a computer to inform the system operator about the present state of the system. Physical measurements are never entirely free of noise and random errors.

A prerequisite of any power system security monitoring or control scheme is a reliable database. In these databases the raw observations are systemically processed to attenuate the effects of uncertainty of the measurement. Any form of filtering implies a loss of information and, consequently, the complete determination of the system state may require additional measurement with inherent extra cost. In power system steady state analysis, for N buses, there are 2N-1 independent variables consisting of voltage magnitudes Vk (k=1,...,N) and voltage angles θk (k=2,...,N) at all buses. Measurements are not adequate for dynamic operation in the presence of measurement errors and possible failure of a section of the real time data collection equipment. Consequently more measurements than the number of unknown state variables are needed. The way in which this redundancy is utilized, gives rise to various techniques for state estimation. State estimation techniques may be broadly divided into two categories, static and dynamic estimation. Static methods mainly have application in determination of load flow in transmission network, where the approximation of steady state over a short period of time is adequate. The assumption will be valid if the system is subjected to relatively low frequency disturbances. Dynamic estimation, on the other hand, is normally associated with transient or dynamic stability problems where the dynamic of the elements of the system must be considered. Rapid data collection and processing must be provided, since the time scale is of the order of seconds.

Static state estimation meets almost all the control center needs due to the fact that dynamic state estimation is computationally intensive. In addition the highly unpredictable and nonlinear nature of power systems makes defining a reliable model of the dynamic power system a very difficult task. For a given power system structure and certain set of parameters and operating data, the purpose of the static estimation is to compute the network operating data and to assemble a complete and reliable database. The unknown independent variables of the estimation scheme are the true node injections $\bar{S}(t)$, the true structure $g(t)$ and the true parameters $\rho(t)$. The structure $g(t)$ describes the interconnection of power system different components.
6

The parameter vector $\rho(t)$ contains the line and meter characteristics. The unknown dependent state vector $\hat{x}$ contains complex voltages $\bar{V}(t)$ which are of dimension $n = 2N-1$, where N is the number of system buses. The known quantities are the measurements $z(t)$ consisting of telemetered line flow, bus injections and bus voltages. At a fixed time instant the measurements are related to the unknown variables as follows

$$z = H(x, g, \rho) + e \tag{1}$$

The measurement error e is a random vector representing the normal difference between the actual observation z and H. Because of the presence of error e, the dimension m of the measurement vector z must be larger than the dimension n of the unknown x. A possible measurement of the redundancy is the ratio $\eta = m/n$. The error e is usually caused by the measurement noise, modeling errors, communication errors, coding errors and the finite meter time constants.

3. Lp ESTIMATION NORMS

Regression analysis is one of the most frequently used methods in empirical science [7]. The description of collected or measured data by linear regression functions with several parameters is needed for modeling and predicting. In electric power system applications, measurements are usually plentiful. The system of linear equations is said to be an over-determined system. The discrete linear L_p approximation for the power system over determined set of linear equation is given by

$$r = Hx - z \tag{2}$$

where r is the residual vector, $H \in R^{mxn}$ is the process matrix, $z \in R^n$ is the measurement vector, m is the number of measurements and n is the number of the unknowns. The length of r in the L_p norm is given by,

$$\| r \|_p = \Big(\sum_{i=1}^{m} | r_i \mid p \Big)^{1/p}, \quad (1 \le p \le \infty) \tag{3}$$

In order for $\hat{x}$ to be the solution the above equation can be written as,

$$\| H\hat{x} - z \|_p = \min_x \| Hx - z \|_p \tag{4}$$

4. Lp ESTIMATION MODELS IN POWER SYSTEMS

The state of an electrical power network may be defined as the flow of complex power through the network branches and the net injection of complex power at each bus. The state of a network at a given moment may be completely described by the set of all complex bus voltages,

$$V_1 (\cos\theta_1 + j \sin\theta_1), \ldots, V_N (\cos\theta_N + j \sin\theta_N) \tag{5}$$

It is assumed that the connections between the nodes and the parameters of the branches are completely known. For a system with N buses, by choosing one reference phase angle, there are therefore $2N-1$ unknown state variables to be determined in order to describe the state of the system. Typically, power systems have far more measurements available than state variables, which allow the use of statistical methods to estimate the state variables of the power system.

A measurement vector z may be created which contains m measurements from the power system. Measurements include real and reactive power line flows and bus injections, voltage magnitudes at buses, tap ratios for transformers, and some times, phase angle measurements. The $2N-1$ state variables constitute the state vector x, which is related to the measurements z in a regression model as,

$$z = h(x) + e \tag{6}$$

The vector e is the error vector, which contains m variables $e_1, \ldots, e_m$. Vector e purpose is to account for the uncertainty in the measurements and the model. Hence, $e_1, \ldots, e_m$ are random variables assumed to have a zero mean and a known diagonal covariance matrix R,

$$R = diag(\sigma_1^2, \ldots, \sigma_m^2). \tag{7}$$

The data pattern in z which is represented by the vector $h(x)$ is formulated such that, for an m sets of observations that are related to a number of state variables, x_1, x_2 and x_N, through a linear regression model, are given by,

$$\begin{aligned} z_1 &= h_{11}x_1 + h_{12}x_2 + \ldots + h_{1n} + e_1 \\ z_2 &= h_{21}x_1 + h_{21}x_2 + \ldots + h_{2n} + e_2 \\ &\quad . \qquad . \qquad . \qquad . \\ z_{1m} &= h_{m1}x_1 + h_{m2}x_2 + \ldots + h_{mn} + e_m \end{aligned} \tag{8}$$

Equations (8) can be written as,

$$z_i=h_{1i}x_1+h_{2i}x_2+\cdots+h_{n-1}x_{n-1}+x_n+e_i,\quad i=1,.....,m \tag{9}$$

The N-1 variables $h_{1i}, h_{2i}...h_{n-1,i}$ are termed as independent, or explanatory, variables, while z_i is termed as the response, or dependent variable. A data point is $(h_{1i}, h_{2i}...h_{n-1,i}, z_i)$, and lies in an N-dimensional space, which is made up of two subspaces, the response space and the factor space also called the design space. The response space is one-dimensional and contains the response variable. The factor space is made up of n-1, h-dimensions. The slopes of the hyperplane in R^n are $x_1,...,x_{N-1}$, while the intercept is x_N. Hence, the task is to find x such that the hyper planes best fits the observations to the independent variables. Equation (9) can be represented in matrix form as $z=Hx+e$, where

$$H=\begin{pmatrix} h_{11} & \ldots\ldots & h_{1,n-1} & 1 \\ h_{21} & \ldots\ldots & h_{2,n-1} & 1 \\ \ldots & \ldots\ldots & \ldots & \ldots \\ h_{m1} & \ldots\ldots & h_{m,n-1} & 1 \end{pmatrix} \tag{10}$$

The matrix H is called the design matrix, and x the parameter vector.
In power systems the task of state estimation is, thus, to use statistical methods to find x such that the measurements given by z best fit the known system model provided by the design matrix.

4.1. Weighted Least Squares (WLS)

Given the regression model of (6), let R be the variance-covariance matrix. The objective function is given by,

$$J(x)=\frac{1}{2}\sum_{j=1}^{m}\left(\frac{r_i}{\sigma_i}\right)^2=\frac{1}{2}r^TR^{-1}r=(z-Hx)^TR^{-1}(z-Hx) \tag{11}$$

where $1/\sigma_i^{\,2}$ acts as a weight. Minimizing $J(x)$ yields

$$\frac{\partial J(x)}{\partial x}=0=-H^TR^{-1}(z-Hx) \tag{12}$$

By taking the pseudo inverse of H_x the estimated $\hat{x}$ is found such that,

$$\hat{x}=(H^T R^{-1} H)^{-1} H^T \hat{x} R^{-1} z\,. \tag{13}$$

This leads to,

$$\hat{z}=H\hat{x}=H(H^TR^{-1}H)^{-1}H^TR^{-1}z=Sz \tag{14}$$

where S is called the hat matrix. This formula is used to form the residual sensitivity matrix W that relates the residuals to the errors,

$$r=z-\hat{z}=z-Sz=(I-S)z=Wz \tag{15}$$

Note that the matrix W is singular, hence there are multiple m-vectors z which can satisfy $r=Wz$. It can be seen that these errors are considered as linear combinations of the residuals. This is important both in WLS residual analysis and in explaining certain properties of the WLS when gross errors are involved. For example, the property of having linear combinations of large errors resulting in small residuals, and large error results in several large residuals.

4.2. Least Absolute Value (LAV)

The least absolute value (LAV) criterion is based on the L_1 norm and has very useful bad data rejection properties. Equation (3) can be solved by substituting the value of $r_i^+ - r_i^-$ for the residual vector r_i, where r_i^+ and r_i^- represent the positive and negative parts of r respectively. The constraints H_x are linearized about the current x^k which give rise to the following,

$$\min\rightarrow\ (e^T)\begin{pmatrix} r^+ \\ r^- \\ x^+ \\ x^- \end{pmatrix} \tag{16}$$

with the constraint

$$(I,\ -I,\ H_x)\begin{pmatrix} r^+ \\ r^- \\ x^+ \\ x^- \end{pmatrix}=z \tag{17}$$

Equation (17) can be solved for Δx using the simplex linear programming method. H_x is the Jacobian matrix. The value of x is then updated, $x^{k+1}=x^k+\Delta x$. The constraints are linearized at the updated operating point and the procedure is repeated until convergence occurs.

4.3. Least squares estimator (LS)

The least squares solution (LS) is obtained when $p=2$. The LS solution has been commonly used in power system applications. It provides one way of dealing with an over-determined system. The LS method is preferred largely because the determination of the desired parameters is mathematically simple. Based on the assumption that the error r in the measurements is independent and normally distributed, Equation (2) can be rewritten as,

$$||H x - z||_2^2 = (||H x - z||^T \; ||H x - z||) \rightarrow min$$

$$H^T Hx = H^T z$$

$$\hat{x} = (H^T H)^{-1} H^T z \tag{18}$$

The quantity $(H^T H)^{-1} H^T$ is the pseudoinverse of matrix H.

4.4. Maximum Absolute Deviation

The maximum absolute value criterion is based on L∞ norm. In this, the optimal estimator minimizes the H∞ norm of the power spectrum density matrix of the estimation error r_i *such* that,

$$min\ s = max\ \ | r_i | = || Hx_i - z_i ||_\infty \tag{19}$$

$$\begin{pmatrix} H_x & e \\ -H_x & e \end{pmatrix} \begin{pmatrix} x \\ f \end{pmatrix} \geq \begin{pmatrix} z \\ -z \end{pmatrix} \tag{20}$$

Simplex linear programming method is used to solve for Δx. As the constraints are inequality constraints, it is necessary to add both surplus and artificial variables to each equation.

5. SIMULATION RESULTS

The IEEE 6, 30 and 57 bus networks were used to evaluate the comparison of L_p (L_1, L_2 and L_∞) estimators. The topologies of the 6-bus, the 30-bus and the 57-bus systems as well as the measurement set are shown in Fig. 1, Fig 2 and Fig 3. Test system data are given in reference [8] and [9]. The methods have been programmed in MATLAB on a Pentium III-800 MHz computer with 516 MB RAM. The proposed algorithm was tested on two different cases in which the measurement sets are assumed contaminated with different levels of noise. The noise level varies between 0% (case 1-without noise) to 20% (case 2-with noise) of random noise. The noise is introduced in both active and reactive power measurements. The two study cases have identical loads and circuit topology. The results obtained both two cases are tabulated and given a case number.

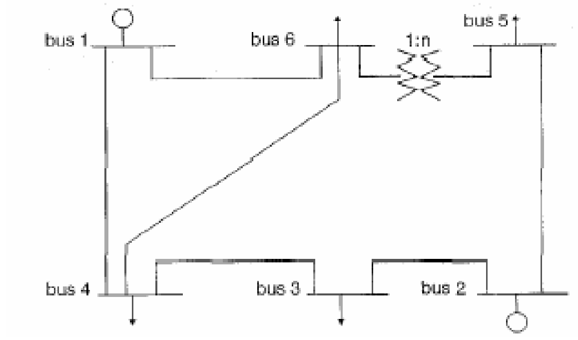

Figure 1. IEEE 6-bus test system [6].

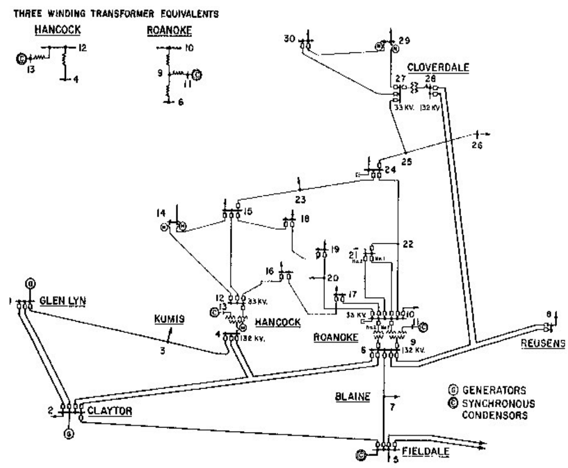

Figure 2. IEEE 30-bus test system [10].

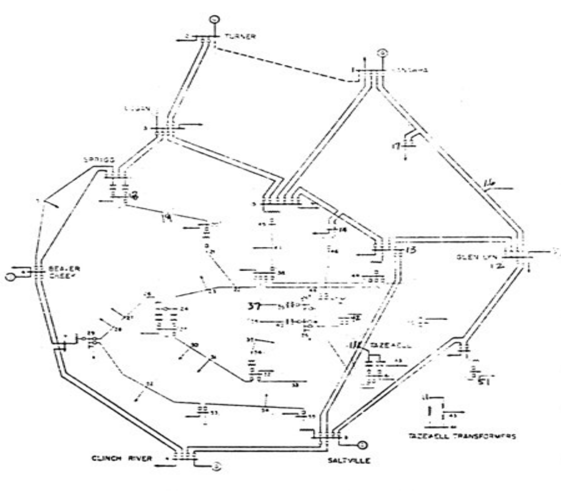

Figure 3. IEEE 57-bus test system [7]

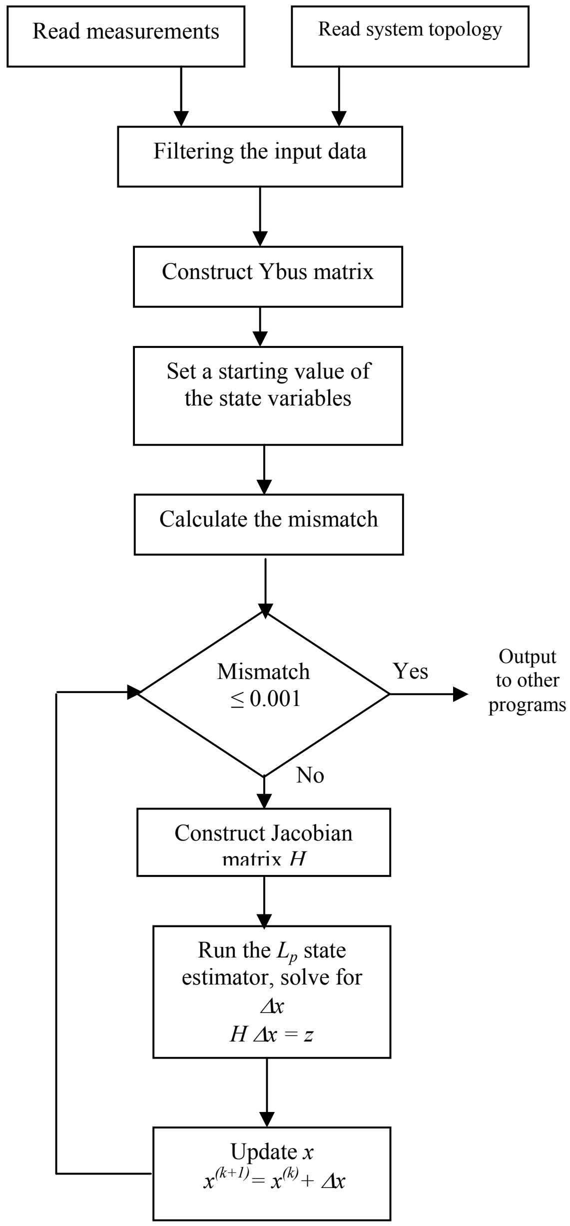

Figure 4. Power flow state estimation [11].

The output for each case includes the bus voltage magnitudes and voltage phase angles. The residuals for both two cases are calculated and plotted. The computational time consumed by each state estimator is measured and compared. Table 1, Table 2 and Table 3 contain the output results for without and with noise cases under study. It can be inferred from Table 1, 2 and 3 that the method of the least absolute value gives the least residual and hence the more accurate result. The computational time is the least in case of the least square estimator.In low noise condition, the two methods give equally accurate results. But with high noise data, which is the practical case in power application, the least absolute value gives better results.

Using a combination of the L_2 estimator with the other estimators can substantially reduce the computational time and the number of iteration needed for L_1 and L_∞. In the combined method, the least square estimator L_2 is executed for one iteration, and then the other estimator is applied. For IEEE 6-bus, 30-bus and 57-bus test systems with the same input data, when this new combination method is applied, the results in Table 1, Table 2 and Table 3 are obtained.

Table 1. Comparison with respect to residuals and computational time of Lp estimators for the 6-bus test system

L_p norm	Residuals		Computational Time (sec.)	
	Without Noise	With Noise	Without Noise	With Noise
L_1	2.7804	2.7837	0.93	0.94
L_2	2.7808	2.7850	0.0600	0.0500
L_∞	2.7832	2.7878	0.3800	0.3900

Table 2. Comparison with respect to residuals and computational time of Lp estimators for the 30-bus test system

L_p norm	Residuals		Computational Time (sec.)	
	Without Noise	With Noise	Without Noise	With Noise
L_1	13.4091	13.4105	2.600	4.010
L_2	13.4416	13.4385	0.600	0.610
L_∞	13.4227	13.4155	5.930	6.150

Table 3. Comparison with respect to residuals and computational time of Lp estimators for the 57-bus test system

L_p norm	Residuals		Computational Time (sec.)	
	Without Noise	With Noise	Without Noise	With Noise
L_1	18,187	18,345	8,320	9,003
L_2	18,304	18,620	1,066	1,072
L_∞	18,450	18,721	11,324	11,938

Table 4 shows that the computational time is much more less than it's needed for L_1 and L_∞ estimators, individually. It is advisable to use the combined L_p method to estimate the voltage angles and voltage magnitudes in buses.

Table 4. Comparison with respect to computational time of the combined Lp estimators for test system

	$L_1 + L_2$				$L_\infty + L_2$			
	Residuals		Computational Time (sec.)		Residuals		Computational Time (sec.)	
System	Without Noise	With Noise	Without Noise	With Noise	Without Noise	With Noise	Without Noise	With Noise
6-bus	2.780	2.7836	0.110	0.170	2.7834	2.7869	0.110	0.110
30-bus	13.40	13.410	1.7000	1.810	13.422	13.415	3.023	3.0260
57-bus	18,18	18,346	2,007	2,134	18,641	18,724	2,340	2,450

Figure 4. Comparison with respect to computational time of the L_∞ and the combined L_p *(L_2+ L_∞) estimators* for the 6-bus test system

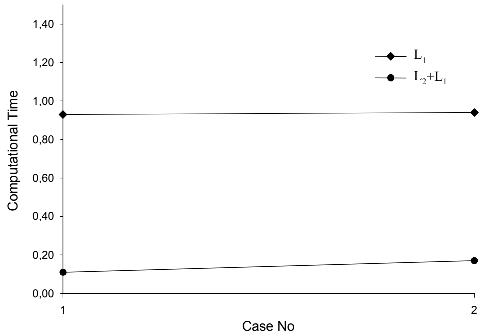

Figure 5. Comparison with respect to computational time of the L_1 and the combined L_p *(L_2+L_1) estimators* for the 6-bus test system

Figure 4, 6 and 7 show comparison with respect to computational time of the L_∞ and the combined L_p *(L_2+ L_∞) estimators* for the 6-bus test system, the 30-bus test system and the 57-bus test system respectively. Figure 5 shows comparison with respect to computational time of the L_1 and the combined L_p *(L_2+L_1) estimators* for the 6-bus test system.

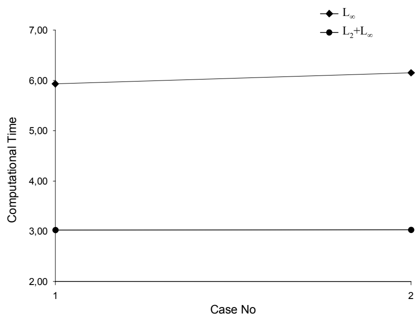

Figure 6. Comparison with respect to computational time of the L_∞ and the combined L_p *(L_2+ L_∞) estimators* for the 30-bus test system

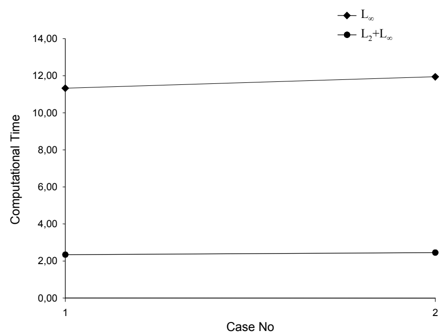

Figure 7. Comparison with respect to computational time of the L_∞ and the combined L_p *(L_2+ L_∞) estimators* for the 57-bus test system

6. CONLUSIONS

This paper presented a new and efficient combined method to estimate the voltage (angles and magnitudes) in system buses. The proposed method based on combination L_p estimators for enhancement of computational speed.. As the proposed method converges much faster than the least absolute value estimator for large power systems, the computational time and iteration numbers needed for convergence substantially reduces. At the same time, residuals obtained using a combination of L_p norms are the same as residuals obtained using L_p norms individual. Results on three sample power systems presented. The state estimation technique based on L_P norms is a very effective tool and can be applied in a variety of areas in engineering.

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