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Hall Effect on Unsteady MHD Free Convection Flow Past an Impulsively Started Porous Plate with Viscous and Joule's Dissipation

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Abstract- An investigation on the nonlinear problem of the effect of Hall current on the unsteady free convection flow of viscous, incompressible, electrically conducting fluid past an impulsively started infinite vertical porous plate is carried out, when a uniform magnetic field is applied transverse to the plate, while the viscous and Joule's dissipations are taken into account. The solutions of the coupled nonlinear partial differential equations have been obtained by using finite difference methods. Hall current effect on primary and secondary velocity, skin friction and rate of heat transfer are analyzed in detail for heating and cooling of the plate by convection currents. Physical interpretations and justifications are rendered for various results obtained.

Keywords- *Hall current; unsteady MHD flow; natural convection; viscous and Joule's dissipation.*

I. Introduction

When a body moves in the upper atmosphere of earth one must consider the dynamics of various physical phenomena into consideration and nevertheless when such a problem is mathematically modeled one usually makes a compromise and neglects a few of them to make the problem amenable for mathematical analysis depending upon the methods employed. Here, we consider the problem of Hall effect on the hydromagnetic free convection flow of ionized air past a impulsively moving vertical plate in the upper atmosphere, taking into consideration the viscous and Joule's dissipation. In what follows we narrate the significance of the various physical phenomena considered. It must be kept in mind that the literature on all these various aspects are so vast that one cannot do justice in an article of this type to provide the complete literature survey and hence only very relevant and recent works are cited.

A. *Free Convection and MHD*

Free convection flows arise when buoyancy forces due to density differences occur and these act as "driving forces". The density differences are caused by temperature differences and hence such problems are mathematically exciting owing to the coupling between momentum equation and heat equation and nonlinearity.

It is well known that it is possible to alter the flow and heat transfer around an object in upper atmosphere by applying a magnetic field provided the air is rarefied and sufficiently ionized. In fact, it has been established [1] that the skin friction and heat transfer can be substantially reduced by applying a transverse magnetic field. In the field of aerodynamic heating, the problem of providing heat protection by MHD effects gained a great momentum after the classical work [2]. Fortunately, the magnetic Reynolds numbers in such flows are not very high so that the induced magnetic field can be neglected. This amounts to saying that the electromagnetic equations are decoupled from momentum and heat equations whereas the latter two involve the electromagnetic variables.

The following references are an excellent source of information in this regard: [3], [4], [5], [6], [7], [8], [9], [10], [11]. One may also refer to the survey article [12] for further details.

B. Hall Effect

The upper atmosphere is ionized and is electrically conducting and its electrical conductivity depends on various parameters like location, time, height, season, etc, and it is anisotropic. Under such situations, when the number density of electrons is relatively small, it becomes pertinent to note that the charged particles are tied to the lines of force when a strong magnetic field is applied, and this prevents their motion transverse to the magnetic field. Then the tendency of the current to flow in a direction normal to both the electric and magnetic fields is called Hall effect and the corresponding current is known as Hall current. .

There are a good number of works on Hall effect and one may refer to [13], [14], [15], [16], [17].

C. Suction

Generally, the low energy fluid in the boundary layer is sucked through the wall to prevent boundary layer separation. Suction is one of the techniques employed in the boundary layer control in aerodynamics and space science and it is effectively used to reduce the skin friction as well as heat transfer around the moving body. Suction is also used effectively to increase the lift on airfoils too. In this connection, one may refer to [4], [5], [6], [18], [19].

D. Viscous Dissipation

The heat generated per unit time and unit volume by internal friction is called the viscous dissipation. In the case of air, for a temperature difference of 10^o F, a velocity of the order 150 ms^{-1} will make the viscous dissipation term comparable with other terms [20]. Indeed, it is known [21] that significant viscous dissipation may occur in natural convection in various devices which are subject to large decelerations or which operate at high rotative speeds. In addition, important viscous dissipation effects may also be present in stronger gravitational fields and in processes wherein the scale of the processes is very large, for example, on larger planets, in large masses of gas in space, and in geological processes in fluids internal to various bodies. One may refer to the works of [7], [21], in this regard.

E. Joule's Dissipation

Apart from the viscous dissipation, in the MHD flows, Joule's dissipation also acts a volumetric heat source [22], [23] and this represents the electromagnetic energy dissipated on account of heating of the medium by the electric current. It depends on the strength of the applied magnetic field. Indeed, in MHD flows there is not only energy transfer between the electromagnetic field and the fluid flow, but also a portion of the kinetic energy is converted to thermal energy by means of Joule heating. When a stronger magnetic field is applied the flow is retarded severely and also there is a considerable heating of the fluid due to Joule effect. Joule heating causes, in general, an increase of temperature and its gradient, mainly in the temperature boundary layer [24]. Hence, one must consider this effect while modeling problems related to atmospheric flights.

To understand the importance of and the current interests on the investigations involving Hall current, viscous and Joule's dissipation, one may refer to [25], [26], [27], [28], [29], [30], [31], [32], [33], [34].

Apart from the application mentioned supra, the investigations of heat transfer in *magnetohydrodynamic* (MHD) flows past a porous plate under the influence of a magnetic field find useful applications in many engineering problems such as MHD generator, plasma studies, nuclear reactors, geothermal energy extractions and the boundary layer control in the field of aerodynamics [35].

However, to the best of the knowledge of the authors, so far no attempt has been made to study the effect of Hall current on the free convection flow past an impulsively started vertical porous plate in the presence of a uniform transverse magnetic field, taking into account viscous and Joule's dissipations. The main focus of the paper is to gain physical insights and hence the nonlinear problem considered here is solved using a simple and straight forward explicit finite difference method. An attempt has been made to provide physical reasoning or justification wherever possible. Researchers interested in rigorous mathematical methods to solve this problem may refer to a related problem addressed in [32] and adopt a similar procedure. However, it should be noted that the perturbation technique employed in the cited work is cumbersome and time-consuming.

II. FORMULATION OF THE PROBLEM

An unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past an impulsively started infinite vertical porous plate, in the presence of a transverse magnetic field with the effect of Hall current is considered. The initial temperature of the fluid is the same as that of the fluid, but at time $t'>0$ the porous plate starts moving impulsively in its own plane with a constant velocity U_0 and its temperature instantaneously rises or falls to $T_w^{'}$ which thereafter is maintained as such. The fluid is assumed to have constant properties except that the influence of the density variations with the temperature, following the well-known Boussinesq approximation [36] is considered only in the body force terms.

The x'-axis is taken along the infinite vertical porous wall in the upward direction and y'-axis normal to the wall. A constant magnetic field of magnitude $B_0^{'}$ is applied in y'- direction. Since the effect of Hall current gives rise to a force in z' direction, which induces a cross flow in that direction, the flow becomes three dimensional. Let u', v' and w' denote the velocity components in the x', y' and z' directions respectively. Let $v_0^{'}$ be the constant suction velocity.

The governing equations of the problem are as follows:

$$\nabla' \cdot \vec{q}' = 0 \tag{1}$$

$$\frac{\partial \vec{q}'}{\partial t'} + (\vec{q} \cdot \nabla')\vec{q}' = -\frac{1}{\rho} + \vec{g}\,\beta(T' - T'_\infty) + \frac{1}{\rho}(\vec{j}' \times \vec{B}') \tag{2}$$

$$\rho C_p \left[\frac{\partial T'}{\partial t'} + (\vec{q}' \cdot \nabla')T'\right] = k\nabla'^2 T' + \mu\,\Phi' + \frac{\vec{j}'^2}{\sigma} \tag{3}$$

$$\vec{j}' = \sigma\left[\vec{E}' + \vec{q}' \times \vec{B}'\right] - \frac{\sigma}{e\, n_e}\left[\vec{j}' \times \vec{B}' - \nabla' p'_e\right] \quad (4)$$

$$\nabla' \cdot \vec{E}' = -\frac{\partial \vec{B}'}{\partial t'} \quad (5)$$

$$\nabla' \cdot \vec{B}' = 0 \quad (6)$$

The above equations are equation of continuity, momentum equation, energy equation, generalized Ohm's Law and the two Maxwell's equations respectively. Here, $\vec{q}'$ is the velocity field, T' the temperature field, T'_∞ the temperature of the fluid at infinity, $\vec{B}'$ the magnetic induction vector, $\vec{E}'$ the electric field vector, $\vec{j}'$ the current density vector, p' the pressure of the fluid, p'_e the electron pressure, ρ the density of the fluid, μ the coefficient of viscosity, ν the kinematic coefficient of viscosity, k the thermal conductivity, e the electron charge, n_e number density of electron, C_p the specific heat capacity at constant pressure, Φ' the viscous dissipation function and t' the time. The viscous dissipation function, for an incompressible fluid, is given by

$$\Phi' = 2\left[\left(\frac{\partial u'}{\partial x'}\right)^2 + \left(\frac{\partial v'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial z'}\right)^2\right] + \left(\frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'} + \frac{\partial v'}{\partial z'}\right)^2 + \left(\frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'}\right)^2 \quad (7)$$

The term $\frac{\vec{j}'^2}{\sigma}$ in the energy equation is the Joule's dissipation.

The magnetic Reynolds number is considered to be small and hence the induced magnetic field is neglected in comparison to the transversely applied magnetic field $\vec{B}' = B'_0\, \hat{j}$, which is assumed to be uniform [37]. Further, since no external electric field is applied, and the effect of polarization of ionized fluid is negligible, it can be assumed that the electric field is zero. As the plate is infinite, all variables in the problem are functions of y' and t' only. Hence, by the usual considerations of the impulsively started vertical flat plate problem, the basic equations become

$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\,\beta(T' - T'_\infty) - \frac{\sigma B_0'^2 u'}{\rho(1+m^2)}(u' + m w') \quad (8)$$

$$\frac{\partial w'}{\partial t'} - v_o \frac{\partial w'}{\partial y'} = \nu \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0'^2 u'}{\rho(1+m^2)}(m u' - w') \quad (9)$$

$$\rho C_p\left(\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'}\right) = k \frac{\partial^2 T'}{\partial y'^2} + \mu\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right] + \frac{\sigma B_0'^2}{\rho(1+m^2)}\left(u'^2 + w'^2\right) \quad (10)$$

where $m = \frac{\sigma B'_0}{e n_e}$ is the Hall current parameter. Obviously, $v' = v'_o$.

The physical quantities are cast in the non-dimensional form by using the following dimensionless scheme:

$$u = \frac{u'}{U_0}, \quad w = \frac{w'}{U_0}, \quad y = \frac{y' v_0}{\nu}, \quad t = \frac{t' v_0^2}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \quad (11)$$

Hence, the governing equations of the problem in the non-dimensional form are given by

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G\theta - \frac{M}{(1+m^2)}(u + m w) \quad (12)$$

$$\frac{\partial w}{\partial t} - \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + \frac{M}{(1+m^2)}(mu - w) \quad (13)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P}\frac{\partial^2 \theta}{\partial y^2} + \frac{ME}{(1+m^2)}(u^2 + w^2) + E\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \quad (14)$$

Here, $G = \frac{\nu g \beta (T'_w - T'_\infty)}{v_0^2 U_0}$ is the Grashof number, $M = \frac{\sigma B_0'^2 \nu}{v_0^2 \rho}$ the square of the Hartmann number, $P = \frac{\mu C_p}{k}$ the Prandtl number and $E = \frac{U_0^2}{C_p (T'_w - T'_\infty)}$ is the Eckert number.

It is evident that the initial and boundary conditions are given by

$u(y,t) = w(y,t) = \theta(y,t) = 0$ for $t = 0$ and
$u(y,t) = 1$; $w(y,t) = 0$; $\theta(y,t) = 1$ for $y = 0$ and $t > 0$
and $u(y,t) = 0$; $w(y,t) = 0$; $\theta(y,t) = 0$ for $y \to \infty$ and $t > 0$. (15)

III. SOLUTION OF THE PROBLEM

It is not possible to find the exact solutions of the coupled non-linear partial differential equations (12) to (14) subject to the initial and boundary conditions (15) and hence the numerical solutions of the above equations are obtained by explicit finite difference scheme. The set of finite difference equations corresponding to equations (12) to (14) are:

$$u(j,k+1) = u(j,k) + \frac{\Delta t}{\Delta y^2}\left[u(j+1,k)-2u(j,k)+u(j-1,k)\right]$$

$$+\Delta t\left[G\theta(j,k)-\frac{M}{(1+m^2)}\{u(j,k)+mw(j,k)\}+\frac{u(j,k)-u(j-1,k)}{\Delta y}\right] \quad (16)$$

$$w(j,k+1) = w(j,k) + \frac{\Delta t}{\Delta y^2}\left[w(j+1,k)-2w(j,k)+w(j-1,k)\right]$$

$$+\Delta t\left[\frac{M}{(1+m^2)}\{mu(j,k)-w(j,k)\}+\frac{w(j,k)-w(j-1,k)}{\Delta y}\right] \quad (17)$$

$$\theta(j,k+1) = \theta(j,k) + \frac{\Delta t}{P\Delta y^2}\left[\theta(j+1,k)-2\theta(j,k)+\theta(j-1,k)\right]$$

$$+\frac{\Delta t}{\Delta y}\left[\theta(j,k)-\theta(j-1,k)\right]+\frac{ME\Delta t}{1+m^2}\left[w^2(j,k)+u^2(j,k)\right]$$
$$+\frac{E\Delta t}{\Delta y^2}\left[\{u(j+1,k)-u(j,k)\}^2+\{w(j+1,k)-w(j,k)\}^2\right] \quad (18)$$

where the index j and k correspond to the variables y and t respectively.

During computations, Δy is taken as 0.1 and Δt is taken as 0.002. The initial conditions in equation (15) take the form $u(j,1)=0$, $w(j,1)=0$ and $\theta(j,1)=0$. The boundary conditions at $y=0$ in equation (15) become $u(1,k)=1$, $w(1,k)=0$ and $\theta(1,k)=1$.

Depending upon the closed form solution obtained in [4] using Laplace Transform method for the hydromagnetic free convection flow of an electrically conducting fluid, with Prandtl number unity, past an impulsively started infinite vertical porous plate with constant suction, $y\rightarrow\infty$ is approximated by $y=4.1$. This approximation was also earlier used in [5] and [6]. Thus, the boundary conditions in equation (15) for $y\rightarrow\infty$ reduce to $u(42,k)=0$, $w(42,k)=0$ and $\theta(42,k)=0$.

The $x-$ component of the velocity at the end of a time step, namely $u(j,k+1)$ for $j=1$ to $j=42$, is computed using equation (16) in terms of velocity components and temperature at gird points on the earlier time steps. Similarly, the values of $w(j,k+1)$ and $\theta(j,k+1)$ are computed using the equations (17) and (18) respectively. This procedure is repeated till $t=0.4$, that is $k=201$.

To judge the accuracy of the results, under appropriate limits, the numerical solutions obtained are compared with the analytical solutions of [4] and [32] at $t=0.2$ and $t=0.4$ for the entire range of y values. The agreement was excellent with a maximum error of less than 5%. Further, to test the convergence part, the program was also run with smaller values of Δt , say $\Delta t=0.001$. There were no significant changes in the results, thus establishing the fact that the finite difference scheme is independent of the mesh size for $\Delta t<0.002$.

IV. RESULTS AND DISCUSSION

The results of our present investigation reduce to those of [5], in the absence of Hall current, when viscous and Joule's dissipation are neglected. The results of [4] can also be recovered from our results by appropriate choice of values for the physical parameters. When our numerical results were compared with the earlier analytical work like [4] and [32], there was an excellent agreement with a maximum error of less than 5%, as stated above. Qualitatively, the results obtained are in good agreement with the earlier works, like those of [7] and [17]. The quantitative differences arise because of different initial and boundary conditions used owing to different physical situations considered. Graphical illustrations of such comparisons are not presented due to paucity of space. For further details on the accuracy of the present numerical method, one may refer to [17].

It must be noted that negative values of the parameters G and E correspond to the case of the plate being heated by the convection currents and similarly their positive values correspond to the case of the plate being cooled by the convection currents. We refer to the values of $G=\pm 0.9$ as moderate cooling and heating and $G=\pm 5$ as greater cooling and greater heating respectively. In the following discussion, the value of the non-dimensional time is fixed at $t=0.4$. The value of Prandtl number is taken as 0.71 which corresponds to air. Air is assumed to be incompressible since all the velocities considered are less than the velocity of sound in the medium (air) so that the Mach number is less than unity [38]. The values for the various parameters are chosen approximately to correspond to a sufficiently ionized air, the flow of which can be modified by an applied magnetic field.

Figures 1 and 2 show that the primary velocity u is diminished and the secondary velocity w is increased due to the applied magnetic field. This is in good agreement with the results of [13]. Indeed, when a transverse magnetic field is applied it is well known that the Lorentz force acts in a direction opposite to the flow and offers resistance to the flow and such a phenomenon is described by the term "magnetic-viscosity".

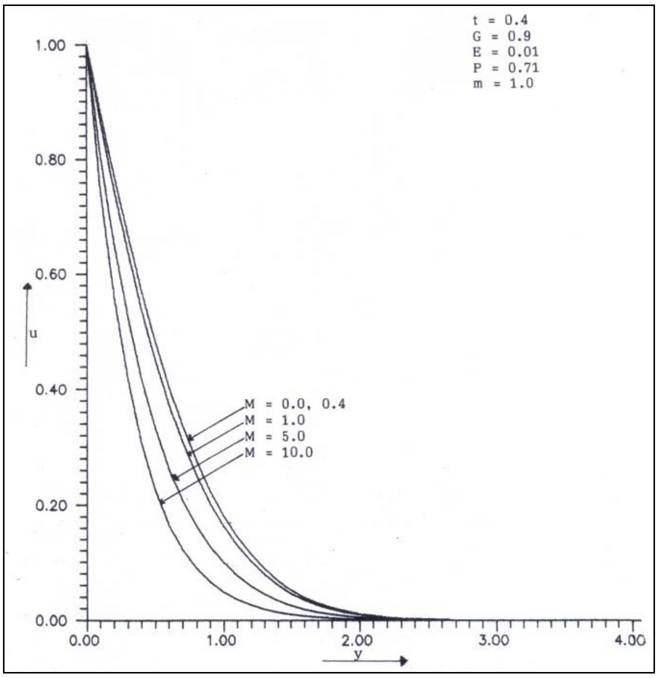

Fig.1. Effect of Magnetic Field on the Primary Velocity

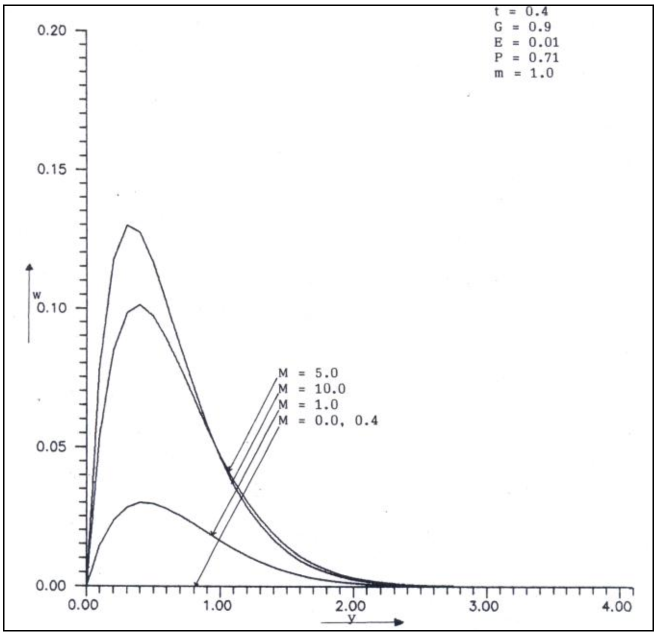

Fig.2. Effect of Magnetic Field on the Secondary Velocity

From Figs. 3 and 4, it is inferred that the Hall current promotes the flow along the plate, both when the fluid is heated or cooled. This is because, in general, the Hall current reduces the resistance offered by the Laurentz force.

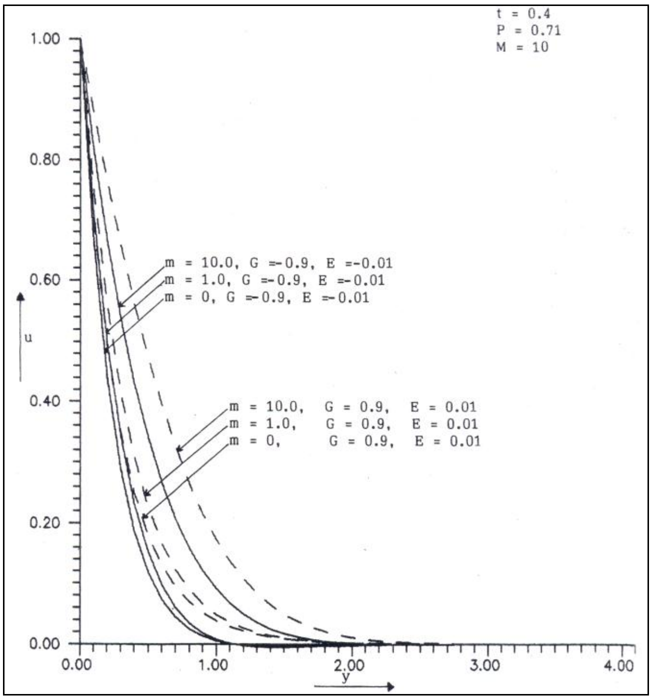

Fig.3. Effect of Hall Current on the Primary Velocity
(For cooling and heating of the plate)

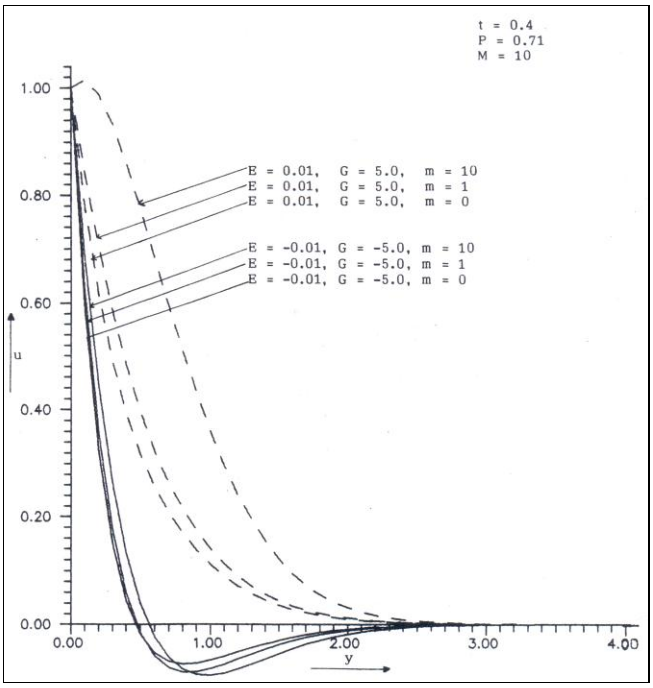

Fig.4. Effect of Hall Current on the Primary Velocity
(For greater cooling and heating of the plate)

It is also observed that the primary velocity u is greater in the case of cooling of the plate than in the case of heating of the plate. Flow reversal is also noticed in the case of greater heating of the plate. The rise and fall in velocity due to cooling and heating of the plate can be explained as follows.

In the process of external cooling of the plate, the free convection currents travel away from the plate. As the fluid is also moving with the plate in the upward direction, the convection currents tend to help the velocity to increase. But, in the case of heating of the plate, as the free convection currents are traveling towards the plate, the motion is opposed by these currents and hence there is a decrease in velocity. In the case of greater heating, this opposition is large enough to counteract the upward push offered by the movement of the plate on the fluid particles just outside the thermal boundary layer and the net force acts downwards and hence the flow becomes downwards.

The effect of Hall current on the secondary velocity w is depicted through Figs. 5 and 6. The secondary velocity is induced by the component of the Lorentz force in the z direction which arises solely due to the Hall current. From equation (13), it is clear that $\frac{M\,m}{(1+m^2)}u$ is the term which decides the flow in the z direction. If the Hall parameter $m=0$, then the term mentioned above is zero and hence there is no force to induce the flow in the z direction. That is, $w \equiv 0$. Further, $\frac{m}{(1+m^2)}$ increases as m increases in the range $0 \le m \le 1$ and it decreases as m increases in the range $m>1$. This means that the magnitude of the component of the Lorentz force in the z direction increases as m increases in the range $0 \le m \le 1$ and hence the secondary velocity w is increased, while it decreases when m increases in the range $m>1$ and hence the secondary velocity w is decreased. These results are observed graphically in Figs. 5 and 6.

The secondary velocity w is observed to be greater in the case of cooling of the plate than in the case of heating of the plate. This result also can be inferred from the term $\frac{M\,m}{(1+m^2)}u$ of equation (13). The primary velocity u is greater in the case of cooling than in the case of heating of the plate and so is the term mentioned above, which is the deciding factor so far as the secondary velocity is concerned. This ultimately results in the secondary velocity being greater in the case of cooling of the plate than in the case of heating of the plate. By similar arguments, the flow reversal observed in the secondary velocity in the case of greater heating can also be attributed to the flow reversal in the primary velocity.

The temperature θ is not significantly affected by the magnetic field and Hall current, except in the very close vicinity of the plate. This is because the effect of the Hartmann number and Hall parameter can be felt only in the Hartmann layer and the thermal boundary layer respectively.

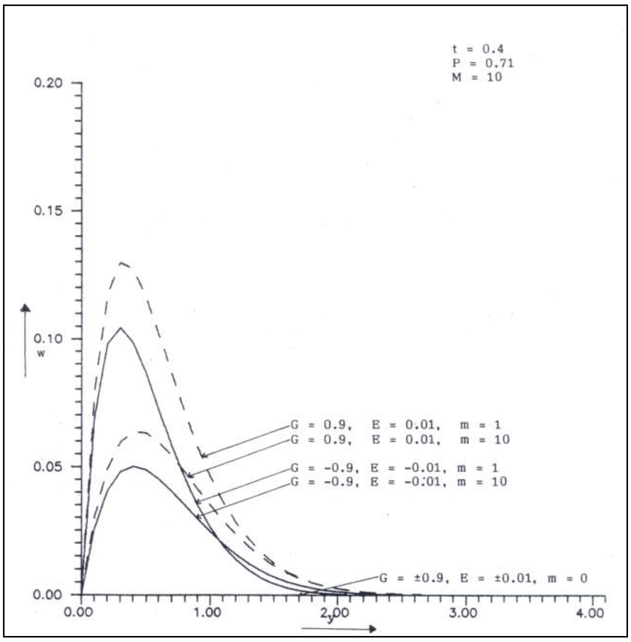

Fig.5. Effect of Hall Current on the Secondary Velocity
(For cooling and heating of the plate)

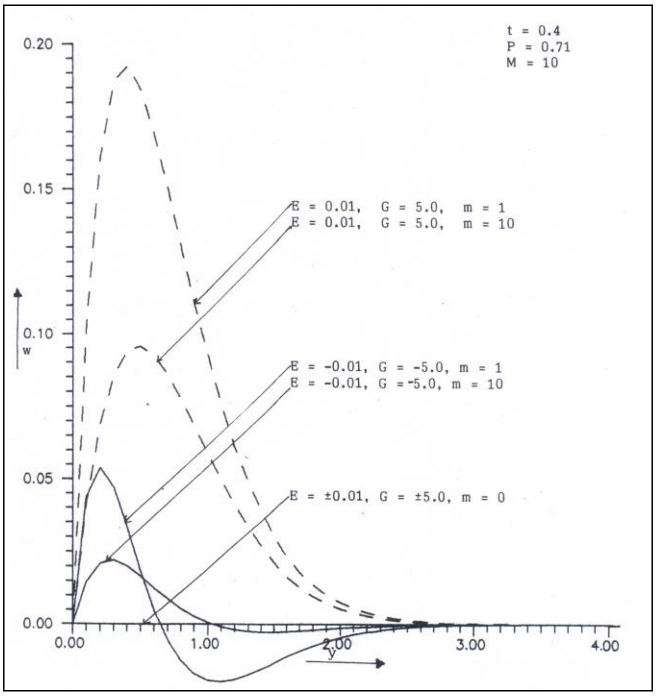

Fig.6. Effect of Hall Current on the Secondary Velocity
(For greater cooling and heating of the plate)

Hence, graphical illustrations of the effect of these parameters on the temperature are not presented. However, appreciable changes in the slope of the temperature profiles very close to the plate, $y=0$, were observed and hence the effect of these parameters on rate of heat transfer are presented below.

The skin friction $\tau = -\left[\frac{\partial u}{\partial y}\right]_{y=0}$ and the rate of heat transfer $q = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0}$ are calculated by numerical differentiation using Newton's forward Interpolation formula. During computation of the above quantities, the non-dimensional time is fixed at $t = 0.4$.

TABLE 1. Values of skin friction and rate of heat transfer for air $(P = 0.71)$ at time $t = 0.4$

G	E	M	m	Skin Friction τ	Rate of Heat Transfer q
-0.9	0.00	0.4	0	1.902441	1.190122
-0.9	0.00	0.4	1	1.902441	1.190122
-0.9	-0.01	0.4	0	1.902073	1.195809
-0.9	-0.01	0.4	1	1.902073	1.195809
-0.9	-0.01	10	0	4.015853	1.206905
-0.9	-0.01	10	1	3.332132	1.203572
-0.9	-0.01	10	10	1.953934	1.196097
0.9	0.00	0.4	0	1.739660	1.190122
0.9	0.00	0.4	1	1.739660	1.190122
0.9	0.01	0.4	0	1.736100	1.185873
0.9	0.01	0.4	1	1.736100	1.185873
0.9	0.01	10	0	3.571448	1.173194
0.9	0.01	10	1	2.826338	1.176920
0.9	0.01	10	10	1.234781	1.185530
-5.0	-0.01	10	0	5.026522	1.207684
-5.0	-0.01	10	1	4.482647	1.205384
-5.0	-0.01	10	10	3.590076	1.200920
5.0	0.01	10	0	2.556977	1.171634
5.0	0.01	10	1	1.671793	1.176159
5.0	0.01	10	10	-0.405391	1.186781

It is observed that the Hartmann number M increases the skin friction τ, irrespective of whether the plate is heated or cooled, both in the presence and absence of Hall current. This is because the effect of transverse magnetic field is to retard the flow by offering additional resistance called magnetic viscosity.

It is noticed that, both in the presence and absence of Hall current, due to magnetic field, the rate of heat transfer q increases when the plate is heated and it decreases when the plate is cooled.

However, the effect of Hall current is to decrease the skin friction in both the cases of heating and cooling of the plate. This is because the effect of Hall current is to decrease the resistance offered by the Lorentz force. Finally, the effect of Hall current is to decrease the rate of heat transfer when the plate is heated and increase the same when the plate is cooled.

V. CONCLUSION

At the outset, our numerical results are in good agreement with those of [5] in the absence of Hall current, when the viscous and Joule's dissipations are neglected. In the non-magnetic case, our results reduce to that of [18]. Qualitatively and quantitatively our results are in good agreement with the earlier analytical results reported in [4] and [32]. The effects of magnetic field and Hall current on the flow and heat transfer are analyzed and physical interpretations or justifications of the results are provided as and when possible. The results obtained can be summarized as follows.

- Applied magnetic field retards the primary flow along the plate and supports the secondary flow induced by the Hall current.
- Hall current promotes the flow along the plate. The secondary flow is supported when the Hall parameter is increased up to unity. If the Hall parameter is increased beyond unity, the secondary flow is retarded. These results are true for both cooling and heating of the plate.
- Both primary and secondary velocities are found to be greater in the case of cooling of the plate than in the case of heating of the plate.
- Flow reversal is observed in both primary and secondary velocity components in the case of greater heating of the plate.
- Magnetic field and Hall current modify only the slope of the temperature profile in the narrow region close to the plate called the thermal boundary layer. Otherwise, their effect on temperature is not significant.
- Skin friction is increased by the magnetic field.
- The effect of magnetic field is to increase the heat transfer rate when the plate is heated and decrease it when the plate is cooled.
- Hall current decreases the skin friction.
- Due to Hall current the heat transfer rate decreases when the plate is heated and it increases when the plate is cooled.

To improve upon the present work, it is suggested that one may consider the effect of rotation. Further, in order to investigate supersonic flows one may consider the effects of compressibility. It is also suggested that the atmosphere may be considered as a stratified fluid. Obviously, these suggestions will lead to more complex problems but are worth investigating.

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