ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2007 (373 - 378)
VOLUME : 7
NUMBER : 1

ANALYSIS OF THE RELATIONSHIP BETWEEN MULTIPATH FADING AND OPTIMUM SUBCARRIER SPACING FOR GENERALIZED MC DS-CDMA SYSTEMS

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ABSTRACT

Generalized multicarrier direct sequence code-division multiple-access (MC DS-CDMA) system is a model which includes a number of specific MC DS-CDMA schemes by varying the normalized subcarrier spacing. In the generalized MC DS-CDMA scheme, for a given system bandwidth and a given channel environment, there exists an optimum subcarrier spacing, which results in a minimum bit error rate. In this paper, as an extension of a previous study, detailed analysis on the variation of the optimum normalized subcarrier spacing for the generalized MC DS-CDMA in multipath Nakagami-m fading channels is performed. The results show that the optimum subcarrier spacing is almost independent of the variation of the fading parameter when the number of subcarriers is small. On the contrary, it is observed that the fading parameter has a clear effect on the settling of the optimum subcarrier spacing for large number of subcarriers.

Keywords: : *MC DS-CDMA, generalized MC DS-CDMA, Nakagami-m fading channels, optimum normalized subcarrier spacing..*

1. INTRODUCTION

Multicarrier code-division multiple-access (MC-CDMA) scheme based on the combination of CDMA and orthogonal frequency division multiplexing (OFDM) has been proposed as an effective candidate for the fourth generation mobile communication systems [1]. This scheme introduces spreading in frequency domain by using several subcarriers and hence provides increased capacity for users [1]-[4]. Two common forms of the MC-CDMA are the multitone DS-CDMA [5], [6] and the orthogonal MC DS-CDMA [7]-[11]. In a multitone DS-CDMA system the subcarrier frequencies are chosen to be orthogonal harmonics of each other with minimum frequency separation among them before DS spreading. Therefore, in multitone DS-CDMA systems, the spacing Δ between two adjacent subcarrier frequencies is $\Delta = 1/T_s$ where T_s represents the symbol duration of the multitone DS-CDMA signal. On the contrary, in an orthogonal MC DS-CDMA system, the subcarrier frequencies are chosen to satisfy the orthogonality condition with the minimum possible frequency separation after DS spreading. For an orthogonal MC DS-CDMA system, the Δ between two adjacent subcarrier

Received Date : 15.05.2005
Accepted Date: 01.02.2007

frequencies is $\Delta = 1/T_c$ where T_c is the chip duration of DS spreading codes.

In [12], Yang and Hanzo defined a class of generalized MC DS-CDMA schemes and evaluated its performance over multipath Nakagami-*m* fading channels [13]. In these schemes, both the multitone DS-CDMA system and the orthogonal MC DS-CDMA system can be viewed as a member of the class of generalized MC DS-CDMA systems having arbitrary subcarrier spacing of λ.

As reported in [12], the generalized MC DS-CDMA system is a general model that includes a number of specific MC DS-CDMA schemes, and the results generated can be extended to different MC DS-CDMA systems by simply varying a single parameter, namely λ. In the generalized MC DS-CDMA scheme, for a given system bandwidth and a given channel environment, there exists an optimum subcarrier spacing, which results in a minimum bit error rate (BER). It is useful to note that the BER versus normalized subcarrier spacing analysis performed in [12] considers the effect of Rayleigh fading and Rician fading for observing the variation of the optimum subcarrier spacing. Our paper will complete this previous study by realizing a detailed analysis on the variation of the optimum normalized subcarrier spacing for various multipath channel scenarios [14].

This paper has the following layout: in the next section, a brief description of the generalized MC DS-CDMA system which was originally introduced in [12] is presented. This section also includes the performance expression to evaluate the average BER of the generalized MC DS-CDMA system. The detailed analysis of the optimum normalized subcarrier spacing of the generalized MC DS-CDMA in multipath Nakagami-*m* fading channels is performed in section 3, and section 4 highlights the conclusion.

2. SYSTEM DESCRIPTION

The generalized MC DS-CDMA system employed in this paper was first described in [12]. In the transmitter, the binary data stream having bit duration of T_b is serial-to-parallel converted to U parallel substreams. Therefore, the symbol duration is $T_s = UT_b$. After serial-to-parallel conversion, the uth substream modulates a subcarrier frequency f_u using binary phase shift keying (BPSK) for $u = 1, 2, \dots, U$. Then, the U subcarrier-modulated substreams are added in order to form the complex modulated signal. Finally, spectral spreading is imposed on the complex signal by multiplying it with a spreading code. Therefore, the transmitted signal of user k can be expressed as

$$s_k(t) = \sum_{u=1}^{U} \sqrt{2P} b_{ku}(t) c_k(t) \cos(2\pi f_u t + \varphi_{ku}) \tag{1}$$

where the following notations are used:

U number of subcarriers;
P average received power of each subcarrier signal;
$b_{ku}(t)$ binary data stream;
$c_k(t)$ binary DS spreading waveform;
f_u uth subcarrier frequency;
φ_{ku} phase angles introduced in the carrier modulation
process;

The processing gain, N_e, of the subcarrier signal can be written as

$$N_e = UN_1 - \frac{(U-1)\lambda}{2} \tag{2}$$

where N_1 is the spreading gain of a corresponding single-carrier DS-CDMA system and λ is the normalized subcarrier spacing. In this study, it is assumed that the channel between the kth transmitter and the corresponding receiver is a multipath Nakagami-*m* fading channel. The complex low-pass equivalent representation of the impulse response experienced by subcarrier u of user k is given by

$$h_{ku}(t) = \sum_{l_p=0}^{L_p-1} \alpha_{ul_p}^{(k)} \delta(t - \tau_{kl_p}) e^{(-j\psi_{ul_p}^{(k)})} \tag{3}$$

where $\alpha_{ul_p}^{(k)}$, τ_{kl_p} and $\psi_{ul_p}^{(k)}$ represent the attenuation factor, delay and phase-shift for the l_pth multipath component of the channel, respectively, while $\delta(t)$ is the Kronecker-Delta function.

The total number of diversity paths, L_p, is given by

$$L_p \approx \left[\frac{2N_e(L_1-1)}{2N_e+(U-1)\lambda} \right] + 1$$

(4)

It is assumed that the phases $\left\{\psi_{ul_p}^{(k)}\right\}$ in Eqn (3) are independent identically distributed (i.i.d.) random variables uniformly distributed in the interval [0, 2π), while the L_p multipath attenuations $\left\{\alpha_{ul_p}^{(k)}\right\}$ are independent Nakagami random variables with a probability density function (pdf) of

$$p\left(\alpha_{ul_p}^{(k)}\right) = M\left(\alpha_{ul_p}^{(k)},\ m,\ \Omega_{ul_p}^{(k)}\right),$$

$$M(R,\ m,\ \Omega) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} e^{(-m/\Omega)R^2} \qquad (5)$$

where Γ(·) is the gamma function, and m is the Nakagami-m fading parameter, which is equal to $m = E^2\left[\left(\alpha_{ul_p}^{(k)}\right)^2\right] / \mathrm{Var}\left[\left(\alpha_{ul_p}^{(k)}\right)^2\right]$. The parameter $\Omega_{ul_p}^{(k)}$ in Eqn (5) is the second moment of $\alpha_{ul_p}^{(k)}$.

A negative exponentially decaying multipath intensity profile (MIP) distribution given below is assumed in this paper

$$\Omega_{ul_p}^{(k)} = \Omega_{u0}^{(k)} \exp(-\eta l_p), \qquad \eta \geq 0 \qquad (6)$$

where $\Omega_{u0}^{(k)}$ is the average signal strength corresponding to the first resolvable path and η is the rate of average power decay.

Considering K asynchronous CDMA users in the system, where all of them use the same U and N_e values, with perfect power control. Consequently, when K signals obeying the form of Eqn (1) are transmitted over the frequency-selective fading channels characterized by Eqn (3), the received signal at the base station can be expressed as

$$r(t) = \sum_{k=1}^{K} \sum_{u=1}^{U} \sum_{l_p=0}^{L_p-1} \sqrt{2P}\alpha_{ul_p}^{(k)} b_{ku}(t-\tau_{kl_p}) c_k(t-\tau_{kl_p}) \cos\left(2\pi f_u t + {}_{ul_p}^{(k)}\right) + n(t)$$

(7)

where ${}_{ul_p}^{(k)} = \varphi_{ku} - \psi_{ul_p}^{(k)} - 2\pi f_u \tau_{kl_p}$, which is assumed to be an i.i.d. random variable having a uniform distribution in [0, 2π), while $n(t)$ represents the additive white Gaussian noise (AWGN) with zero mean and double–sided power spectral density of $N_0/2$.

Assuming that the first user be the user-of-interest and consider the correlator-based RAKE receiver with maximum ratio combining (MRC). L, $1 \leq L \leq L_p$, in this work indicates the number of diversity branches used by the receiver. Further assumptions are: (*i*) the receiver is capable of acquiring perfect time-domain synchronization with each path of the reference signal; (*ii*) the multipath attenuations and phases are assumed to be perfect estimates of the channel parameters. As a result, after MRC the decision variable Z_v of the zeroth data bit corresponding to the vth substream of the reference user can be expressed as

$$Z_v = \sum_{l=0}^{L-1} Z_{vl}\ , \quad v = 1,\ 2, \ldots, U \qquad (8)$$

$$Z_{vl} = \int_{\tau_l}^{T_s+\tau_l} r(t) \cdot \alpha_{vl} c(t-\tau_l) \cos(2\pi f_v t + {}_{vl})\, dt$$

(9)

According to the decision variable Z_v, $v = 1,\ 2, \ldots, U$, the current data bit of the vth substream is decided to be 0 or 1, depending on whether Z_v is higher than zero. Finally, the U number of parallel data substreams are parallel-to-serial converted, in order to output the serial data bits. Based on the performance analysis derived in [12], the average BER for the generalized MC DS-CDMA system is defined as [12, Eqn (47)]

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} \prod_{l=0}^{L-1} \left(\frac{m\sin^2\theta}{\left[\left(\frac{\Omega_0 E_b}{N_0}\right)^{-1} + \frac{2(KL_p-1)q(L_p,\eta)}{L_p} \cdot \left(\frac{1}{3N_e} + (U-1)\bar{I}_M\right)\right]^{-1} e^{-\eta l} + m\sin^2\theta} \right)^m d\theta$$

(10)

where $q(L_p,\eta) = (1-e^{-\eta L_p})/(1-e^{-\eta})$, Ω_0 is the average value of Ω_{ul} $(= q(L_p,\eta)/L_p)$. $\bar{I}_M$ in Eqn (10) can be expressed as [12, Eqn (35)]

$$\bar{I}_M = \frac{1}{U(U-1)} \sum_{v=1}^{U} \sum_{\substack{u=1 \\ u \neq v}}^{U} \frac{N_e}{2\pi^2(u-v)^2\lambda^2} \cdot \left[1 - \sin c\left(\frac{2\pi(u-v)\lambda}{N_e}\right)\right] \qquad (11)$$

3. ANALYSIS AND RESULTS

In this section, the detailed analysis of the variation of the optimum normalized subcarrier spacing for the generalized MC DS-CDMA over Nakagami-*m* fading channels is performed. The system parameters which are fixed in all figures are summarized as follows: The spreading gain and the number of resolvable paths of the corresponding single-carrier DS-CDMA system are N_1 = 128 and L_1 = 32, respectively. The MIP decay-factor is η = 0.2, the number of simultaneous users is K = 10, the SNR per bit is E_b/N_0 = 15 dB. The number of subcarriers used in this section is U = 2, 8, and 32. We assume that the receiver was capable of combining all the resolvable paths ($L = L_p$).

The Nakagami-*m* multipath fading channels has been demonstrated to closely model various multipath channels [13], [15]-[17]. The probability density functions belonging to Nakagami-*m* spans the range from the multipath fading channel that is worse than the Rayleigh fading channel to nonfading AWGN channels by varying the parameter *m* from 1/2 to infinity.

Figure 1 shows the influence of the normalized subcarrier spacing, λ, on the average BER of the generalized MC DS-CDMA system for different values of the fading parameter, *m*, when the number of subcarriers is equal to 2. As can be seen, a common value of the optimum subcarrier spacing, λ_{opt}, is carried out. Therefore, variation of λ_{opt} is almost independent of the fading parameter *m* when the number of subcarrier is small.

Figure 2, shows the BER versus the normalized subcarrier spacing, λ, when U = 8. The variations observed on this figure are the sings belonging to the effect of the fading parameter on the variation of λ_{opt}. According to Figure 2, a common λ_{opt} for the all values of *m* are no longer possible. It should be noted that the λ_{opt} values calculated for the all values of *m* are different from each other.

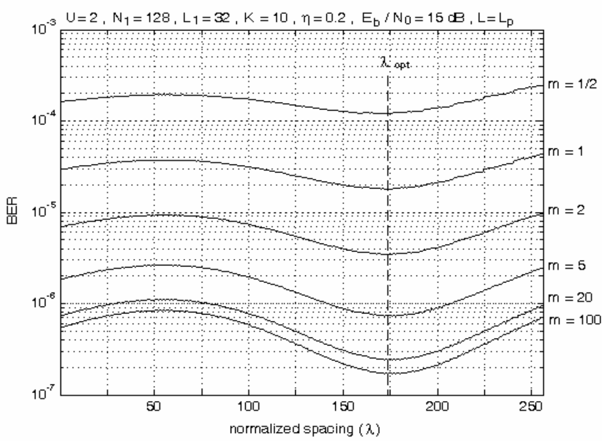

***Fig. 1.** BER versus the normalized subcarrier spacing, λ for the generalized MC DS-CDMA system over multipath Nakagami-m fading channel: U = 2 and m ∈ {1/2, 1, 2, 5, 20, 100}.*

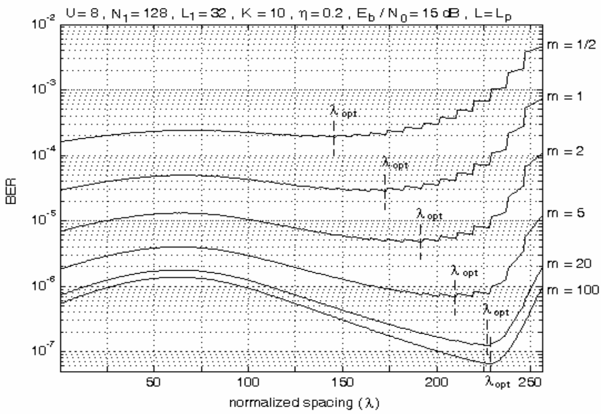

***Fig. 2.** BER versus the normalized subcarrier spacing, λ for the generalized MC DS-CDMA system over multipath Nakagami-m fading channel: U = 8 and m ∈ {1/2, 1, 2, 5, 20, 100}.*

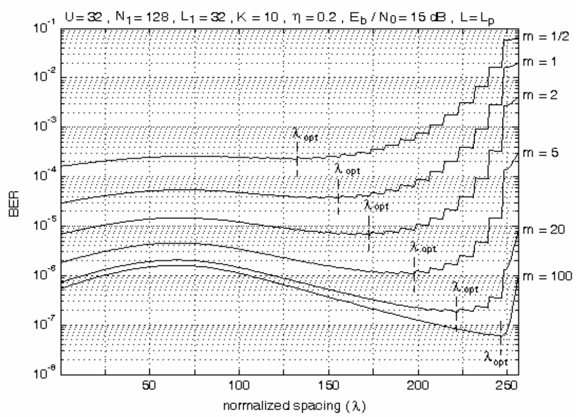

***Fig. 3.** BER versus the normalized subcarrier spacing, λ for the generalized MC DS-CDMA system over multipath Nakagami-m fading channel: U = 32 and m ∈ {1/2, 1, 2, 5, 20, 100}*

The most impressive results about the effect of the m on the variation of λ_{opt} were obtained when the number of subcarriers is large. As can be seen in Figure 3, the values of λ_{opt}, which results in a minimum BER are quite different for the all m considered in this paper. To illustrate the importance of using individual λ_{opt} values instead of a common λ_{opt}, let us focus on a numerical example. Based on Figure 3, the λ_{opt} for m = 5 and m = 100 are approximately λ_{opt} = 197 and λ_{opt} = 247, respectively, and the achieved BER with these values are approximately 1×10^{-6} and 6×10^{-8}, respectively. If λ_{opt} = 197 is employed for m = 100, based on the reality that a common λ_{opt} can be obtained for all values m as indicated in Figure 1, then the achieved BER will equal to approximately 2×10^{-7} instead of 6×10^{-8}. As a result, a "*spacing-optimized*" MC DS-CDMA scheme will not be obtained for this case.

4. CONCLUSION

As an extension of a previous study, a detailed analysis of the variation of the optimum normalized subcarrier spacing for the generalized MC DS-CDMA in multipath Nakagami-*m* fading channels is performed in this paper. The influence of the multipath fading on the variation of the optimum normalized subcarrier spacing is analyzed for the generalized MC DS-CDMA system. The results indicate that the variation of the optimum normalized subcarrier spacing is almost independent of the fading parameter m when the number of subcarriers is small. On the contrary, the fading parameter m plays an important role on the settling of the optimum normalized subcarrier spacing when the number of subcarriers is large.

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