ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING
YEAR : 2004 (1131-1136)
VOLUME : 4
NUMBER : 2

INSTANTANEOUS FREQUENCY ESTIMATION USING A LEAST SQUARES TIME-FREQUENCY METHOD

Mahmut ÖZTÜRK [1] **Aydın AKAN [2]**

[1,2]Istanbul University, Engineering Faculty,
Department of Electrical and Electronics Engineering,
34850, Avcılar, Istanbul
TURKEY

[1]E-mail: mahmutoz@istanbul.edu.tr [2]E-mail: akan@istanbul.edu.tr

ABSTRACT

We present a method for estimating the instantaneous frequency of a signal. This method involves the calculation of a time-frequency energy density of the signal, then obtaining an instantaneous frequency estimation from this joint density. Time-frequency energy density is calculated as a least squares optimal combination of multi-window Gabor based evolutionary spectra. The optimal weights are obtained by minimizing an error criterion that is the difference between a reference time-frequency distribution and the combination of evolutionary spectra. Then instantaneous frequency of the signal is estimated from the final evolutionary spectrum as time conditional average frequency. Examples are given to illustrate the performance of our method.

Keywords: : *Instantaneous frequency, Time-frequency analysis, Evolutionary spectrum.*

1. INTRODUCTION

Instantaneous frequency (IF) of a signal, ω(t), is defined as the derivative of the phase of its corresponding analytic signal, $x(t) = A(t)e^{j\phi(t)}$ [1]. Moreover, from a joint time--frequency (TF) perspective, the IF of a signal is defined as the average of frequencies at a given time (or time conditional mean frequency) [2]:

$$< \omega >_t = \omega(t) = \int \omega \frac{S(t,\omega)}{S(t)} d\omega \qquad (1)$$

where

$$S(t) = \int S(t,\omega) d\omega$$

is the density in time $(|x(t)|^2)$ or time marginal of the TF density $S(t,\omega)$. Estimating the IF of a signal is an important issue in many signal processing applications such as communications, radar, bioengineering, etc. [3,4]. For instance, in spread spectrum communication systems, jammers can be eliminated by estimating their IF and removing them by a time-varying filter [5].

[2] *This work was supported by The Research Fund of The University of Istanbul, Project number: B-833/08022001.*

Received Date : 24.04.2002
Accepted Date: 15.06.2004

In our approach the IF is estimated using a least squares multi-window evolutionary spectrum as the TF energy density for the signal. TF signal analysis is a helpful tool for analyzing the time-varying frequency content of a non-stationary signal [2]. The Wigner-Ville Spectrum (WVS) is defined as a time-dependent spectrum for non-stationary stochastic process $x(t)$ and given by [6]:

$$P(t,\omega) = E\{W(t,\omega)\}$$
$$= E\left\{ \int_{-\infty}^{\infty} \left[x\left(t - \frac{\tau}{2}\right) x^*\left(t + \frac{\tau}{2}\right) \right] e^{-j\omega\tau} d\tau \right\}$$

where $W(t,\omega)$ denotes the Wigner Distribution (WD) and the above is the statistical average of the WDs of the realizations of the process. When we have several observations of the non-stationary process $x(t)$, we can use an ensemble average of the individual WDs of these observations to estimate the WVS. However, this is not the case in general; we are only given a single realization of the process. In that case, Time-Frequency Distributions (TFDs) with a smoothing kernel function is used to estimate the WVS [2]. A good amount of research has been done to design kernels with desired properties yielding unbiased and low variance WVS estimates [6, 8].

A new estimate of the WVS is proposed as the optimal average of multiple-window spectrograms of the process in [9, 10]. In this work we use a WVS estimate that is an optimal combination of evolutionary spectra obtained by a multi-window Gabor expansion [7]. The optimal combination coefficients are obtained by minimizing the squared error between a reference TFD (which is taken to be the Wigner-Ville Distribution of the signal) and the multi-window spectral estimate.

2. EVOLUTIONARY SPECTRAL ANALYSIS BY MULTI-WINDOW GABOR EXPANSION

Given a non-stationary signal, $x(n), 0 \le n \le N-1$, a discrete Wold-Cramer representation [12] for it is given by

$$x(n) = \sum_{k=0}^{K-1} A(n,\omega_k) e^{j\omega_k n}, \tag{2}$$

where $\omega_k = 2\pi k / K$, K is the number of frequency samples, and $A(n,\omega_k)$ is an evolutionary kernel. The evolutionary spectrum is obtained from this kernel as $S(n,\omega_k) = \frac{1}{K}\left|A(n,\omega_k)\right|^2$. In [7] we show that the kernel can be obtained from the coefficients of a Gabor expansion. The multi-window Gabor expansion is given by [7]

$$x(n) = \sum_{m=0}^{M-1}\sum_{k=0}^{K-1} a_{i,m,k} h_i(n-mL) e^{j\omega_k n} \tag{3}$$
$$= \sum_{k=0}^{K-1} A_i(n,\omega_k) e^{j\omega_k n} \tag{4}$$

where $\{a_{i,m,k}\}$ are the Gabor coefficients, $\{h_{i,m,k}\}$ are the Gabor basis functions that are obtained by scaling, translating and modulating with a sinusoid a window function:

$$h_{i,m,k}(n) = h_i(n-mL) e^{j\omega_k n} \tag{5}$$

and the synthesis window $h_i(n)$ is obtained by scaling a unit-energy mother window $g(n)$ as

$h_i(n) = 2^{i/2} g(2^i n), i = 0,1,..., I-1.$

The multi-window Gabor coefficients are evaluated by

$$a_{i,m,k} = \sum_{n=0}^{N-1} x(n) \gamma_i^*(n-mL) e^{-jw_k n}, \tag{6}$$

where the analysis window $\gamma_i(n)$ is solved from the bi-orthogonality condition between $h_i(n)$ and $\gamma_i(n)$ [7]. Hence by comparing the representations of the signal in (3) and (4) we obtain the evolutionary kernel as

$$A_i(n,\omega_k) = \sum_{m=0}^{M-1} a_{i,m,k} h_i(n-mL) \tag{7}$$

Replacing for the coefficients $\{a_{i,m,k}\}$, we obtain also that

$$A_i(n,\omega_k) = \sum_{l=0}^{N-1} x(l) w_i(n,l) e^{-j\omega_k l}, \quad (8)$$

where we defined the time-varying window for scale i as

$$w_i(n,l) = \sum_{m=0}^{M-1} \gamma_i^*(l-mL) h_i(n-mL).$$

Then the evolutionary spectrum of $x(n)$ calculated using the window $h_i(n)$ is obtained by

$$S_i(n,\omega_k) = \frac{1}{K}\left|A_i(n,\omega_k)\right|^2,$$

where the factor $1/K$ is used for proper energy normalization. We should mention that normalizing the $w_i(n,l)$ to unit energy, the total energy of the signal is preserved thus justifying the use of $S_i(n,\omega_k)$ as a TF energy density for $x(n)$. Furthermore, $S_i(n,\omega_k)$ is always non--negative and approximates the marginal conditions [2]; hence, in contrast to many TFDs, interpretable as TF energy density function [7].

3. IF ESTIMATION BY MULTI-WINDOW LEAST SQUARES EVOLUTIONARY SPECTRUM

Given a realization of a discrete-time, nonstationary process corrupted by additive noise $x(n) = s(n) + \eta(n)$ where $s(n)$ and $\eta(n)$ denotes the signal and noise processes respectively. We intend to obtain a high resolution evolutionary spectral estimate with good performance in low signal to noise ratio (SNR) conditions such that the IF of the signal $s(n)$ can be estimated. We calculate a weighted average combination of evolutionary spectra $S_i(n,\omega_k)$ that is closest to a reference TFD in a least squares sense. Given the signal $x(n)$, we calculate evolutionary spectra $S_i(n,\omega_k)$ for $i = 0,1,\ldots,I-1$ as

$$S_i(n,\omega_k) = \frac{1}{K}\left|\sum_{l=0}^{N-1} x(l) W_i(n,l) e^{-j\omega_k l}\right|^2. \quad (9)$$

Gauss windows are used as $h_i(n)$, for their optimal concentration in the TF plane [7]. Then we estimate the WVS of the process $x(n)$ as a weighted average of the evolutionary spectra

$$\hat{P}(n,\omega_k) = \sum_{i=0}^{I-1} c_i S_i(n,\omega_k) \quad (10)$$

where the weights $\{c_i\}$ are obtained by minimizing the error function

$$\varepsilon_i = \sum_{n=0}^{N-1}\sum_{k=0}^{K-1}\left|P_R(n,\omega_k) - \sum_{i=0}^{I-1} c_i S_i(n,\omega_k)\right|^2 \quad (11)$$

and $P_R(n,\omega_k)$ is a reference TFD which is taken here as Wigner-Ville Distribution of the signal for its optimal TF resolution.
By using a matrix notation, the minimization problem in (11) can be rewritten as

$$\min_{c_i} \left\|P_R - Sc\right\|^2 \quad (12)$$

The solution of this least squares minimization problem is

$$c^o = (S^T S)^{-1} S^T P_R$$

where the superscript 'o' stands for optimum. Then a WVS estimate is obtained as optimal weighted average using $\{c_i^o\}$ in equation (10). Finally, we mask or threshold our estimate $\hat{P}(n,\omega_k)$ to eliminate any possible negative values caused by any negative c_i^o coefficient, and result in a non-negative time-varying spectrum, i.e.,

$$\hat{P}(n,\omega_k)^+ = \begin{cases} \hat{P}(n,\omega_k), & \hat{P}(n,\omega_k) \geq 0; \\ 0, & \hat{P}(n,\omega_k) < 0. \end{cases} \quad (13)$$

where $\hat{P}(n,\omega_k)^+$ denotes the positive-only part of the spectrum. Then the IF of the signal can be calculated from this TF density according to equation (1) as time conditional mean frequency. In our simulations we obtain both TF density and IF estimate of several nonstationary signals.

4. EXAMPLES

To illustrate the performance of our proposed method, we consider a quadratic FM signal. In Fig.1, we show the least squares evolutionary spectral estimate $\hat{P}(n,\omega_k)^+$ of this signal. Fig. 2 shows the IF estimate (the solid line) obtained from this evolutionary spectrum, and the true IF of the signal (the dashed line). As shown the IF estimate of the signal is very close to the true IF. To compare our result with that of another method, we applied Cakrak and Loughlin's [10] optimal combination of spectrograms method and obtained the result given in Fig. 3. As another example, we consider a sinusoidal FM signal and estimate its IF using our method. Fig. 4 shows the least squares evolutionary spectral estimate of this signal. The IF estimate (the solid line), and the true IF of the signal (the dashed line) are given in Fig. 5. IF estimate obtained by the least squares combination of spectrograms is shown in Fig. 6. It is clear from the figures that our least squares combination of multi-window evolutionary spectra gives better IF estimates than Cakrak and Loughlin's [10] spectrogram method.

5. CONCLUSIONS

In this work, we present a new method for obtaining the Instantaneous Frequency of non-stationary signals. Our method uses the optimal combination in the least squares sense, of evolutionary spectra that are calculated by multi-window Gabor expansion. The optimal weights are obtained by minimizing the squared error between the combination of evolutionary spectra and a reference TFD. Examples show that our method combines the advantages of multiple-window evolutionary spectral analysis and high resolution TFDs, i.e., it provides non-negative and high resolution time-varying spectral estimates as such the IF estimate is sufficiently close to the correct IF of the signal.

REFERENCES

[1] Cohen, L., and Lee, C., ``Instantaneous Frequency, Its Standard Deviation and Multicomponent Signals,'' SPIE, Advanced Algorithms and Architectures for Signal Processing III}, Vol. 975, pp. 186--208, 1988.

[2] Cohen, L., Time-Frequency Analysis}. Prentice Hall, Englewood Cliffs, NJ, 1995.

[3] Mandel, L., ``Interpretation of Instantaneous Frequencies,'' AJP, Vol. 42, pp. 840-846, Oct. 1974.

[4] Cakrak, F., Loughlin, P.J., ``Multiwindow Time-varying Spectrum with Instantaneous Bandwidth and Frequency Constraints,'' IEEE Trans. on Signal Proc., Vol. 49, No. 8, pp. 1656-1666, Aug. 2001.

[5] Akan, A., and Cekic, Y., ``Interference Suppression in DSSS Communication Systems Using Instantaneous Frequency Estimation,'' The 6th IEEE International Conference on Electronics, Circuits and Systems - ICECS'99, pp. 461-464, Paphos, Cyprus, Sep. 5 - 8, 1999.

[6] Martin, W., and Flandrin, P., ``Wigner-Ville spectral analysis of nonstationary processes,'' IEEE Trans. on ASSP, Vol. 33, No. 6, pp. 1461-1470, Dec. 1985.

[7] Akan, A., and Chaparro, L.F.,``Multi-window Gabor Expansion for Evolutionary Spectral Analysis,'' Signal Processing, Vol. 63, pp. 249-262, Dec. 1997.

[8] Jones, D.L. and Baraniuk, R.G., ``An Adaptive Optimal-Kernel Time--Frequency Representation,'' IEEE Trans. on Signal Process., Vol. 43, No. 10, pp. 2361-2371, May 1995.

[9] Bayram, M., and Baraniuk, R.G., ``Multiple Window Time--Frequency Analysis,'' IEEE SP Intl. Symposium on Time-Frequency and Time-Scale Analysis, TFTS'96, Paris, France, pp. 173-176, Jun. 1996.

[10] Cakrak, F., and Loughlin, P., ``Multiwindow Time-Varying Spectrum with Instantaneous Bandwidth and Frequency Constraints,'' IEEE Trans. on Signal Process., Vol. 49, pp. 1656-1666, Aug. 2001.

[11] Akan, A., and Chaparro, L.F., ``Evolutionary Chirp Representation of Non-stationary Signals via Gabor Transform,,''

Signal Processing, Vol. 81, No. 11, pp. 2429-2436, Nov. 2001.
[12] Priestley, M.B., Non-linear and Non-stationary Time Series Analysis. Academic Press, London, 1988.

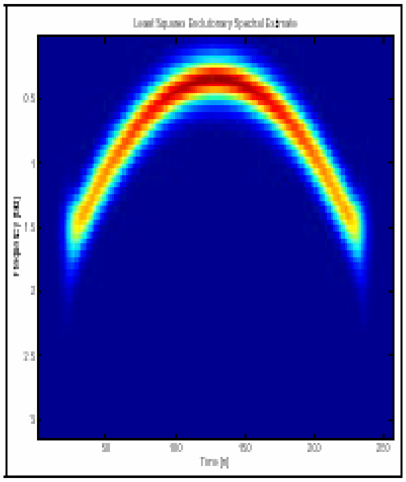


Fig. 1. Least Squares Multi-window Evolutionary Spectral Estimate.

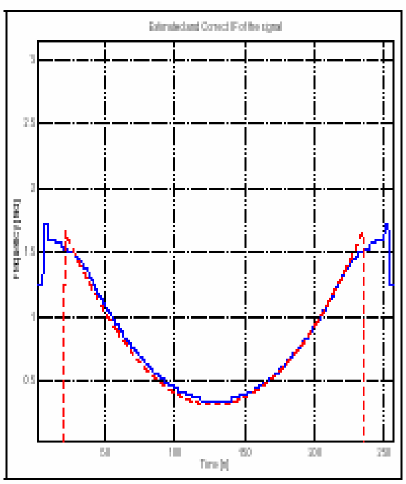


Fig. 2. Estimated (solid line) and true (dashed line) IF.

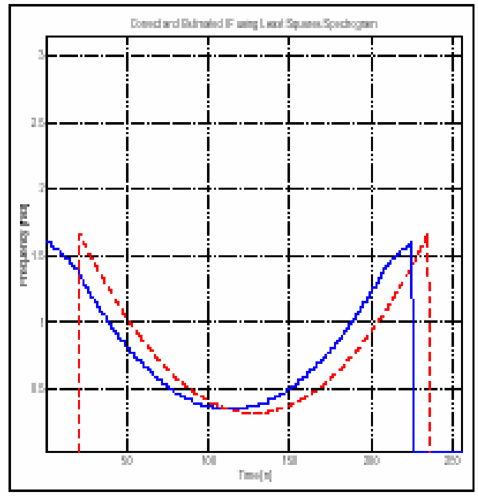


Fig. 3. Correct (dashed line) and estimated (solid line) IF by spectrogram combinations.

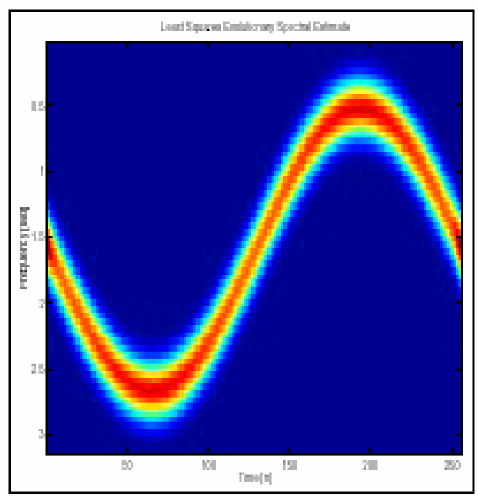


Fig. 4. Least Squares Evolutionary Spectral Estimate of the signal.

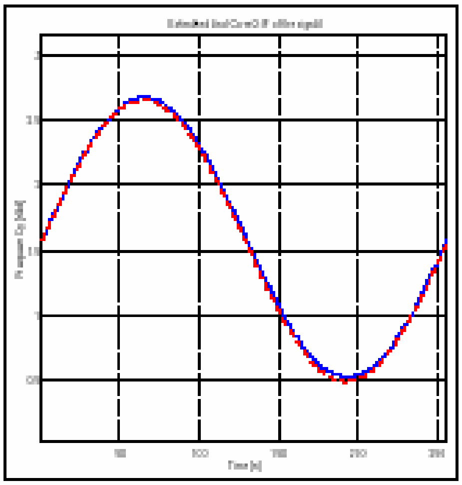


Fig. 5. Estimated (solid line) and true (dashed line) IF.

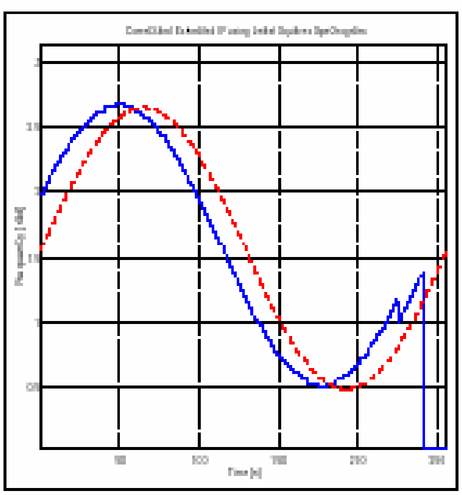
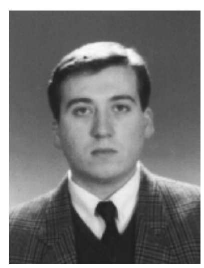


Fig. 6. Correct (dashed line) and estimated (solid line) IF by spectrogram combinations.

Authors' Biographies:

Mahmut ÖZTÜRK was born in Istanbul, Turkey in 1977. He received the B.Sc. degree in 2000 from Istanbul University, Istanbul, Turkey. He is currently an M.Sc. student at the same university. He is also a research assistant at the department of Electrical and Electronics Engineering, Istanbul University. His research interests are digital signal processing, and time-frequency signal analysis and its applications.



Aydın AKAN was born in Bursa, Turkey in 1967. He received the B.Sc. degree in 1988 from the University of Uludag, Bursa, Turkey in 1988, the M.Sc. degree from the Technical University of Istanbul, Istanbul, Turkey in 1991, and the Ph.D. degree from the University of Pittsburgh, Pittsburgh, PA, USA, in 1996 all in electrical engineering. He has been with the Department of Electrical and Electronics Engineering, University of Istanbul since 1996 where he currently holds an Associate Professor position. His current research interests are digital signal processing, statistical signal processing, time-frequency analysis methods and applications of time-frequency methods to communications and bioengineering. Dr. Akan is a member of the IEEE Signal Processing Society since 1994.