

Transportation Problem: South-East Corner Method and a Comparative Study on the North-West Corner, South-East Corner, North-East Corner and South-West Corner Methods

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Abstract- The North – West Corner Method (NWCM), the South – East Corner Method (SECM), the North – East Corner Method (NECM) and the South – East Corner Method (SECM), are adopted to compute the Initial Basic Feasible Solution (IBFS) of the transportation problem. In this paper, after giving the procedure of the SECM, we show that the NWCM and the SECM lead to the same solution, as well as the NECM and the SWCM.

Keywords- NWCM, SECM, NECM, SECM, IBFS, Transportation Problem

I. INTRODUCTION

The linear program to minimize the transportation costs from different origins to the different destinations in respecting the constraints of availability and demand, is called the transportation problem. In this problem, the availability can be equal to the demand (balanced problem), the availability may be superior to the demand and the availability may be less than the demand. One of the first and important applications of the linear programming techniques, was the formulation and the solution of the transportation problem. The basic transportation problem was originally stated by Hitchcock [1] and later discussed in detail by Koopman [2]. An earlier approach was provided by Kantorovich [3]. The linear programming formulation and the associated systematic method for solution were first given in Dantzig [4]. The recent approaches were respectively given by Polaniyappa and Venoba [5], Lakshmi & Anantha [6] and Anantha & Lakshmi [7]. The general transportation problem can be represented in a table form [8, 9] with mn cells:

TABLE I. TABLE OF THE GENERAL TRANSPORTATION PROBLEM

Destinations Origins	D_1	D_2	...	D_j	...	D_n	Supply : a_i
O_1	c_{11}	c_{12}	...	c_{1j}	...	c_{1n}	a_1
O_2	c_{21}	c_{22}	...	c_{2j}	...	c_{2n}	a_2
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
O_i	c_{i1}	c_{i2}	...	c_{ij}	...	c_{in}	a_i
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
O_m	c_{m1}	c_{m2}	...	c_{mj}	...	c_{mn}	a_m
Demand : b_j	b_1	b_2	...	b_j	...	b_n	$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

Where a_i is the quantity of commodities available at the origin i , b_j is the quantity of commodities requested at the destination j and c_{ij} is the transportation cost from the origin i to the destination j . A set of non negative values, $i = 1, \dots, m$; $j = 1, \dots, n$; that satisfies the constraints is called a feasible solution to the transportation problem [8, 10, 11].

A feasible solution is said to be optimal if it minimizes the total transportation cost. A non-degenerate basic feasible solution is a basic feasible solution to a $(m \times n)$ transportation problem that contains exactly $m+n-1$ allocations in independent positions.

II. METHODOLOGY

Followings steps are involved in the SECM:

Step 1: Draw the general transportation problem table and verify that the problem is balanced.

Step 2: For the south-east corner, take $x_{mn} = \min(a_m, b_n)$.

Step 3: If $x_{mn} = a_m$, then the row m is deleted. Replace b_n by $b_n - a_m$.

If $x_{mn} = b_n$, the column n is deleted. Replace a_m by $a_m - b_n$.

Step 4: If $a_m = b_n$, then $x_{mn} = a_m = b_n$: the row m and the column n are deleted. We have a degenerate basic feasible solution.

Step 5: A new matrix of order $(m-1) \times n$, or $m \times (n-1)$ or $(m-1) \times (n-1)$. These are reduced matrices.

Repeat steps 1-3 till all quantities are exhausted.

For the NWCM, the NECM and the SWCM we begin respectively with the north-west cell [6, 8], the north-east cell [10] and the south-west cell [7]. For those NWCM, NECM, SWCM and SECM, the costs c_{ij} are not necessary to find an IBFS.

III. ILLUSTRATION, RESULTS AND DISCUSSIONS

Find the IBFS to the following transportation problem by the SECM :

TABLE II. DATA OF THE TRANSPORTATION PROBLEM FOR THE SECM

$O_i \backslash D_j$	D_1	D_2	D_3	D_4	a_i
O_1					6
O_2					8
O_3					10
b_j	4	6	8	6	$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 24$

It is a balanced transportation problem as:

$$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 24.$$

At the beginning, we have a matrix of order 3×4 .

$$x_{34} = \min(a_3, b_4) = \min(10, 6) = 6;$$

$$b_4 = 6 - 6 = 0 \text{ and we replace } a_3 \text{ by } a_3 - b_4 = 10 - 6 = 4.$$

The column 4 is deleted and a new matrix of order 3×3 will appear. $x_{33} = \min(a_3, b_3) = \min(4, 8) = 4;$

$$a_3 = 4 - 4 = 0. \text{ We replace } b_3 \text{ by } b_3 - a_2 = 8 - 4 = 4.$$

The row 3 is deleted and a new matrix of order 2×3 will appear.

$$x_{23} = \min(a_2, b_3) = \min(8, 4) = 4;$$

$$b_3 = 4 - 4 = 0. \text{ We replace } a_2 \text{ by } a_2 - b_3 = 8 - 4 = 4.$$

The column 3 is deleted and a new matrix of order 2×2 will appear.

If we follow this logic, we find $x_{22} = 4, x_{12} = 2$ and $x_{11} = 4$.

In short, for the SECM, we have the following table:

TABLE III. TRANSPORTATION PROBLEM FOR THE SECM

$O_i \backslash D_j$	D_1	D_2	D_3	D_4	a_i
O_1	$x_{11} = 4$	$x_{12} = 2$			6
O_2		$x_{22} = 4$	$x_{23} = 4$		8
O_3			$x_{33} = 4$	At first $x_{34} = 6$	10
b_j	4	6	8	6	$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 24$

We get a non-degenerate basic feasible solution. Using the NWCM [6, 8], we get the next table:

TABLE IV. TRANSPORTATION PROBLEM FOR THE NWCM

$O_i \backslash D_j$	D_1	D_2	D_3	D_4	a_i
O_1	At first $x_{11} = 4$	$x_{12} = 2$			6
O_2		$x_{22} = 4$	$x_{23} = 4$		8
O_3			$x_{33} = 4$	$x_{34} = 6$	10
b_j	4	6	8	6	$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 24$

This gives us a non-degenerate basic feasible solution.

Therefore, the SECM and the NWCM yield the same result.

With the NECM [10] we have the next table:

TABLE V. TRANSPORTATION PROBLEM FOR THE NECM

$O_i \backslash D_j$	D_1	D_2	D_3	D_4	a_i
O_1				At first $x_{14} = 6$	6
O_2			$x_{23} = 8$		8
O_3	$x_{31} = 4$	$x_{32} = 6$			10
b_j	4	6	8	6	$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 24$

We get a degenerate basic feasible solution. Using the SWCM [7], we get the next table:

TABLE VI. TRANSPORTATION PROBLEM FOR THE SWCM

$O_i \backslash D_j$	D_1	D_2	D_3	D_4	a_i
O_1				$x_{14} = 6$	6
O_2			$x_{23} = 8$		8
O_3	At first $x_{31} = 4$	$x_{32} = 6$			10
b_j	4	6	8	6	$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 24$

This gives us a degenerate basic feasible solution. Thus, the NECM and the SWCM give the same result.

IV. CONCLUSION

In this paper, we gave the SECM which is a new method used in solving transportation problem. For this SECM, we proved that the values of x_{ij} move, on or around the diagonal from the cell x_{mn} to the cell x_{11} . We also showed that, on the one hand the SECM and NWCM, and on the other hand the SECM and NECM, always have the same IBFS.

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