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Hypercyclic Weighted Composition Operators on $\ell^2(\mathbb{Z})$

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Abstract: A bounded linear operator T on a separable Hilbert space $\mathscr{H}$ is called *hypercyclic* if there exists a vector $x \in \mathscr{H}$ whose orbit $\{T^n x : n \in \mathbb{N}\}$ is dense in $\mathscr{H}$. In this paper, we characterize the hypercyclicity of the weighted composition operators $C_{u,\varphi}$ on $\ell^2(\mathbb{Z})$ in terms of their weight functions and symbols. First, a necessary and sufficient condition is given for $C_{u,\varphi}$ to be hypercyclic. Then, it is shown that the finite direct sums of the hypercyclic weighted composition operators are also hypercyclic. In particular, we conclude that the class of the hypercyclic weighted composition operators is weakly mixing. Finally, several examples are presented to illustrate the hypercyclicity of the weighted composition operators.

Keywords: Hypercyclic operators, Orbit, Weighted composition operators.

1. Introduction

A bounded linear operator T on a separable Hilbert space $\mathscr{H}$ is called *hypercyclic* if there exists a vector $x \in \mathscr{H}$ whose orbit under T, $\{T^n x : n \in \mathbb{N}\}$ is dense in $\mathscr{H}$. Such a vector is called a *hypercyclic vector* for an operator T. The concept of the hypercyclicity is intimately related to the invariant subset problem in such a way that an operator T has no nontrivial invariant closed subsets if and only if all nonzero vectors are hypercyclic for T. This outstanding fact has attracted many mathematicians' attention to study the hypercyclicity and the dynamics of the linear operators in the recent two decades. Especially, the hypercyclic phenomena for some types of operators such as the weighted shifts and the composition operators has been observed by many authors. S. Rolewicz [10] gave a historical example. Indeed, for every scalar λ with $|\lambda| > 1$, he showed that λB is hypercyclic, where B is the backward shift operator on $\ell^2(\mathbb{N})$. The satisfactory and comprehensive discussions of the main concepts of the linear dynamics such as cyclic, supercyclic, hypercyclic and weakly mixing operators may be found in the survey articles [1, 8, 12, 13, 15] and in the interesting books [2, 7].

Recall that a bounded linear operator T on a separable Hilbert space $\mathscr{H}$ is *weakly mixing* if and only if $T \oplus T$ is hypercyclic [2]. In a separable F-space setting, T is weakly mixing if and only if T

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satisfies the hypercyclicity criterion [13]. It should be mentioned that not all hypercyclic operators are necessarily weakly mixing (c.f. [11]).

The main idea of this paper is based upon H. N. Salas's work [12] who has widely characterized the hypercyclic weighted shifts. For example, we remind one of his interesting results below.

Theorem 1. Let T be a bilateral weighted shift with a positive weight sequence $\{w_n\}$ i.e., $Te_n = w_n e_{n+1}$. Then T is hypercyclic if and only if for given $\varepsilon > 0$ and $m \in \mathbb{N}$, there exists n arbitrarily large such that for all $|j| \leq m$

$$\prod_{i=0}^{n-1} w_{i+j} < \varepsilon \quad \text{and} \quad \prod_{i=1}^{n} w_{j-i} > \frac{1}{\varepsilon}.$$

As a consequence, Salas has also shown that every hypercyclic weighted shift is weakly mixing. Furthermore, the finite direct sums of the hypercyclic weighted shifts have been studied by Salas in that paper (Theorem 2.5). In [9], a necessary and sufficient condition for the weighted shifts on the various complex sequence spaces is given to possess a hypercyclic subspace. Moreover, G. Costakis and A. Manoussos have studied the dynamics of weighted shifts on the space of bounded sequences of complex numbers [5].

We will confine our attention to the infinite-dimensional separable Hilbert space $\ell^2(\mathbb{Z})$ consisting of the square-summable sequences of complex numbers. The usual Schauder basis of $\ell^2(\mathbb{Z})$ is denoted by $\{e_n\}_{n=1}^{\infty}$ that is, $e_i(j) = \delta_{ij}$, where δ_{ij} is Kronecker delta. A weight function u on $\mathbb{Z}$ and a non-singular transformation $\varphi : \mathbb{Z} \to \mathbb{Z}$ induce the weighted composition operator $C_{u,\varphi} : \ell^2(\mathbb{Z}) \to \ell^2(\mathbb{Z})$ defined by

$$C_{u,\varphi} f := u \cdot f \circ \varphi.$$

The transformation φ is usually called the symbol of the weighted composition operator in the literature. The weighted composition operator $C_{u,\varphi}$ is bounded if and only if $\sup\{H(n) : n \in \mathbb{N}\} < \infty$, where $H(n) = \sum_{j \in B(n)} |u(j)|^2$, $B(n) = \{j \in supp(u) : \varphi(j) = n\}$ and $supp(u) = \{j \in \mathbb{Z} : u(j) \neq 0\}$. For more details the reader is referred to [14]. The hypercyclicity of the weighted composition operators and their adjoints on the holomorphic functions spaces have been studied in [15] in detail. Before this work, in [13] it was shown that, if C_φ is hypercyclic on $H(U)$, the F-space of all holomorphic functions on an open unit disk U, then φ is univalent and has no fixed point in U. Further, Gallardo and Montes [6] have obtained a complete characterization of the cyclic, supercyclic and hypercyclic scalar multiples of composition operators with the linear fractional symbols acting on the weighted Dirichlet spaces. In [3], the dynamic behavior of the weighted composition operators on the space of holomorphic functions, defined on a simply connected domain, has been studied.

In this paper, we characterize the hypercyclic weighted composition operators in terms of their weight functions and symbols. We give a necessary and sufficient condition for the weighted composition operator $C_{u,\varphi}$ to be hypercyclic. Then, it is shown that the finite direct sums of the hypercyclic weighted composition operators are also hypercyclic. In particular, we conclude that the class of the hypercyclic weighted composition operators is weakly mixing. Finally, to illustrate the obtained results, some examples are presented.

2. Hypercyclic Weighted Composition Operators

In this section, we are going to extend Theorem 1 and prove some other results related to the hypercyclicity of the weighted composition operators. The necessary and sufficient conditions are given such that $C_{u,\varphi}$ and $\oplus_{k=1}^{s} C_{u_k,\varphi_k}$ are hypercyclic. For this, in what follows, we shall assume that φ is invertible and $u(k) \neq 0$ for each $k \in \mathbb{Z}$. Actually, these conditions do not imply the invertibility of $C_{u,\varphi}$. To distinguish between these facts, consider the following example. Define $\varphi : \mathbb{Z} \to \mathbb{Z}$ by $\varphi(k) = k$, define $u : \mathbb{Z} \to [0,\infty)$ by $u(k) = 2$ if $k \leq 0$ and $u(k) = \frac{1}{k}$ if $k \geq 1$. Note that φ is invertible and $u(k) \neq 0$ for each $k \in \mathbb{Z}$. However in this case, $C_{u,\varphi} = D$ where D is the diagonal operator given by $De_k = 2e_k$ if $k \leq 0$ and $De_k = \frac{1}{k} e_k$ otherwise. Subsequently in this circumstance, it is clear that D is not invertible. Remind that a weighted composition operator $C_{u,\varphi}$ is invertible if and only if

- $\inf_{k\in\mathbb{Z}} |u(k)| > 0$,
- φ is injective,
- φ has dense range.

In other words, $C_{u,\varphi}$ is invertible if and only if φ is invertible and $\inf_{k\in\mathbb{Z}} |u(k)| > 0$. Throughout this paper, without loss of generality, we may and will assume that the weight function u is non-negative. Further, we adopt the notations $\varphi^n = \underbrace{\varphi \circ \varphi \circ \cdots \circ \varphi}_{n \text{ times}}$ and $\varphi^{-n} = \underbrace{\varphi^{-1} \circ \varphi^{-1} \circ \cdots \circ \varphi^{-1}}_{n \text{ times}}$. We now turn to investigate the hypercyclicity of the weighted composition operators.

Lemma 1. Let $C_{u,\varphi}$ be a hypercyclic weighted composition operator on $\ell^2(\mathbb{Z})$. Then φ cannot be the identity transformation and $u(k) \neq 0$ for all $k \in \mathbb{Z}$.

Proof. We use the well known fact that the adjoint of a hypercyclic operator has no eigenvalue (c.f. [2]). Let f, g be arbitrary elements of $\ell^2(\mathbb{Z})$. Then

$$
\begin{aligned}
\langle C_{u,\varphi}^* f, g\rangle = \langle f, C_{u,\varphi} g\rangle &= \sum_{n=-\infty}^{\infty} f_n \overline{u_n g_{\varphi(n)}} \\
&= \sum_{n=-\infty}^{\infty} \overline{u_n} f_n \overline{g_{\varphi(n)}} \\
&= \langle \overline{u} f, g \circ \varphi \rangle.
\end{aligned}
$$

Suppose on contrary that φ is the identity transformation. In this case, $C^*_{u,\varphi}$ has an eigenvalue which is a contradiction. Moreover, if $u(k_0) = 0$ for some $k_0 \in \mathbb{Z}$, then $C_{u,\varphi}$ cannot be hypercyclic. To see this, note that for every integer $n \geq 1$ and $f \in \ell^2(\mathbb{Z})$, we have

$$\begin{aligned} C^n_{u,\varphi}(f)(k_0) &= [\prod_{i=0}^{n-1} u(\varphi^i(k_0))] f(\varphi^n(k_0)) \\ &= u(k_0)u(\varphi(k_0)) \cdots u(\varphi^{n-1}(k_0)) f(\varphi^n(k_0)) \\ &= 0, \end{aligned}$$

because $u(k_0) = 0$. Thus, there will never exist a vector f and a strictly increasing sequence (n_k) such that $C^{n_k}_{u,\varphi} \to e_{k_0}$ and so the weighted composition operator $C_{u,\varphi}$ is never hypercyclic. ■

Definition 1. Let X be a topological vector space and $T : X \to X$ be a bounded linear operator. We say that T satisfies the **hypercyclicity criterion** if there exist an increasing sequence of integers (n_k), two dense subsets $D_1, D_2 \subset X$ and a sequence of maps $S_{n_k} : D_2 \to X$(not necessarily linear or continuous) such that

- $T^{n_k}(x) \to 0$ for every $x \in D_2$;
- $S_{n_k}(y) \to 0$ for every $y \in D_2$;
- $T^{n_k} S_{n_k}(y) \to 0$ for every $y \in D_2$.

For the possible setting, $n_k = k$ and $D_1 = D_2$, it is called **Kitai's hypercyclicity criterion.**

Remark 2.1. Note that neither every hypercyclic operator T on a separable F-space X satisfies the hypercyclicity criterion nor every such hypercyclic operator T is necessarily weakly mixing. While if T satisfies the hypercyclicity criterion then so does $T \oplus T$, i.e., T is weakly mixing. Many years these facts had been remained as the most exciting open problems which has been recently solved by M. De La Rosa and C. J. Read in [11].

Theorem 2. Let $T : X \to X$ be a bounded linear operator on a separable F-space X. Assume that T satisfies the hypercyclicity criterion. Then T is hypercyclic.

Proof. See [2]. ■

In the following theorem, we give a necessary and sufficient condition for the weighted composition operator to be hypercyclic.

Theorem 3. Suppose φ is a monotonic map and $C_{u,\varphi}$ is a bounded weighted composition operator on $\ell^2(\mathbb{Z})$. Then $C_{u,\varphi}$ is hypercyclic if and only if for each $\varepsilon > 0$ and $m \in \mathbb{N}$, there exists $n \in \mathbb{N}$ arbitrarily large such that

$$\prod_{i=1}^{n} u \circ \varphi^{-i} < \varepsilon \quad \text{and} \quad \prod_{i=0}^{n-1} \frac{1}{u \circ \varphi^i} < \varepsilon$$

on $[-m,m]$.

Proof. First suppose that $C_{u,\varphi}$ is hypercyclic and $\varepsilon > 0$, $m \in \mathbb{N}$ are given. Since the set of all hypercyclic vectors for $C_{u,\varphi}$ is dense in $\ell^2(\mathbb{Z})$ [2], then for each $\acute{\varepsilon} > 0$ there exists a hypercyclic vector f for $C_{u,\varphi}$ such that

$$\|f - \Sigma_{j\in[-m,m]} e_j\| < \acute{\varepsilon}. \tag{1}$$

By virtue of the inequality (1), the coefficients of f, $|\langle f, e_j\rangle| > 1-\acute{\varepsilon}$ whenever $j \in [-m,m]$ and $|\langle f, e_j\rangle| < \acute{\varepsilon}$ otherwise.

First, we deal with the case that φ is an increasing transformation. Therefore, one may find a $n \in \mathbb{N}$ sufficiently large, indeed $\varphi^n(j) > 2m$ and $\varphi^{-n}(j) < -2m$ for each $j \in [-m,m]$, such that

$$\|C^n_{u,\varphi} f - \Sigma_{j\in[-m,m]} e_j\| < \acute{\varepsilon}. \tag{2}$$

Consequently, we have

$$\|C^n_{u,\varphi}(\langle f, e_j\rangle e_j)\| = |\langle f, e_j\rangle| [\prod_{i=1}^{n} u\circ\varphi^{-i}](j) < \acute{\varepsilon},$$

since it is assumed that $\varphi^{-n}(j) < -2m$ for each $j \in [-m,m]$. Further, we may assume that $\acute{\varepsilon} < 1$. Hence,

$$\prod_{i=1}^{n} u\circ\varphi^{-i}(j) < \frac{\acute{\varepsilon}}{|\langle f, e_j\rangle|} < \frac{\acute{\varepsilon}}{1-\acute{\varepsilon}} \tag{3}$$

for each $j \in [-m,m]$. Moreover, for all $j \in [-m,m]$, one may similarly claim that

$$\begin{aligned}
\acute{\varepsilon} &> \|C^n_{u,\varphi} f - e_{\varphi^n(j)}\| \\
&= \|[\prod_{i=0}^{n-1} u\circ\varphi^{i}] f\circ\varphi^n - e_{\varphi^n(j)}\| \\
&\geq |[\prod_{i=0}^{n-1} u\circ\varphi^{i}](j) f\circ\varphi^n(j) - 1| \\
&\geq 1 - [\prod_{i=0}^{n-1} u\circ\varphi^{i}](j) |f\circ\varphi^n(j)|.
\end{aligned}$$

Since $\varphi^n(j) > 2m$ for each $j \in [-m,m]$, thus giving $|f\circ\varphi^n(j)| < \acute{\varepsilon}$ and hence

$$\prod_{i=0}^{n-1} u\circ\varphi^{i}(j) > \frac{1-\acute{\varepsilon}}{|f\circ\varphi^n(j)|} > \frac{1-\acute{\varepsilon}}{\acute{\varepsilon}}. \tag{4}$$

Eventually, by choosing $\acute{\varepsilon}$ such that $\frac{\acute{\varepsilon}}{1-\acute{\varepsilon}} < \varepsilon$, the inequalities (3) and (4) yield that both necessary statements of the theorem are satisfied.

Now we come to the case that φ is a decreasing transformation. In this case, we may choose a $n \in \mathbb{N}$ sufficiently large, $\varphi^n(j) < -2m$ and $\varphi^{-n}(j) > 2m$ for each $j \in [-m,m]$, such that the inequality (2) holds. Then the inequalities (3) and (4) are established whenever $\varphi^n(j) < -2m$ and $\varphi^{-n}(j) > 2m$ for each $j \in [-m,m]$, respectively. The rest of the proof proceeds in a similar way.

For the reverse implication, the proof is basically done by Kitai's hypercyclicity criterion (Definition 1). Take two dense sets $D_1 = D_2 = span\{e_j : j \in \mathbb{Z}\}$. Define the following maps

$$S_n e_j := [\prod_{i=0}^{n-1} \frac{1}{u(\varphi^i(j))}] e_{\varphi^n(j)}\ , \qquad j \in \mathbb{Z}$$

and extend them linearly to $span\{e_j : j \in \mathbb{Z}\}$. One can easily examine the following statements:

$$C^n_{u,\varphi} e_j = [\prod_{i=1}^{n} u(\varphi^{-i}(j))] e_{\varphi^{-n}(j)} \quad \text{and} \quad C^n_{u,\varphi} S^n e_j = e_j.$$

Now assume that for each $\varepsilon > 0$ and each integer $m \geq 0$, there exists an arbitrarily large integer n such that

$$\prod_{i=1}^{n} u \circ \varphi^{-i} < \varepsilon \quad \text{and} \quad \prod_{i=0}^{n-1} \frac{1}{u \circ \varphi^i} < \varepsilon$$

for each $j \in [-m,m]$. Then, for each $f \in span\{e_j : j \in \mathbb{Z}\}$, we can find a strictly increasing sequence (n_k) of positive integers such that

$$C^{n_k}_{u,\varphi} f \to 0,\ \ S^{n_k} f \to 0 \quad \text{and} \quad C^{n_k}_{u,\varphi} S^{n_k} f = f$$

as $k \to \infty$. Thus, $C_{u,\varphi}$ satisfies the hypercyclicity criterion and hence is hypercyclic. ■

Remark 2.2. By letting $u = 1$ in the previous theorem, it is easily understood that the composition operator C_φ on $\ell^2(\mathbb{Z})$ cannot be hypercyclic itself.

Theorem 4. Let $\{C_{u_k,\varphi_k}\}_{k=1}^s$ be a sequence of weighted composition operators. Then $\oplus_{k=1}^s C_{u_k,\varphi_k}$ is hypercyclic if and only if for given $\varepsilon > 0$ and $m \in \mathbb{N}$ there exists a $n \in \mathbb{N}$ sufficiently large such that

$$\max\{\prod_{i=1}^{n} u_k \circ \varphi_k^{-i} : 1 \leq k \leq s\} < \varepsilon$$

and

$$\max\{\prod_{i=0}^{n-1} \frac{1}{u_k \circ \varphi_k^i} : 1 \leq k \leq s\} < \varepsilon$$

on $[-m,m]$.

Proof. By the same argument to that of Theorem 3, the proof can be developed similarly to the case of the finite direct sums of the weighted composition operators. First assume that $\oplus_{k=1}^s C_{u_k,\varphi_k}$ is hypercyclic. Let $\varepsilon > 0$ and $m \in \mathbb{N}$ be arbitrary. Then, for any k ($1 \leq k \leq s$) there exist $f_k \in \ell^2(\mathbb{Z})$ and $n_k \in \mathbb{N}$ sufficiently large such that for each $j \in [-m,m]$, we have

$$\|C^{n_k}_{u_k,\varphi_k}(\langle f_k, e_j \rangle e_j)\| = |\langle f_k, e_j \rangle| [\prod_{i=1}^{n_k} u_k \circ \varphi_k^{-i}](j) < \varepsilon$$

and

$$\|C^{n_k}_{u_k,\varphi_k} f_k - e_{\varphi_k{}^{n_k}(j)}\| < \varepsilon.$$

Now, let $n = \max\{n_k : 1 \le k \le s\}$. Therefore, by the above inequalities, the assertions can be inferred as argued in the proof of Theorem 3.

Conversely, for each k, $(1 \le k \le s)$ define

$$S_{n,k} e_j := [\prod_{i=0}^{n-1} \frac{1}{u_k(\varphi_k^i(j))}] e_{\varphi_k^n(j)}$$

on $D_1 = D_2 = span\{e_j : j \in \mathbb{Z}\}$. By the hypothesis, there exists a n^k sufficiently large, such that

$$\prod_{i=1}^{n^k} u_k \circ \varphi_k^{-i} < \varepsilon \text{ and } \prod_{i=0}^{n^k-1} \frac{1}{u_k \circ \varphi_k^i} < \varepsilon$$

on $[-m,m]$. Now we may construct a strictly increasing sequence (n_p^k) of positive integers such that

$$C^{n_p^k}_{u_k,\varphi_k} f \to 0, \;\; S^{n_p^k} f \to 0 \quad \text{and} \quad C^{n_p^k}_{u_k,\varphi_k} S^{n_p^k} f = f$$

for each $f \in span\{e_j : j \in \mathbb{Z}\}$ as $p \to \infty$. Thus, each C_{u_k,φ_k} satisfies the hypercyclicity criterion and hence is hypercyclic. ∎

Corollary 1. Let $C_{u,\varphi}$ be a hypercyclic weighted composition operator. Then $C_{u,\varphi} \oplus C_{u,\varphi} \oplus \cdots \oplus C_{u,\varphi}$ is also hypercyclic. Furthermore, $C_{u,\varphi}$ is weakly mixing.

Proof. The proof is straightforward by Theorem 4. In addition, it is already followed from the proof of Theorem 3, as in our setting any operator T satisfying the hypercyclicity criterion satisfies that $\underbrace{T \oplus \cdots \oplus T}_{n \text{ times}}$ is hypercyclic. ∎

Recall that a bounded linear operator T on a separable F-space X is weakly mixing if and only if T satisfies the hypotheses of the hypercyclicity criterion which is proved by J. P. Bès and A. Peris [4]. By scrutinizing the proof of Theorem 3, it is seen that how the assumptions

$$\prod_{i=1}^{n} u \circ \varphi^{-i} < \varepsilon \quad \text{and} \quad \prod_{i=0}^{n-1} \frac{1}{u \circ \varphi^i} < \varepsilon$$

would result in the $C_{u,\varphi}$ to satisfy the hypotheses of the hypercyclicity criterion.

3. Examples

Example 3.1. In some setting, the weighted composition operators can be recast as weighted shifts. For if, define $\varphi(n) = n+1$. Then $C_{u,\varphi}$ is the bilateral backward shift given by $B_w e_n =$

$w_n e_{n-1}$, where $w = (w_n) = (u(n-1))$ for all $n \in \mathbb{Z}$. Indeed,

$$\begin{aligned} C_{u,\varphi} e_n(j) &= u(j)(e_n \circ \varphi)(j) \\ &= u(j) e_n(j+1) \\ &= \begin{cases} u(n-1), & \text{if j=n-1;} \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Then Theorem 1 is easily deduced from Theorem 3.

Example 3.2. Define

$$u(k) = \begin{cases} \frac{1}{k}, & \text{k=1, 2, ...;} \\ 2, & \text{k=0, -1, -2, ...} \end{cases}$$

and for each $k \in \mathbb{Z}$, define $\varphi(k) = k - t$, where t is an arbitrary positive integer. Then by Theorem 3, the corresponding weighted composition operator $C_{u,\varphi}$ is hypercyclic. For this, consider that

$$\lim_{n \to +\infty} u\, u \circ \varphi \ldots u \circ \varphi^{n-1} = +\infty$$

and

$$\lim_{n \to +\infty} u \circ \varphi^{-1} \ldots u \circ \varphi^{-n} = 0$$

on any symmetric interval of $\mathbb{Z}$ about zero.

Example 3.3. Consider the transformation $\varphi : \mathbb{Z} \to \mathbb{Z}$, defined by $\varphi(k) = -k+1$. Then $\varphi^{2i}(k) = k$ and $\varphi^{2i+1}(k) = -k+1$ for every $i \in \mathbb{N}$. Now, if the weight sequence u is defined in such a way that $u(k)u(-k+1) = 1$, then by Theorem 3, $C_{u,\varphi}$ cannot be hypercyclic.

Example 3.4. Let $\varphi : \mathbb{Z} \to \mathbb{Z}$ be defined as $\varphi(k) = -k$. Then, $C_{u,\varphi}$ commutes the k-th weight of the basis vector e_k with the ones of e_{-k}, for every $k \in \mathbb{Z}$. In this case, two conditions of Theorem 3 cannot be held simultaneously. Even though, if we define

$$u(k) = \begin{cases} \frac{1}{k}, & \text{k=} \pm 1, \pm 2, \text{...;} \\ 1, & \text{k=0.} \end{cases}$$

Then, for each $k \in [-1, 1]$, it should be noticed that

$$\prod_{i=0}^{n-1} \frac{1}{u \circ \varphi^i(k)} = \underbrace{\frac{1}{u(k)u(-k)u(k)\ldots u(-k)}}_{n \text{ times}} \nrightarrow 0$$

as $n \to +\infty$. Therefore, $C_{u,\varphi}$ is not hypercyclic. It is, in fact, not hypercyclic since $\|C_{u,\varphi}\| \leq 1$.

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