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Generalized Jacobi Elliptic Function Method for Traveling Wave Solutions of (2+1)-Dimensional Breaking Soliton Equation

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Özet. Bu çalışmada, genelleştirilmiş Jakobi eliptik fonksiyon metodu kullanılarak (2+1) boyutlu breaking soliton denkleminin periyodik çözümleri ve çok katlı soliton çözümleri sembolik bilgisayar programı yardımıyla elde edilmiştir.

Anahtar Kelimeler. (2+1)-boyutlu breaking soliton denklemi, Jakobi eliptik fonksiyon metodu, periyodik çözümler, çok katlı soliton çözümler, hareket eden dalga çözümleri.

Abstract. In this study, we implemented the generalized Jacobi elliptic function method with symbolic computation to construct periodic and multiple soliton solutions for the (2+1)-dimensional breaking soliton equation.

Keywords. (2+1)-dimensional breaking soliton equation, generalized Jacobi elliptic function method, periodic solutions, multiple soliton solutions, traveling wave solutions.

1. Introduction

The theory of nonlinear dispersive wave motion has recently been the subject of much study. We do not attempt to characterize the general form of nonlinear dispersive wave equations [1,2]. Nonlinear phenomena play a crucial role in applied mathematics and physics. Furthermore, when an original nonlinear equation is directly calculated, the solution will preserve the actual physical characteristics of solutions [3]. Explicit solutions to the nonlinear equations are of fundamental importance. Various methods for obtaining explicit solutions to nonlinear evolution equations have been proposed, and many explicit exact methods have been introduced in literature [4-36]. Among them are the generalized Miura transformation, Darboux

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transformation, Cole-Hopf transformation, Hirota's dependent variable transformation, inverse scattering transform, and Bäcklund transformation, tanh method, sine-cosine method, Painleve method, homogeneous balance method, similarity reduction method, improved tanh method, and so on. In fact, recently a direct algebraic approach has been constructed for an automated tanh-function method by Parkes and Duffy [12]. The authors present a Mathematica package that deals with complicated algebra and outputs directly the required solutions for particular nonlinear equations.

In this study, we implemented the generalized Jacobi elliptic function method [28] with symbolic computation to construct new double-periodic solutions and multiple soliton solutions for the (2+1)-dimensional breaking soliton equation.

2. An Analysis of the Method and Applications

Before starting to use a generalized Jacobi elliptic function method [28], we will give a simple description of the method. For doing this, one can consider a two variables general form of nonlinear PDE

$$Q(u, u_t, u_x, u_{xx}, ...) = 0, \tag{1}$$

and transform (1) with $u(x,t) = u(\xi)$, $\xi = x + \alpha y + \beta t$, where α, β is a constant. After transformation, we get a nonlinear ODE for $u(\xi)$

$$Q'(u', u'', u''', ...) = 0. \tag{2}$$

The solution of (2) we are looking for is expressed in the form

$$U_i(\xi) = a_0 + \sum_{i=1}^{n} \left[a_i F^i(\xi) + b_i F^{-i}(\xi) \right], \tag{3}$$

where $\xi = x + \alpha y + \beta t$, n is a positive integer that can be determined by balancing the highest order derivative with the highest nonlinear terms in the equation $\alpha, \beta, a_0, a_i, b_i$ and ξ can be determined. Substituting (3) into (2) yields a set of algebraic equations for

$$F^i \left(\sqrt{A + BF^2 + CF^4} \right)^j, \quad j = 0,\ 1, \quad i = 0,\ 1,\ 2, \dots .$$

In this way, all coefficients of $F^i \left(\sqrt{A + BF^2 + CF^4} \right)^j$ have to vanish. After this separated algebraic equation, we could find the coefficients $\alpha, \beta, a_0, a_i, b_i$ and α.

In this work, we will consider the solution of the (2+1)-dimensional breaking soliton equation by using the generalized Jacobi elliptic function method which is introduced by Huai-Tang and Hong-Qing [28]. The fundamental of their method is to take full advantage of the elliptic equation and use its solutions F. The desired elliptic equation is given as

$$F'^2 = A + BF^2 + CF^4, \tag{4}$$

where $F' = \dfrac{dF}{d\xi}$ and a, b, c are constants. Some of the solutions are given in [28]. In this study we have given several extra cases so that we have obtained double-periodic solutions and multiple soliton solutions of (2) in the form of Jacobi elliptic functions (4).

Example. Consider a (2+1)-dimensional breaking soliton equation

$$\begin{aligned} u_t + 4buv_x + 4bu_xv + bu_{xxy} &= 0, \\ v_x - u_y &= 0. \end{aligned} \tag{5}$$

For doing this example, we can use transformation with (1) then (5) becomes

$$\begin{aligned} \beta u' + 4buv' + 4bu'v + b\alpha u''' &= 0, \\ v' - \alpha u' &= 0, \end{aligned} \tag{6}$$

and integrating (6) yields,

$$\begin{aligned} \beta u + 4buv + b\alpha u'' &= 0, \\ v - \alpha u &= 0. \end{aligned} \tag{7}$$

If $v = \alpha u$ and substituting into (7)

$$\beta u + 4b\alpha u^2 + b\alpha u'' = 0. \tag{8}$$

Balancing u^2 with u'' then gives $n = 2$. Therefore, we may choose

$$u = a_0 + a_1F + a_2F^2 + \frac{b_1}{F} + \frac{b_2}{F^2}. \tag{9}$$

Substituting (9) into (8) yields a set of algebraic equations for $\alpha, \beta, a_0, a_i, b_i$. These systems are found to be

$$\begin{aligned} 4a_0^2b\alpha + 2Aa_2b\alpha + 8a_1bb_1\alpha + 8a_2bb_2\alpha + 2bb_2C\alpha + a_0\beta &= 0, \\ 6Abb_2\alpha + 4bb_2^2\alpha &= 0, \end{aligned}$$

$$2Abb_1\alpha + 8bb_1b_2\alpha = 0,$$
$$4bb_1^2\alpha + 8a_0bb_2\alpha + 4bBb_2\alpha + b_2\beta = 0,$$
$$8a_0bb_1\alpha + bBb_1\alpha + 8a_1bb_2\alpha + b_1\beta = 0,$$
$$8a_0ba_1\alpha + bBa_1\alpha + 8a_2bb_1\alpha + a_1\beta = 0,$$
$$4a_1^2b\alpha + 8a_0a_2b\alpha + 4a_2bB\alpha + a_2\beta = 0,$$
$$8a_1a_2b\alpha + 2a_1bC\alpha = 0,$$
$$4a_2^2b\alpha + 6a_2bC\alpha = 0.$$

From the solutions of the system, we can find

$$a_0 = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right),\ a_1 = 0,\ a_2 = -\frac{3C}{2},\ b_1 = 0,$$
$$b_2 = -\frac{3A}{2},\ A \neq 0,\ b \neq 0,\ \alpha \neq 0,\ \beta = -4b\sqrt{B^2 + 12AC}. \quad (10)$$

With the aid of Mathematica substituting (10) into (9), we have obtained the following double-periodic solutions of (8). If $v = \alpha u$, these solutions are

(i) If $A = 1$, $B = -(1 + m^2)$, $C = m^2$, $\xi = x + \alpha y - \left(4b\sqrt{B^2 + 12AC}\right)t$.

$$u_1 = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}sn^2(\xi) - \frac{3A}{2}\left(\frac{1}{sn^2(\xi)}\right),$$
$$v_1 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2 + 12AC\alpha}\right) - \frac{3C\alpha}{2}sn^2(\xi) - \frac{3A\alpha}{2}\left(\frac{1}{sn^2(\xi)}\right).$$
$$u_2 = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}\left(\frac{cn(\xi)}{dn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{dn(\xi)}{cn(\xi)}\right)^2,$$
$$v_2 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2 + 12AC\alpha}\right) - \frac{3C\alpha}{2}\left(\frac{cn(\xi)}{dn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{dn(\xi)}{cn(\xi)}\right)^2.$$

(ii) If $A = 1 - m^2$, $B = 2m^2 - 1$, $C = -m^2$, $\xi = x + \alpha y - \left(4b\sqrt{B^2 + 12AC}\right)t$.

$$u_3 = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}cn^2(\xi) - \frac{3A}{2}\left(\frac{1}{cn^2(\xi)}\right),$$
$$v_3 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2 + 12AC}\alpha\right) - \frac{3C\alpha}{2}cn^2(\xi) - \frac{3A\alpha}{2}\left(\frac{1}{cn^2(\xi)}\right).$$

(iii) If $A = m^2 - 1$, $B = 2 - m^2$, $C = -1$, $\xi = x + \alpha y - \left(4b\sqrt{B^2 + 12AC}\right)t$.

$$u_4 = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}dn^2(\xi) - \frac{3A}{2}\left(\frac{1}{dn^2(\xi)}\right),$$

$$v_4 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2} dn^2(\xi) - \frac{3A\alpha}{2}\left(\frac{1}{dn^2(\xi)}\right).$$

(iv) If $A = -m^2(1-m^2)$, $B = 2m^2-1$, $C = 1$, $\xi = x + \alpha y - \left(4b\sqrt{B^2+12AC}\right)t$.

$$u_5 = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}\left(\frac{dn(\xi)}{sn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{sn(\xi)}{dn(\xi)}\right)^2,$$

$$v_5 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{dn(\xi)}{sn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{sn(\xi)}{dn(\xi)}\right)^2.$$

(v) If $A = 1-m^2$, $B = 2-m^2$, $C = 1$, $\xi = x + \alpha y - \left(4b\sqrt{B^2+12AC}\right)t$.

$$u_6 = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}\left(\frac{cn(\xi)}{sn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{sn(\xi)}{cn(\xi)}\right)^2,$$

$$v_6 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{cn(\xi)}{sn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{sn(\xi)}{cn(\xi)}\right)^2.$$

(vi) If $A = \frac{1}{4}$, $B = \frac{m^2-2}{2}$, $C = \frac{m^2}{4}$, $\xi = x + \alpha y - \left(4b\sqrt{B^2+12AC}\right)t$.

$$u_7 = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}\left(\frac{sn(\xi)}{1 \pm dn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{1 \pm dn(\xi)}{sn(\xi)}\right)^2,$$

$$v_7 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{sn(\xi)}{1 \pm dn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{1 \pm dn(\xi)}{sn(\xi)}\right)^2.$$

(vii) If $A = \frac{m^2}{4}$, $B = \frac{m^2-2}{2}$, $C = \frac{m^2}{4}$, $\xi = x + \alpha y - \left(4b\sqrt{B^2+12AC}\right)t$.

$$u_8 = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}(sn(\xi) \pm icn(\xi))^2 - \frac{3A}{2}\left(\frac{1}{(sn(\xi) \pm icn(\xi))^2}\right),$$

$$v_8 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}(sn(\xi) \pm icn(\xi))^2 - \frac{3A\alpha}{2}\left(\frac{1}{(sn(\xi) \pm icn(\xi))^2}\right),$$

$$u_9 = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}\left(\frac{dn(\xi)}{i\sqrt{1-m^2}sn(\xi) \pm cn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{i\sqrt{1-m^2}sn(\xi) \pm cn(\xi)}{dn(\xi)}\right)^2.$$

$$v_9 = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{dn(\xi)}{i\sqrt{1-m^2}sn(\xi) \pm cn(\xi)}\right)^2$$
$$- \frac{3A\alpha}{2}\left(\frac{i\sqrt{1-m^2}sn(\xi) \pm cn(\xi)}{dn(\xi)}\right)^2.$$

(viii) If $A = \frac{1}{4}$, $B = \frac{1-2m^2}{2}$, $C = \frac{1}{4}$, $\xi = x + \alpha y - \left(4b\sqrt{B^2+12AC}\right)t$.

$$u_{10} = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}(msn(\xi) \pm idn(\xi))^2$$
$$- \frac{3A}{2}\left(\frac{1}{(msn(\xi) \pm idn(\xi))^2}\right).$$

$$v_{10} = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}(msn(\xi) \pm idn(\xi))^2$$
$$- \frac{3A\alpha}{2}\left(\frac{1}{(msn(\xi) \pm idn(\xi))^2}\right).$$

$$u_{11} = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}\left(\frac{dn(\xi)}{mcn(\xi) \pm i\sqrt{1-m^2}}\right)^2$$
$$- \frac{3A}{2}\left(\frac{mcn(\xi) \pm i\sqrt{1-m^2}}{dn(\xi)}\right)^2.$$

$$v_{11} = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{dn(\xi)}{mcn(\xi) \pm i\sqrt{1-m^2}}\right)^2$$
$$- \frac{3A\alpha}{2}\left(\frac{mcn(\xi) \pm i\sqrt{1-m^2}}{dn(\xi)}\right)^2.$$

$$u_{12} = \frac{1}{2}\left(-B + \sqrt{B^2+12AC}\right) - \frac{3C}{2}\left(\frac{sn(\xi)}{1 \pm cn(\xi)}\right)^2$$
$$- \frac{3A}{2}\left(\frac{1 \pm cn(\xi)}{sn(\xi)}\right)^2.$$

$$v_{12} = \frac{1}{2}\left(-B\alpha + \sqrt{B^2+12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{sn(\xi)}{1 \pm cn(\xi)}\right)^2$$
$$- \frac{3A\alpha}{2}\left(\frac{1 \pm cn(\xi)}{sn(\xi)}\right)^2.$$

$$u_{13} = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}\left(\frac{cn(\xi)}{\sqrt{1-m^2}sn(\xi) \pm dn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{\sqrt{1-m^2}sn(\xi) \pm dn(\xi)}{cn(\xi)}\right)^2.$$

$$v_{13} = \frac{1}{2}\left(-B\alpha + \sqrt{B^2 + 12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{cn(\xi)}{\sqrt{1-m^2}sn(\xi) \pm dn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{\sqrt{1-m^2}sn(\xi) \pm dn(\xi)}{cn(\xi)}\right)^2.$$

(ix) If $A = \frac{m^2-1}{4}$, $B = \frac{m^2+1}{2}$, $C = \frac{m^2-1}{4}$, $\xi = x + \alpha y - \left(4b\sqrt{B^2 + 12AC}\right) t$.

$$u_{14} = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}\left(\frac{dn(\xi)}{1 \pm msn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{1 \pm msn(\xi)}{dn(\xi)}\right)^2.$$

$$v_{14} = \frac{1}{2}\left(-B\alpha + \sqrt{B^2 + 12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{dn(\xi)}{1 \pm msn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{1 \pm msn(\xi)}{dn(\xi)}\right)^2.$$

(x) If $A = \frac{1-m^2}{4}$, $B = \frac{m^2+1}{2}$, $C = \frac{1-m^2}{4}$, $\xi = x + \alpha y - \left(4b\sqrt{B^2 + 12AC}\right) t$.

$$u_{15} = \frac{1}{2}\left(-B + \sqrt{B^2 + 12AC}\right) - \frac{3C}{2}\left(\frac{cn(\xi)}{1 \pm sn(\xi)}\right)^2 - \frac{3A}{2}\left(\frac{1 \pm sn(\xi)}{cn(\xi)}\right)^2.$$

$$u_{15} = \frac{1}{2}\left(-B\alpha + \sqrt{B^2 + 12AC}\alpha\right) - \frac{3C\alpha}{2}\left(\frac{cn(\xi)}{1 \pm sn(\xi)}\right)^2 - \frac{3A\alpha}{2}\left(\frac{1 \pm sn(\xi)}{cn(\xi)}\right)^2.$$

(xi) If $A=-\dfrac{(1-m^2)^2}{4}$, $B=\dfrac{m^2+1}{2}$, $C=-\dfrac{1}{4}$, $\xi=x+\alpha y-\left(4b\sqrt{B^2+12AC}\right)t$.

$$u_{16}=\frac{1}{2}\left(-B+\sqrt{B^2+12AC}\right)-\frac{3C}{2}(mcn(\xi)\pm dn(\xi))^2 -\frac{3A}{2}\left(\frac{1}{(mcn(\xi)\pm dn(\xi))^2}\right).$$

$$v_{16}=\frac{1}{2}\left(-B\alpha+\sqrt{B^2+12AC}\alpha\right)-\frac{3C\alpha}{2}(mcn(\xi)\pm dn(\xi))^2 -\frac{3A\alpha}{2}\left(\frac{1}{(mcn(\xi)\pm dn(\xi))^2}\right).$$

(xii) If $A=\dfrac{1}{4}$, $B=\dfrac{m^2+1}{2}$, $C=\dfrac{(1-m^2)^2}{4}$, $\xi=x+\alpha y-\left(4b\sqrt{B^2+12AC}\right)t$.

$$u_{17}=\frac{1}{2}\left(-B+\sqrt{B^2+12AC}\right)-\frac{3C}{2}\left(\frac{sn(\xi)}{dn(\xi)\pm cn(\xi)}\right)^2 -\frac{3A}{2}\left(\frac{dn(\xi)\pm cn(\xi)}{sn(\xi)}\right)^2.$$

$$v_{17}=\frac{1}{2}\left(-B\alpha+\sqrt{B^2+12AC}\alpha\right)-\frac{3C\alpha}{2}\left(\frac{sn(\xi)}{dn(\xi)\pm cn(\xi)}\right)^2 -\frac{3A\alpha}{2}\left(\frac{dn(\xi)\pm cn(\xi)}{sn(\xi)}\right)^2.$$

(xiii) If $A=\dfrac{1}{4}$, $B=\dfrac{m^2-2}{2}$, $C=\dfrac{m^4}{4}$, $\xi=x+\alpha y-\left(4b\sqrt{B^2+12AC}\right)t$.

$$u_{18}=\frac{1}{2}\left(-B+\sqrt{B^2+12AC}\right)-\frac{3C}{2}\left(\frac{cn(\xi)}{\sqrt{1-m^2}\pm dn(\xi)}\right)^2 -\frac{3A}{2}\left(\frac{\sqrt{1-m^2}\pm dn(\xi)}{cn(\xi)}\right)^2.$$

$$v_{18}=\frac{1}{2}\left(-B\alpha+\sqrt{B^2+12AC}\alpha\right)-\frac{3C\alpha}{2}\left(\frac{cn(\xi)}{\sqrt{1-m^2}\pm dn(\xi)}\right)^2 -\frac{3A\alpha}{2}\left(\frac{\sqrt{1-m^2}\pm dn(\xi)}{cn(\xi)}\right)^2.$$

Here $sn(\xi,m), cn(\xi,m), dn(\xi,m)$ are Jacobi elliptic functions and m denotes the modulus of the Jacobi elliptic functions. If $m\to 1$, then $sn\xi\to\tanh\xi$, $cn\xi\to\sec h\xi$,

$dn\xi \to \sec h\xi$. We can obtain the following multiple soliton solutions of (5).

$$\begin{cases} u_{19} &= \frac{3}{4} - \frac{3}{8}\left(\frac{\tanh(\xi)}{1 \pm \sec h(\xi)}\right)^2 - \frac{3}{8}\left(\frac{1 \pm \sec h(\xi)}{\tanh(\xi)}\right)^2 \\ v_{19} &= \frac{3\alpha}{4} - \frac{3\alpha}{8}\left(\frac{\tanh(\xi)}{1 \pm \sec h(\xi)}\right)^2 - \frac{3\alpha}{8}\left(\frac{1 \pm \sec h(\xi)}{\tanh(\xi)}\right)^2 \end{cases}$$

where $\xi = x + \alpha y - 4bt$.

$$\begin{cases} u_{21} = 3 - \frac{3}{2}\tanh^2(\xi) - \frac{3}{2}\left(\frac{1}{\tanh^2(\xi)}\right) \\ v_{21} = 3\alpha - \frac{3\alpha}{2}\tanh^2(\xi) - \frac{3\alpha}{2}\left(\frac{1}{\tanh^2(\xi)}\right) \end{cases}$$

where $\xi = x + \alpha y - 16bt$.

If $m \to 0$, then $sn\xi \to \sin\xi$, $cn\xi \to \cos\xi$, $dn\xi \to 1$. We can obtain the following triangular periodic solutions of (5).

$$\begin{cases} u_{22} &= 1 - \frac{3}{2}\cot^2(\xi) - \frac{3}{2}\tan^2(\xi) \\ v_{22} &= \alpha - \frac{3\alpha}{2}\cot^2(\xi) - \frac{3\alpha}{2}\tan^2(\xi) \end{cases}$$

where $\xi = x + \alpha y - 16bt$,

$$\begin{cases} u_{23} = \frac{1}{4} - \frac{3}{8}\left(\frac{\sin(\xi)}{1 \pm \cos(\xi)}\right)^2 - \frac{3}{8}\left(\frac{1 \pm \cos(\xi)}{\sin(\xi)}\right)^2 \\ v_{23} = \frac{\alpha}{4} - \frac{3\alpha}{8}\left(\frac{\sin(\xi)}{1 \pm \cos(\xi)}\right)^2 - \frac{3\alpha}{8}\left(\frac{1 \pm \cos(\xi)}{\sin(\xi)}\right)^2 \end{cases}$$

where $\xi = x + \alpha y - 4bt$.

3. Conclusion

In this paper, we present the generalized Jacobi elliptic function method by using ansatz (3) and, with aid of Mathematica, implement it in a computer algebraic system. An implementation of the method is given by applying it to the (2+1)-dimensional breaking soliton equation. We also obtain some new double-periodic solutions and multiple soliton solutions at the same time. The method can be used for many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculations on a computer.

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