SYNTHESIS OF LOSSLESS LADDER NETWORKSWITH SIMPLE LUMPED ELEMENTS CONNECTED VIACOMMENSURATE TRANSMISSION LINES

Metin ŞENGÜL

Kadir Has University, Engineering Faculty, Cibali-Fatih,Istanbul, Turkey
msengul@khas.edu.tr

Abstract: *An algorithm has been proposed, to synthesize low-pass, high-pass, band-pass and band-stop lossless ladder networks with simple lumped elements connected via commensurate transmission lines (Unit elements, UEs). First, the type of the element that will be extracted is determined from the given mixed-element network function. After obtaining element value, it is extracted, and the two-variable reflection function of the remaining mixed-element network is obtained. This process is repeated until extracting all the elements. For each network type, an example is included, to illustrate the implementation of the related algorithm.*

Keywords: *Synthesis, Mixed lumped and distributed elements, Ladder networks, Scattering matrix.*

1. Introduction

The design problem with mixed lumped and distributed elements requires the characterization of the mixed-element structures using transcendental or multivariable functions. The first approach deals with non-rational single variable transcendental functions and is based on the study of cascaded non-commensurate transmission lines by Kinarivala [1]. First results on the synthesis of a transcendental driving-point impedance function as a cascade of lumped, lossless, two-ports and commensurate transmission lines were given by Riederer and Weinberg [2]. The other approach to describe mixed lumped and distributed two-ports is based on Richards transformation, $\lambda = \tanh(p\tau)$ which converts the transcendental functions of a distributed network into rational functions [3]. The attempts to generalize this approach to mixed lumped and distributed networks led to the multivariable synthesis procedures, where the Richards variable λ is used for distributed-elements and the original frequency variable p for lumped-elements. In this way, all the network functions could be written as rational functions of two complex variables. After the work of Ozaki and Kasami [4] on the multivariable positive real functions, the network design problem with mixed lumped and distributed elements is attempted to be solved by many researchers especially using the multivariable approach. In this context, although there have been valuable contributions, a complete theory for the approximation and synthesis problems of mixed lumped and distributed networks is still not available.

At the end of a circuit design process, after getting the network function which meets the design specifications, it is necessary to obtain the circuit structure and element values. So here, a synthesis algorithm has been proposed to extract the element values from the given two-variable network function of a special type structure called lossless ladder networks with simple lumped elements connected via commensurate transmission line.

In the following section, the characterization of two-variable networks is introduced. Subsequently, after giving the synthesis algorithm, examples are presented, to illustrate the utilization of the proposed algorithm.

2. Characterization of Two-variable Networks

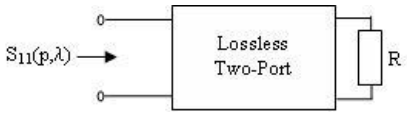

Figure 1. Lossless two-port with input reflectance function $S_{11}(p,\lambda)$.

Let $\{S_{kl}; k,l = 1,2\}$ designate the scattering parameters of a lossless two-port like the one depicted in figure 1. For a mixed lumped and distributed element, reciprocal, lossless two-port, the scattering parameters may be expressed in Belevitch form as follows [5-8].

$$S(p,\lambda) = \begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \frac{1}{g(p,\lambda)} \begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ f(p,\lambda) & -\mu h(-p,-\lambda) \end{bmatrix} \quad (1)$$

where $\mu = \dfrac{f(-p,-\lambda)}{f(p,\lambda)}$.

In (1), $p = \sigma + j\omega$ is the usual complex frequency variable associated with lumped-elements, and $\lambda = \Sigma + j\Omega$ is the Richards variable associated with equal-length transmission lines or so called commensurate transmission

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lines ($\lambda = \tanh p\tau$, where τ is the commensurate delay of the distributed elements).

$g(p,\lambda)$ is $(n_p + n_\lambda)^{th}$ degree scattering Hurwitz polynomial with real coefficients such that $g(p,\lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P}$, where

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p 0} & \cdots & \cdots & g_{n_p n_\lambda} \end{bmatrix},$$

$$\mathbf{P}^T = \begin{bmatrix} 1 \; p \; p^2 \ldots p^{n_p} \end{bmatrix}$$
$$\boldsymbol{\lambda}^T = \begin{bmatrix} 1 \; \lambda \; \lambda^2 \ldots \lambda^{n_\lambda} \end{bmatrix}. \quad (2)$$

Similarly, $h(p,\lambda)$ is a $(n_p + n_\lambda)^{th}$ degree polynomial with real coefficients $h(p,\lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$, where

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p 0} & \cdots & \cdots & h_{n_p n_\lambda} \end{bmatrix}. \quad (3)$$

$f(p,\lambda)$ is a real polynomial which includes all the transmission zeros of the two-port network.

Since the network is considered as a lossless two-port terminated in a resistance, then energy conversation requires that

$$S(p,\lambda) S^T(-p,-\lambda) = I, \quad (4)$$

where I is the identity matrix. The open form of (4) is given as

$$g(p,\lambda) g(-p,-\lambda) = h(p,\lambda) h(-p,-\lambda) + f(p,\lambda) f(-p,-\lambda). \quad (5)$$

The fundamental properties of this kind of mixed-element structures can be found in [6, 9-10]. In the following section, the proposed synthesis algorithm is given.

3. Algorithm

In the flowchart seen in figure 2, the proposed synthesis algorithm has been given. Shortly, after calculating the constants ($\alpha_1 = \pm$ and $\alpha_2 = \pm 1$) from the given two-variable reflection function, the types of the first and the second components are decided. Then the value of the first element and the reflection function of the remaining network are calculated. After extracting the second component, it is not necessary to decide the type of the next element, since the network has a ladder structure. After calculating all the element values, the algorithm is stopped.

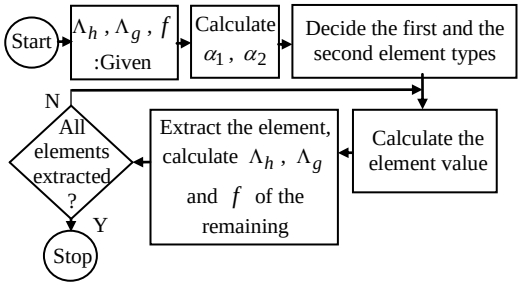

Figure 2. Flowchart of the proposed synthesis algorithm.

In the following part, synthesis algorithm has been detailed for low-pass, high-pass, band-pass and band-stop mixed-element structures.

3.1. Low-pass Ladders Connected with Unit Elements

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element low-pass structure are as follows,

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & h_{02} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & h_{12} & \cdots & h_{1n_\lambda} \\ h_{20} & h_{21} & \cdots & \cdots & 0 \\ \vdots & \vdots & 0 & 0 & \vdots \\ h_{n_p 0} & \cdot & 0 & \cdots & 0 \end{bmatrix}, \quad \Lambda_g = \begin{bmatrix} g_{00} & g_{01} & g_{02} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & g_{12} & \cdots & g_{1n_\lambda} \\ g_{20} & g_{21} & \cdots & \cdots & 0 \\ \vdots & \vdots & 0 & 0 & \vdots \\ g_{n_p 0} & \cdot & 0 & \cdots & 0 \end{bmatrix}, \quad f(p,\lambda) = (1-\lambda^2)^{n_\lambda/2}.$$

Step 1:

$\alpha_1 = \pm 1$	$\alpha_2 = \pm 1$	
	If $h(n_p,1) \neq 0$, $g(n_p,1) \neq 0$	If $h(n_p,1) = 0$, $g(n_p,1) = 0$
$\frac{h(n_p,0)}{g(n_p,0)}$	$\frac{h(n_p,1)}{g(n_p,1)}$	$\frac{h(n_p-1,1)}{g(n_p-1,1)}$

Step 2:

α_1	α_2	First component	Next component
+1	-1	UE	Series inductor (L)
-1	+1	UE	Parallel capacitor (C)

+1	+1	Series inductor (L)	UE
-1	-1	Parallel capacitor (C)	UE

Step 3:

Element Type	Unit Element	Series Inductor	Parallel Capacitor
Element Value	$Z=\frac{1+S_{11}^{(\lambda)}}{1-S_{11}^{(\lambda)}}$ where $S_{11}^{(\lambda)}=\frac{\sum_{i=0}^{n_\lambda} h(0,i)}{\sum_{i=0}^{n_\lambda} g(0,i)}$	$L=\frac{g(n_p,0)+\alpha_1 h(n_p,0)}{g(n_p-1,0)-\alpha_1 h(n_p-1,0)}$	$C=\frac{g(n_p,0)+\alpha_1 h(n_p,0)}{g(n_p-1,0)-\alpha_1 h(n_p-1,0)}$
Polynomial g of the extracted element	$\frac{Z^2+1}{2Z}\lambda+1$	$\frac{L}{2}p+1$	$\frac{C}{2}p+1$
Polynomial h of the extracted element	$\frac{Z^2-1}{2Z}\lambda$	$\frac{L}{2}p$	$-\frac{C}{2}p$
Polynomial f of the extracted element	$(1-\lambda^2)^{1/2}$	1	1
Polynomial $g^{(RN)}(p,\lambda)$ of the remaining network	$-h(-\lambda)h(p,\lambda)+g(-\lambda)g(p,\lambda)$	$-h(-p)h(p,\lambda)+g(-p)g(p,\lambda)$	$-h(-p)h(p,\lambda)+g(-p)g(p,\lambda)$
Polynomial $h^{(RN)}(p,\lambda)$ of the remaining network	$g(\lambda)h(p,\lambda)-h(\lambda)g(p,\lambda)$	$g(p)h(p,\lambda)-h(p)g(p,\lambda)$	$g(p)h(p,\lambda)-h(p)g(p,\lambda)$
Polynomial $f^{(RN)}(p,\lambda)$ of the remaining network	$(1-\lambda^2)^{(n_\lambda-1)/2}$	$(1-\lambda^2)^{n_\lambda/2}$	

Step 4: Set new $h(p,\lambda)$, $g(p,\lambda)$ and $f(p,\lambda)$ two-variable polynomials as $h(p,\lambda)=h^{(RN)}(p,\lambda)$, $g(p,\lambda)=g^{(RN)}(p,\lambda)$, $f(p,\lambda)=f^{(RN)}(p,\lambda)$, and go to Step 3.

3.2. High-Pass Ladders Connected with Unit Elements

The polynomial $f(p,\lambda)$ and the coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$ describing the mixed-element high-pass structure are as follows,

$$f(p,\lambda)=p^{n_p}(1-\lambda^2)^{n_\lambda/2},\ \Lambda_h=\begin{bmatrix} h_{00} & h_{01} & 0 & \cdots & 0 \\ h_{10} & \vdots & h_{12} & \cdots & \vdots \\ h_{20} & h_{n_p-21} & \ddots & \cdots & h_{n_p-2n_\lambda} \\ \vdots & h_{n_p-11} & h_{n_p-12} & \vdots & h_{n_p-1n_\lambda} \\ h_{n_p0} & h_{n_p1} & h_{n_p2} & \cdots & h_{n_pn_\lambda} \end{bmatrix},\ \Lambda_g=\begin{bmatrix} g_{00} & g_{01} & 0 & \cdots & 0 \\ g_{10} & \vdots & g_{12} & \cdots & \vdots \\ g_{20} & g_{n_p-21} & \ddots & \cdots & g_{n_p-2n_\lambda} \\ \vdots & g_{n_p-11} & g_{n_p-12} & \vdots & g_{n_p-1n_\lambda} \\ g_{n_p0} & g_{n_p1} & g_{n_p2} & \cdots & g_{n_pn_\lambda} \end{bmatrix}.$$

Step 1:

$\alpha_1 = \pm 1$	$\alpha_2 = \pm 1$	
	If $h(0,1) \neq 0$ $g(0,1) \neq 0$	If $h(0,1) = 0$ $g(0,1) = 0$
$\frac{h(0,0)}{g(0,0)}$	$\frac{h(0,1)}{g(0,1)}$	$\frac{h(1,1)}{g(1,1)}$

Step 2:

α_1	α_2	First component	Next component
+1	-1	UE	Series capacitor (C)
-1	+1	UE	Parallel inductor (L)
+1	+1	Series capacitor (C)	UE
-1	-1	Parallel inductor (L)	UE

Step 3:

Element Type	Unit Element	Parallel Inductor	Series Capacitor
Element Value	$Z = \frac{1+S_{11}^{(\lambda)}}{1-S_{11}^{(\lambda)}}$ where $S_{11}^{(\lambda)} = \frac{\sum_{i=0}^{n_\lambda} h(n_p,i)}{\sum_{i=0}^{n_\lambda} g(n_p,i)}$	$L = \frac{g(1,0)-\alpha_1 h(1,0)}{g(0,0)+\alpha_1 h(0,0)}$	$C = \frac{g(1,0)-\alpha_1 h(1,0)}{g(0,0)+\alpha_1 h(0,0)}$
Polynomial g of the extracted element	$\frac{Z^2+1}{2Z}\lambda + 1$	$p + \frac{1}{2L}$	$p + \frac{1}{2C}$
Polynomial h of the extracted element	$\frac{Z^2-1}{2Z}\lambda$	$-\frac{1}{2L}$	$\frac{1}{2C}$
Polynomial f of the extracted element	$(1-\lambda^2)^{1/2}$	p	p
Polynomial $g^{(RN)}(p,\lambda)$ of the remaining network	$-h(-\lambda)h(p,\lambda) + g(-\lambda)g(p,\lambda)$	$h(-p)h(p,\lambda) - g(-p)g(p,\lambda)$	$h(-p)h(p,\lambda) - g(-p)g(p,\lambda)$
Polynomial $h^{(RN)}(p,\lambda)$ of the remaining network	$g(\lambda)h(p,\lambda) - h(\lambda)g(p,\lambda)$	$g(p)h(p,\lambda) - h(p)g(p,\lambda)$	$g(p)h(p,\lambda) - h(p)g(p,\lambda)$
Polynomial $f^{(RN)}(p,\lambda)$ of the remaining network	$(1-\lambda^2)^{(n_\lambda - 1)/2}$	$p^{n_p - 1}(1-\lambda^2)^{n_\lambda/2}$	

Step 4: Set new $h(p,\lambda)$, $g(p,\lambda)$ and $f(p,\lambda)$ two-variable polynomials as $h(p,\lambda) = h^{(RN)}(p,\lambda)$, $g(p,\lambda) = g^{(RN)}(p,\lambda)$, $f(p,\lambda) = f^{(RN)}(p,\lambda)$, and go to Step 3.

3.3. Band-Pass Ladders Connected with Unit Elements

The polynomial $f(p,\lambda)$ and the coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$ describing the mixed-element band-pass structure are as follows,

$$f(p,\lambda)=p^{n_p/2}(1-\lambda^2)^{n_\lambda/2},\quad \Lambda_h=\begin{bmatrix} h_{00} & 0 & 0 & \cdots & 0\\ h_{10} & h_{11} & 0 & \ddots & 0\\ \vdots & \vdots & \ddots & 0 & 0\\ h_{\frac{n_p}{2}0} & h_{\frac{n_p}{2}1} & h_{\frac{n_p}{2}2} & \cdots & h_{\frac{n_p}{2}n_\lambda}\\ \vdots & \vdots & \iddots & 0 & 0\\ h_{n_p-10} & h_{n_p-11} & 0 & \iddots & 0\\ h_{n_p0} & 0 & 0 & \cdots & 0 \end{bmatrix},\quad \Lambda_g=\begin{bmatrix} g_{00} & 0 & 0 & \cdots & 0\\ g_{10} & g_{11} & 0 & \ddots & 0\\ \vdots & \vdots & \ddots & 0 & 0\\ g_{\frac{n_p}{2}0} & g_{\frac{n_p}{2}1} & g_{\frac{n_p}{2}2} & \cdots & g_{\frac{n_p}{2}n_\lambda}\\ \vdots & \vdots & \iddots & 0 & 0\\ g_{n_p-10} & g_{n_p-11} & 0 & \iddots & 0\\ g_{n_p0} & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Step 1:

$\alpha_1=\pm1$	$\alpha_2=\pm1$			
	If $h(0,1)\neq0$, $g(0,1)\neq0$	If $h(0,1)=0$, $g(0,1)=0$	If $h(1,1)=0$, $g(1,1)=0$	If $h(n_p,1)=0$, $g(n_p,1)=0$
$\frac{h(0,0)}{g(0,0)}$	$\frac{h(0,1)}{g(0,1)}$	$\frac{h(1,1)}{g(1,1)}$	$\frac{h(n_p,1)}{g(n_p,1)}$	$\frac{h(n_p-1,1)}{g(n_p-1,1)}$

Step 2:

α_1	α_2	First component	Next component
+1	-1	UE	Series-LC section in series
-1	+1	UE	Parallel-LC section in parallel
+1	+1	Series-LC section in series	UE
-1	-1	Parallel-LC section in parallel	UE

Step 3:

Element Type	Unit Element	Series LC Section in Series		Parallel LC Section in Parallel	
		Inductor	Capacitor	Inductor	Capacitor
Element Value	$Z=\frac{1+S_{11}^{(\lambda)}}{1-S_{11}^{(\lambda)}}$ where $S_{11}^{(\lambda)}=\frac{\sum_{i=0}^{n_\lambda}h(\frac{n_p}{2},i)}{\sum_{i=0}^{n_\lambda}g(\frac{n_p}{2},i)}$	$L=\frac{g(n_p,0)+\alpha_1h(n_p,0)}{g(n_p-1,0)-\alpha_1h(n_p-1,0)}$	$C=\frac{g(1,0)-\alpha_1h(1,0)}{g(0,0)+\alpha_1h(0,0)}$	$L=\frac{g(1,0)-\alpha_1h(1,0)}{g(0,0)+\alpha_1h(0,0)}$	$C=\frac{g(n_p,0)+\alpha_1h(n_p,0)}{g(n_p-1,0)-\alpha_1h(n_p-1,0)}$
Polynomial g of the extracted element	$\frac{Z^2+1}{2Z}\lambda+1$	$\frac{L}{2}p+1$	$p+\frac{1}{2C}$	$p+\frac{1}{2L}$	$\frac{C}{2}p+1$
Polynomial h of the extracted element	$\frac{Z^2-1}{2Z}\lambda$	$\frac{L}{2}p$	$\frac{1}{2C}$	$-\frac{1}{2L}$	$-\frac{C}{2}p$
Polynomial f of the extracted element	$(1-\lambda^2)^{1/2}$	1	p	p	1

Polynomial $g^{(RN)}(p,\lambda)$ of the remaining network	$-h(-\lambda)h(p,\lambda)$ $+g(-\lambda)g(p,\lambda)$	$-h(-p)h(p,\lambda)$ $+g(-p)g(p,\lambda)$	$h(-p)h(p,\lambda)$ $-g(-p)g(p,\lambda)$	$h(-p)h(p,\lambda)$ $-g(-p)g(p,\lambda)$	$-h(-p)h(p,\lambda)$ $+g(-p)g(p,\lambda)$
Polynomial $h^{(RN)}(p,\lambda)$ of the remaining network	$g(\lambda)h(p,\lambda)$ $-h(\lambda)g(p,\lambda)$	$g(p)h(p,\lambda)$ $-h(p)g(p,\lambda)$	$g(p)h(p,\lambda)$ $-h(p)g(p,\lambda)$	$g(p)h(p,\lambda)$ $-h(p)g(p,\lambda)$	$g(p)h(p,\lambda)$ $-h(p)g(p,\lambda)$
Polynomial $f^{(RN)}(p,\lambda)$ of the remaining network	$(1-\lambda^2)^{(n_\lambda-1)/2}$	$p^{n_p/2}(1-\lambda^2)^{n_\lambda/2}$	$p^{(n_p/2)-1}(1-\lambda^2$	$p^{(n_p/2)-1}(1-\lambda^2)^{n_\lambda/2}$	$p^{n_p/2}(1-\lambda^2)^{n_\lambda/2}$

Step 4: Set new $h(p,\lambda)$, $g(p,\lambda)$ and $f(p,\lambda)$ two-variable polynomials as $h(p,\lambda)=h^{(RN)}(p,\lambda)$, $g(p,\lambda)=g^{(RN)}(p,\lambda)$, $f(p,\lambda)=f^{(RN)}(p,\lambda)$, and go to Step 3.

3.4. Band-Stop Ladders Connected with Unit Elements

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element band-stop structure are as follows,

$$\Lambda_h=\begin{bmatrix} 0 & h_{01} & h_{02} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & h_{12} & \ddots & h_{1n_\lambda} \\ \vdots & \vdots & \ddots & 0 & \vdots \\ h_{\frac{n_p}{2}0} & h_{\frac{n_p}{2}1} & h_{\frac{n_p}{2}2} & \cdots & h_{\frac{n_p}{2}n_\lambda} \\ \vdots & \vdots & \iddots & 0 & \vdots \\ h_{n_p-10} & h_{n_p-11} & h_{n_p-12} & \iddots & h_{n_p-1n_\lambda} \\ 0 & h_{n_p1} & h_{n_p2} & \cdots & h_{n_pn_\lambda} \end{bmatrix},\quad \Lambda_g=\begin{bmatrix} 1 & g_{01} & g_{02} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & g_{12} & \ddots & g_{1n_\lambda} \\ \vdots & \vdots & \ddots & 0 & \vdots \\ g_{\frac{n_p}{2}0} & g_{\frac{n_p}{2}1} & g_{\frac{n_p}{2}2} & \cdots & g_{\frac{n_p}{2}n_\lambda} \\ \vdots & \vdots & \iddots & 0 & \vdots \\ g_{n_p-10} & g_{n_p-11} & g_{n_p-12} & \iddots & g_{n_p-1n_\lambda} \\ g_{n_p0} & g_{n_p1} & g_{n_p2} & \cdots & g_{n_pn_\lambda} \end{bmatrix},$$

$$f(p,\lambda)=\left(\prod_{i=1}^{n_r}(p+p_i)\right)(1-\lambda^2)^{n_\lambda/2}.$$

Step 1:

$\alpha_1=\pm1$		$\alpha_2=\pm1$
If $h(\frac{n_p}{2},0)>0$	If $h(\frac{n_p}{2},0)<0$	$\frac{h(1,n_\lambda)}{g(1,n_\lambda)}$
-1	+1	

Step 2:

α_1	α_2	First component	Next component
+1	-1	Series-LC section in parallel	UE
-1	+1	Parallel-LC section in series	UE
+1	+1	UE	Series-LC section in parallel
-1	-1	UE	Parallel-LC section in series

Step 3:

Element Type	Unit Element	Parallel LC Section in Series		Series LC Section in Parallel	
		Inductor	Capacitor	Inductor	Capacitor
Element Value	$Z=\frac{1+S_{11}^{(\lambda)}}{1-S_{11}^{(\lambda)}}$ where $S_{11}^{(\lambda)}=\frac{\sum_{i=0}^{n_\lambda}h(\frac{n_p}{2},i)}{\sum_{i=0}^{n_\lambda}g(\frac{n_p}{2},i)}$	$L=\frac{g(1,0)}{\frac{-\alpha_1h(1,0)}{g(0,0)}+\alpha_1h(0,0)}$	$C=\frac{g(n_p,0)+\alpha_1h(n_p,0)}{g(n_p-1,0)-\alpha_1h(n_p-1,0)}$	$L=\frac{g(n_p,0)+\alpha_1h(n_p,0)}{g(n_p-1,0)-\alpha_1h(n_p-1,0)}$	$C=\frac{g(1,0)}{\frac{-\alpha_1h(1,0)}{g(0,0)}+\alpha_1h(0,0)}$

Polynomial g of the extracted element	$\frac{Z^2+1}{2Z}\lambda+1$	$2LCp^2+Lp+2$	$2LCp^2+Cp+2$
Polynomial h of the extracted element	$\frac{Z^2-1}{2Z}\lambda$	Lp	$-Cp$
Polynomial f of the extracted element	$(1-\lambda^2)^{1/2}$	LCp^2+1	LCp^2+1
Polynomial $g^{(RN)}(p,\lambda$ of the remaining network	$-h(-\lambda)h(p,\lambda)$ $+g(-\lambda)g(p,\lambda)$	$-h(-p)h(p,\lambda)+g(-p)g(p,\lambda)$	$-h(-p)h(p,\lambda)+g(-p)g(p,\lambda)$
Polynomial $h^{(RN)}(p,\lambda$ of the remaining network	$g(\lambda)h(p,\lambda)$ $-h(\lambda)g(p,\lambda)$	$g(p)h(p,\lambda)-h(p)g(p,\lambda)$	$g(p)h(p,\lambda)-h(p)g(p,\lambda)$
Polynomial $f^{(RN)}(p,\lambda$ of the remaining network	$(1-\lambda^2)^{(n_\lambda-1)/2}$	$\left[f(p)/(LCp^2+1)\right](1-\lambda^2)^{(n_\lambda-1)/2}$	$\left[f(p)/(LCp^2+1)\right](1-\lambda^2)^{(n_\lambda-1)/2}$

Step 4: Set new $h(p,\lambda)$, $g(p,\lambda)$ and $f(p,\lambda)$ two-variable polynomials as $h(p,\lambda)=h^{(RN)}(p,\lambda)$, $g(p,\lambda)=g^{(RN)}(p,\lambda)$, $f(p,\lambda)=f^{(RN)}(p,\lambda)$, and go to Step 3.

4. Examples

In this section, four examples are presented, to illustrate the implementation of the proposed algorithm.

The given coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$ and polynomial $f(p,\lambda)$ in the following examples are fictitious. They are obtained by multiplying the transfer scattering matrices of the cascaded elements and it is desired to get the same element values after synthesis process via the proposed algorithm. So the element values may not be realizable after a de-normalization step. Also the structures do not describe a filter, a matching network or something else, since they are also fictitious. It is not aimed to build and measure the network since it is not the main idea of this work.

4.1. Low-Pass Case

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element low-pass structure are given as,

$$\Lambda_h=\begin{bmatrix}0 & 3.15 & -1.05\\ 3.5 & -3.8 & 23.3\\ 3 & 65.4 & 0\\ 36 & 0 & 0\end{bmatrix},\Lambda_g=\begin{bmatrix}1 & 3.85 & 1.45\\ 6.5 & 14 & 23.3\\ 15 & 65.4 & 0\\ 36 & 0 & 0\end{bmatrix},$$

$$f(p,\lambda)=(1-\lambda^2).$$

Step 1: $\alpha_1=\frac{h(n_p,0)}{g(n_p,0)}=\frac{h(3,0)}{g(3,0)}=\frac{36}{36}=+1$. $h(3,1)=0$ and $g(3,1)=0 \Rightarrow \alpha_2=\frac{h(n_p-1,1)}{g(n_p-1,1)}=\frac{h(2,1)}{g(2,1)}=\frac{65.4}{65.4}=+1$.

Step 2: $\alpha_1=+1$ and $\alpha_2=+1$, so the first component that will be extracted is an inductor, and the element value is

$$L=\frac{g(n_p,0)+\alpha_1 h(n_p,0)}{g(n_p-1,0)-\alpha_1 h(n_p-1,0)}$$
$$=\frac{g(3,0)+h(3,0)}{g(2,0)-h(2,0)}=\frac{36+36}{15-3}=6,\quad l_1=6.$$

The polynomials $g(p), h(p)$ and $f(p)$ of the inductor are $g(p)=\frac{L}{2}p+1=3p+1$, $h(p)=\frac{L}{2}p=3p$, $f(p)=1$.

The polynomial $f^{(RN)}(p,\lambda)$ and coefficient matrices Λ_h and Λ_g of the remaining network are $f^{(RN)}(p,\lambda)=(1-\lambda^2)$,

$$\Lambda_h=\begin{bmatrix}0 & 3.15 & -1.05\\ 0.5 & -5.9 & 15.8\\ -6 & 12 & 0\end{bmatrix},\Lambda_g=\begin{bmatrix}1 & 3.85 & 1.45\\ 3.5 & 11.9 & 15.8\\ 6 & 12 & 0\end{bmatrix}.$$

If the same algorithm is used, the extracted element values and the remaining network coefficient matrices and polynomial $f^{(RN)}(p,\lambda)$ are as follows,

$z_1=2$ and $\Lambda_h=\begin{bmatrix}0 & 2.4\\ 0.5 & 7.9\\ -6 & 0\end{bmatrix},\Lambda_g=\begin{bmatrix}1 & 2.6\\ 3.5 & 7.9\\ 6 & 0\end{bmatrix}$,

$f^{(RN)}(p,\lambda)=(1-\lambda^2)^{1/2}$.

$c_1 = 3$ and $\Lambda_h = \begin{bmatrix} 0 & 2.4 \\ 2 & -0.4 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 2.6 \\ 2 & 0.4 \end{bmatrix}$,

$f^{(RN)}(p,\lambda) = (1-\lambda^2)^{1/2}$,

z_2 =5 and $\Lambda_h = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, f^{(RN)}(p,\lambda) = 1$.

$l_2 = 4$ and $\Lambda_h = 0, \Lambda_g = 1, f^{(RN)}(p,\lambda) = 1$, and the termination resistor $r = 1$. The obtained network is given in figure 3.

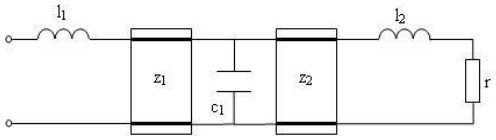

Figure 3. Synthesized low-pass mixed-element network, normalized element values: $l_1 = 6, l_2 = 4, c_1 = 3, z_1 = 2, z_2 = 5, r = 1$.

4.2. High-Pass Case

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element high-pass structure are given as

$$\Lambda_h = \begin{bmatrix} 0.0139 & 0 & 0 \\ -0.0556 & 0.4910 & 0 \\ 0.1667 & -0.2417 & 3.6375 \\ 0 & 4.2750 & -0.2250 \end{bmatrix},$$

$$\Lambda_g = \begin{bmatrix} 0.0139 & 0 & 0 \\ 0.1111 & 0.4910 & 0 \\ 0.5 & 1.6500 & 3.6375 \\ 1 & 4.7250 & 1.0250 \end{bmatrix}, \quad f(p,\lambda) = p^3(1-\lambda^2).$$

Step 1: $\alpha_1 = \dfrac{h(0,0)}{g(0,0)} = \dfrac{0.0139}{0.0139} = +1$. $h(0,1) = 0$ and

$g(0,1) = 0 \Rightarrow \alpha_2 = \dfrac{h(1,1)}{g(1,1)} = \dfrac{0.4910}{0.4910} = +1$.

Step 2: $\alpha_1 = +1$ and $\alpha_2 = +1$, so the first component that will be extracted is a capacitor, and the element value is $C = \dfrac{g(1,0) - \alpha_1 h(1,0)}{g(0,0) + \alpha_1 h(0,0)} = \dfrac{0.1111 + 0.0556}{0.0139 + 0.0139} = 6$, $c_1 = 6$. The polynomials $g(p), h(p)$ and $f(p)$ of the capacitor are $g(p) = p + \dfrac{1}{2C} = p + \dfrac{1}{12}$, $h(p) = \dfrac{1}{2C} = \dfrac{1}{12}$, $f(p) = p$. The polynomial $f^{(RN)}(p,\lambda)$ and coefficient matrices Λ_h and Λ_g of the remaining network are

$f^{(RN)}(p,\lambda) = p^2(1-\lambda^2)$,

$$\Lambda_h = \begin{bmatrix} -0.0833 & 0.3333 & 0 \\ 0.0833 & -0.2792 & 3.5333 \\ 0 & 4.2750 & -0.2250 \end{bmatrix},$$

$$\Lambda_g = \begin{bmatrix} 0.0833 & 0.3333 & 0 \\ 0.4167 & 1.6125 & 3.5333 \\ 1 & 4.7250 & 1.0250 \end{bmatrix}.$$

If the same algorithm is used, the extracted element values and the remaining network coefficient matrices and polynomial $f^{(RN)}(p,\lambda)$ are as follows,

z_1 =4 and $\Lambda_h = \begin{bmatrix} -0.0833 & 0 \\ 0.0833 & -0.8833 \\ 0 & 2.4 \end{bmatrix}$,

$\Lambda_g = \begin{bmatrix} 0.0833 & 0 \\ 0.0833 & 0.8833 \\ 1 & 2.6 \end{bmatrix}$, $f^{(RN)}(p,\lambda) = p^2(1-\lambda^2)^{1/2}$.

$l_1 = 3$ and

$\Lambda_h = \begin{bmatrix} 0.25 & -0.05 \\ 0 & 2.4 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 0.25 & 0.05 \\ 1 & 2.6 \end{bmatrix}$,

$f^{(RN)}(p,\lambda) = p(1-\lambda^2)^{1/2}$.

z_2 =5 and $\Lambda_h = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 0.25 \\ 1 \end{bmatrix}, f^{(RN)}(p,\lambda) = p$

$c_2 = 2$ and $\Lambda_h = 0, \Lambda_g = 1, f^{(RN)}(p,\lambda) = 1$, and the termination resistor $r = 1$. The obtained network is given in figure 4.

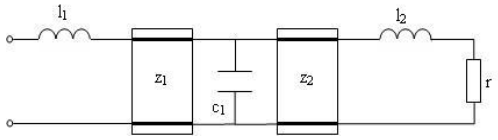

Figure 4. Synthesized high-pass mixed-element network, normalized values: $c_1 = 6, c_2 = 2, l_1 = 3, z_1 = 4, z_2 = 5, r = 1$.

4.3. Band-Pass Case

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element band-pass structure are given as

$f(p,\lambda) = p^2(1-\lambda^2)$,

$$\Lambda_h = \begin{bmatrix} 1 & -0.1429 & 0 \\ -2 & 65.0714 & -4.3571 \\ 26 & 186.6429 & 4.6429 \\ 30 & 391.4286 & -27.1429 \\ 120 & -17.1429 & 0 \end{bmatrix},$$

$$\Lambda_g = \begin{bmatrix} 1 & 0.1429 & 0 \\ 8 & 65.9286 & 4.3571 \\ 56 & 203.3571 & 30.3571 \\ 90 & 408.5714 & 27.1429 \\ 120 & 17.1429 & 0 \end{bmatrix}.$$

Step 1: $\alpha_1 = \dfrac{h(0,0)}{g(0,0)} = \dfrac{1}{1} = +1$,

$\alpha_2 = \dfrac{h(0,1)}{g(0,1)} = \dfrac{-0.1429}{0.1429} = -1$

Step 2: $\alpha_1 = +1$ and $\alpha_2 = -1$, so the first component that will be extracted is a UE, and the component value is

$$S_{11}^{(\lambda)} = \frac{\sum_{i=0}^{n_\lambda} h(n_p/2,i)}{\sum_{i=0}^{n_\lambda} g(n_p/2,i)} = \frac{26+186.6429+4.6429}{56+203.3571+30.3571} = 0.75$$

$z_1 = \frac{1+S_{11}^{(\lambda)}}{1-S_{11}^{(\lambda)}} = 7$. The polynomials $g(\lambda), h(\lambda)$ and $f(\lambda)$ are $g(\lambda) = \frac{z_1^2+1}{2z_1}\lambda + 1 = \frac{50}{14}\lambda + 1$,

$h(\lambda) = \frac{z_1^2-1}{2z_1}\lambda = \frac{48}{14}\lambda$, $f(\lambda) = (1-\lambda^2)^{1/2}$. The polynomial $f^{(RN)}(p,\lambda)$ and coefficient matrices Λ_h and Λ_g of the remaining network are

$f^{(RN)}(p,\lambda) = p^2(1-\lambda^2)^{1/2}$,

$$\Lambda_h = \begin{bmatrix} 1 & 0 \\ -2 & 30.5 \\ 26 & 87.5 \\ 30 & 190 \\ 120 & 0 \end{bmatrix}, \quad \Lambda_g = \begin{bmatrix} 1 & 0 \\ 8 & 30.5 \\ 56 & 92.5 \\ 90 & 190 \\ 120 & 0 \end{bmatrix}.$$

If the same algorithm is used, the extracted element values and the remaining network coefficient matrices and polynomial $f^{(RN)}(p,\lambda)$ are as follows,

l_1=4and

$$\Lambda_h = \begin{bmatrix} 1 & 0 \\ -2 & 30.5 \\ 6 & 87.5 \\ -30 & 180 \end{bmatrix}, \quad \Lambda_g = \begin{bmatrix} 1 & 0 \\ 8 & 30.5 \\ 36 & 92.5 \\ 30 & 180 \end{bmatrix},$$

$f^{(RN)}(p,\lambda) = p^2(1-\lambda^2)^{1/2}$

$c_1 = 5$ and $\Lambda_h = \begin{bmatrix} -1 & 6 \\ 0 & 17.5 \\ -6 & 36 \end{bmatrix}$, $\Lambda_g = \begin{bmatrix} 1 & 6 \\ 6 & 18.5 \\ 6 & 36 \end{bmatrix}$,

$f^{(RN)}(p,\lambda) = p(1-\lambda^2)^{1/2}$

z_2 =6 and $\Lambda_h = \begin{bmatrix} -1 \\ 0 \\ 6 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix}$, $f^{(RN)}(p,\lambda) = p$.

$l_2 = 3$ and $\Lambda_h = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $f^{(RN)}(p,\lambda) = 1$,

$c_2 = 2$ and $\Lambda_h = 0, \Lambda_g = 1$, $f^{(RN)}(p,\lambda) = 1$, and the termination resistor $r = 1$. The obtained network is given in figure 5.

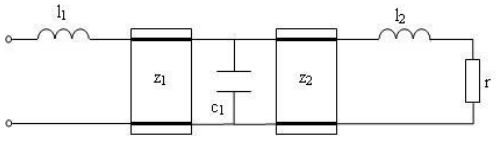

Figure 5. Synthesized band-pass mixed-element network , normalized values:
$z_1 = 7, z_2 = 6, l_1 = 4, l_2 = 3, c_1 = 5, c_2 = 2, r = 1$.

4.3. Band-Stop Case

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element band-stop structure are given as

$$\Lambda_h = \begin{bmatrix} 0 & 3.15 & -1.05 \\ 2 & -5.1 & 11.2 \\ -6 & 163.2 & -50.4 \\ -24 & -138.6 & 427.2 \\ 0 & 793.8 & -264.6 \end{bmatrix}, \quad \Lambda_g = \begin{bmatrix} 1 & 3.85 & 1.45 \\ 4 & 9.1 & 11.2 \\ 54 & 196.8 & 69.6 \\ 60 & 306.6 & 427.2 \\ 252 & 970.2 & 365.4 \end{bmatrix},$$

$f(p,\lambda) = (p^4 + 0.1905p^2 + 0.004)(1-\lambda^2)$.

Step

1: $h(2,0) = -6 < 0 \rightarrow \alpha_1 = +1, \alpha_2 = \frac{h(1,2)}{g(1,2)} = \frac{11.2}{11.2} = +1$.

Step 2: $\alpha_1 = +1$ and $\alpha_2 = +1$, so the first component that will be extracted is a UE, and the component value is

$$S_{11}^{(\lambda)} = \frac{\sum_{i=0}^{n_\lambda} h(n_p/2,i)}{\sum_{i=0}^{n_\lambda} g(n_p/2,i)} = \frac{-6+163.2-50.4}{54+196.8+69.6} = 0.3333,$$

$z_1 = \frac{1+S_{11}^{(\lambda)}}{1-S_{11}^{(\lambda)}} = 2$. The polynomials $g(\lambda), h(\lambda)$ and $f(\lambda)$ are $g(\lambda) = \frac{z_1^2+1}{2z_1}\lambda + 1 = \frac{5}{4}\lambda + 1$,

$h(\lambda) = \frac{z_1^2-1}{2z_1}\lambda = \frac{3}{4}\lambda$, $f(\lambda) = (1-\lambda^2)^{1/2}$. The polynomial $f^{(RN)}(p,\lambda)$ and coefficient matrices Λ_h and Λ_g of the remaining network are

$f^{(RN)}(p,\lambda) = (p^4 + 0.1905p^2 + 0.004)(1-\lambda^2)^{1/2}$,

$$\Lambda_h = \begin{bmatrix} 1 & 2.4 \\ -2 & -5.6 \\ -6 & 115.2 \\ -24 & -213.6 \\ 0 & 604.8 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 2.6 \\ 4 & 5.6 \\ 54 & 124.8 \\ 60 & 213.6 \\ 252 & 655.2 \end{bmatrix}.$$

If the same algorithm is used, the extracted element values and the remaining network coefficient matrices and polynomial $f^{(RN)}(p,\lambda)$ are as follows,

l_1 =3 and $c_1 = 2$, $\Lambda_h = \begin{bmatrix} 0 & 2.4 \\ 3 & -0.6 \\ 0 & 100.8 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 2.6 \\ 3 & 0.6 \\ 42 & 109.2 \end{bmatrix}$,

$f^{(RN)}(p,\lambda) = (p^2 + \frac{1}{42})(1-\lambda^2)^{1/2}$,

z_2 =5 and $\Lambda_h = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 \\ 3 \\ 42 \end{bmatrix}$, $f^{(RN)}(p,\lambda) = (p^2 + \frac{1}{42})$.

$l_2 = 6$ and $c_2 = 7$ and $\Lambda_h = 0, \Lambda_g = 1$, $f^{(RN)}(p,\lambda) = 1$, and the termination resistor $r = 1$. The obtained network is given in figure 6.

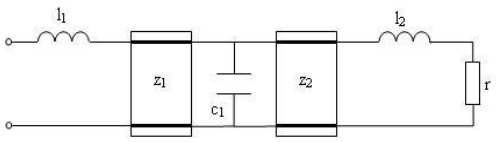

Figure 6. Synthesized band-stop mixed-element network, normalized values:
$z_1 = 2, z_2 = 5, l_1 = 3, l_2 = 6, c_1 = 2, c_2 = 7, r = 1$.

5. Conclusions

The unavoidable connections between lumped-elements destroy the performance of the lumped-element networks at high frequencies. But these connection lines can be used as circuit components. In this case, the circuits must be composed of mixed lumped and distributed elements. But a complete theory for the approximation and synthesis problems of mixed-element networks is still not available.

In this paper, synthesis of lossless ladder networks with simple lumped elements connected with UEs is examined. The synthesis of mixed-element networks may be realized by using single variable boundary polynomials, namely the polynomials $h(p,0), g(p,0), f(p,0)$ for lumped-element section, and $h(0,\lambda), g(0,\lambda), f(0,\lambda)$ for distributed-element section [11-12]. In this case, synthesis is carried out for lumped and distributed sections separately. Then, the components are mixed, to construct the mixed-element network. But in the presented algorithm, the synthesis of mixed-element network is carried out directly by using the two-variable reflection function of the mixed-element network, and components are extracted according to the connection order in the mixed-structure. The implementation of the proposed algorithms is utilized by the given examples. As a result, simple mixed-element network synthesis algorithm has been presented, which is necessary for the applications using this ladder-type of mixed-element networks, e.g. design of filters, broadband matching networks and amplifiers [13-15].

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Metin Şengül received his B.Sc. and M.Sc. degrees in Electronics Engineering from Istanbul University, Turkey in 1996 and 1999, respectively. He completed his Ph.D. in 2006 at Işık University in Istanbul, Turkey. He worked as a technician at Istanbul University from 1990 to 1997 and was a circuit design engineer at the R&D Labs of the Prime Ministry Office of Turkey between 1997 and 2000. He was employed as a lecturer and assistant professor at Kadir Has University, Istanbul, Turkey between 2000 and 2010. Dr. Şengül was a visiting researcher at the Institute for Information Technology, *Technische Universität Ilmenau*, Ilmenau, Germany in 2006 for six months. Currently, he is an associate professor at Kadir Has University, Istanbul, Turkey and working on microwave matching networks/amplifiers, device modeling, circuit design via modeling and network synthesis.