*Journal of Naval Science and Engineering*
*2010, Vol. 6 , No.1, pp. 1-14*

# EFFICIENCY MEASURES IN THE HEALTH SERVICES WITH DEA - AN OVERVIEW

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## Abstract

*Efficiency which could be related to the performance of the processes is one of the main concerns of the organizations. It is therefore important to measure and perform continuous improvement in the efficiency of the processes. This is also valid for the health services as in all others. In literature, there have been several modeling and assessment approaches developed for this purpose. This paper presents a review of the respective researches with related findings in field. Main focus of the review is given to data envelopment analysis which one of the very popular assessment tool for decision making systems. In addition, the review is extended not only to the modeling approaches but also respective software tools as well as strengths and weaknesses of this technique.*

# SAĞLIK HİZMETLERİNDE VZA İLE ETKİNLİK ÖLÇÜMÜ- BİR GENEL BAKIŞ

**Özetçe**

*Süreçlerin performansı ile ilgili olabilen etkinlik organizasyonların önemli ilgi alanlarından biridir. Bu nedenle, süreçlerin etkinliklerinin ölçüm ve sürekli gelişimlerinin sağlanması önem arz etmektedir. Bu durum, diğer alanlarda olduğu kadar sağlık hizmetleri içinde geçerlidir. Bu amaçla literatürde birçok modelleme ve değerlendirme yaklaşımları geliştirilmiştir. Bu çalışmada bahse konu alanla ilgili araştırma ve ilgili sonuçlarını özetlemektedir. Çalışmada temel olarak, karar vermede yaygın bir değerlendirme aracı olan veri zarflama analizine odaklanılmıştır. Bunlara ek olarak çalışmada modelleme yaklaşımlarının yanı sıra, ilgili yazılım ve yöntemin kuvvetli/ zayıf tarafları da gösterilmiştir.*

**Keywords:** Efficiency Measurement, Data Envelopment Analysis, Health Services
**Anahtar Kelimeler:** Etkinlik Ölçümü, Veri Zarflama Analizi (VZA), Sağlık Hizmetleri

## 1. INTRODUCTION

Efficiency has been the subject of research in a wide range of production activities. It is expressed as a percentage which can calculate as the ratio total output power to total input power under specified conditions. Efficiency analysis has always been linked to the relative difficulty encountered is assessing the performance of decision-making units (DMUs) [1] to find its weakness so that subsequent improvements can be made.

Modeling efficiency measurement is a non-parametric way was introduced first by Farrell (1957) [2] including that measurement of price and technical efficiencies and the derivation of the efficient production function. Method summarizes all inputs and outputs into a single virtual input and single virtual output [3].

Efficiency Measurement techniques in general consist of four classes:

- Parametric
- Non-parametric
- Deterministic
- Stochastic.

Each set of techniques has its own strengths and weaknesses. Parametric techniques are regression-based approaches in general, and assume a specific functional form for the frontier, where as non-parametric techniques do not [4]. Parametric techniques are susceptible to model misspecification, as the efficiency scores are sensitive to distributional assumptions regarding the error term [4, 5].

Deterministic methods do not contain random error component. Hence, they may be sensitive to extreme observations since they assume that the observed distance to the frontier is due to inefficiency. Stochastic methods are less sensitive to outliers since part of the distance to the frontier can be attributed to random error [4].

Most relevant applications deal with activities in which some outputs and / or inputs are intangible, and therefore, the efficiency analysis of such activities can hardly be performed by aggregating benefits and costs in accounting terms [6].

## 2. EFFICIENCY MEASUREMENT OF HEALTH SERVICES

The first study of hospital efficiency studies sought to assess hospital efficiency by concentrating on the development of single productivity indices [1, 7]. Econometric theory helped to appreciate the multi-product nature of hospital outcomes and became possible to formulation of multiple inputs and outputs in the assessment of efficiency [1, 8, and 9]. Donabedian *et al.* [32, 33] stated that efficiency in health care can be categorized in to two

I. *Production efficiency* (i.e., how services are produced);

II. *Clinical efficiency* (i.e.; how services are integrated with efficient strategies of care).

The inputs for the efficiency measurement usually are (but not limited) labour (doctors, nurses, physicians, housekeepers, and capital inputs and etc.) and capital inputs (land, buildings, medical equipment and etc.) [33]. Optimization based methodologies such development of performance indicators is improved [1, 10]. Today both parametric and non-parametric are used to measure production efficiency related to the utilization of available resources such as facilities, technologies and workforce [1, 11, and 12].

## 3. DATA ENVELOPMENT ANALYSIS

Data envelopment analysis (DEA) is an empirically based methodology that eliminates the need for some of the assumptions and limitations of traditional efficiency measurement approaches. The basic DEA model as introduced by Farrell in 1957 and later developed by Charnes, Cooper and Rhodes (CCR Model) uses an oriented radial measure of efficiency, which identifies a point on the boundary with the same mix of inputs (input orientation) or outputs (output orientation) of that of the observed unit [1].

DEA is a technique to measure relative efficiency of a set of decision-making units (DMUs) having similar multiple inputs to produce similar multiple outputs. The relative efficiency of a DMU is defined as the ratio of the sum of its weighted outputs, to the sum of its weighted inputs. The objectives are to identify units that are relatively inefficient and setting targets for them based on examining the operational practices of the units classified as efficient. The underlying concept of DEA is based on Pareto optimality (Charnes et. al., 1985). A DMU is considered relatively efficient if there is no other DMU or a combination of DMUs which can produce at least the same amount of all outputs with less of one input and not more of any other input [13]. It computes the comparative ratio of outputs to inputs for each unit, with the score expressed as 0–1 or 0–100%. A DMU with a score less than 100% is inefficient compared to other units. It is used to identify best practices and is increasingly becoming a popular and practical management tool. DEA has been initially used to investigate the relative efficiency of nonprofit organizations but now, its use has spread to hospitals, school, banks, and network industries, among others.

In the first stage, DEA assesses efficiency by estimating a frontier based on input or output orientation. Then, each DMU is assigned an efficiency score by comparing the output and input ratio of the DMU on the efficient frontier. Mathematically, technical efficiency for each DMU is computed as [14]:

Consider a system under evaluation, consisting of n DMUs. The inputs and outputs of every DMU are all nonnegative and every DMU has at least one positive input and one positive output, i.e., $x \geq 0$, $x \neq 0$ and $y \geq 0$, $y \neq 0$. Then, the economic efficiency of DMU is defined as follows:

$$Efficiency = \frac{\sum_{r=i}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_i} = \frac{\text{Weighted sum of inputs of DMU}_\text{p}}{\text{Weighted sum of outputs of DMU}_\text{p}} \tag{1}$$

In this case, the DMUs can be easily compared. However, since the input costs and output prices are not always precisely available, DEA models are generally utilized for this purpose [15].

It can be solved by one of two linear programming formulations. The first formulation maximizes the outputs that can be obtained and constrains the sum of the inputs to be unity [11].

$$\max \quad \frac{\sum_{r=i}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_o}$$

*st*

$$\frac{\sum_{r=i}^{s} u_r y_{ri}}{\sum_{i=1}^{m} v_i x_{ii}} \leq 1 \quad \text{for j = 1,..., n.}$$

$$u_r \geq 0, v_i \geq 0 \text{ for r = 1,..., s; i = i,..., m}$$

This linear fractional programming model is called the CCR ratio model, which can be linearized using Charnes-Cooper's transformations as follows [15]:

$$\max \quad \sum_{r=1}^{s} \mu_r y_{r0}$$

*st*

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \text{for j=1,..,n,}$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\mu_r \geq 0, v_i \geq 0 \quad \text{for} \quad r = 1,\ldots,s; i=1,\ldots,m.$$

xi are inputs, yi are outputs and u and v are scalar values chosen for each production unit such that the efficiencies of each unit are maximized and no efficiencies are greater than one.

The second formulation minimizes the inputs needed and constrains the sum of the weighted output at unity. The adopted DEA model represents the dual of the first linear programming formulation. The linear programming dual is expressed as the following [11]:

$$\min \theta$$

$$\begin{aligned} & st \\ & \sum_{j=1}^{n} x_{ij}\lambda_j \le \theta\, x_{i0} \quad \text{for } i = 1,\ldots,m, \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{r0} \quad \text{for } r = 1,\ldots,s, \\ & \lambda_j \ge 0 \quad \text{for} \quad j = 1,\ldots,n. \end{aligned}$$

where; $\theta_0$ represents the radial efficiency of $DMU_0$ and $\lambda_j$ is the weight placed by DMU0 on DMUj. DMUs for which the optimal solution $\theta<1$ are inefficient.

For some applications, the goal is to increase outputs for a fixed quantity of inputs. The corresponding dual form of the output-oriented model is given by

$$\max \phi$$

$$\begin{aligned} & \text{st} \\ & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{i0} \quad \text{for } i = 1,\ldots,m, \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \ge \phi\, y_{r0} \quad \text{for } r = 1,\ldots,s, \\ & \lambda_j \ge 0 \quad \text{for} \quad j = 1,\ldots,n. \end{aligned}$$

Here, DMUs for which the optimal solution $\Phi>1$ are inefficient. The vast majority of DEA studies in health care use the input-oriented model since the goal is to reduce costs rather than increase the volume of services provided.

### 3.1 DEA Models

The CCR model was introduced by Charnes, Cooper and Rhodes in 1978, which became the basis for a branch of operations research called data envelopment analysis (DEA). After the introduction of the CCR model, other models such as the BCC, RAM, SBM, additive, FDH, models were introduced to enrich DEA [15]. Table 1-1 presents the CCR model in input- and output versions, each in the form of a pair of dual linear programs.

Table 1: CCR DEA Model

| Input Oriented | |
|---|---|
| Envelopment Model | Multiplier Model |
| $\min \theta - \varepsilon(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+)$ | $\max Z=\sum_{r=1}^{s} \mu_r y_{r0}$ |
| subject to | subject to |
| $\sum_{j=1}^{n} x_{ij}\lambda_j + S_i^- = \theta x_{i0}$ For i =1,2,…,m | $\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0$ |
| $\sum_{j=1}^{n} y_{rj}\lambda_j - S_r^+ = y_{r0}$ | $\sum_{i=1}^{m} v_i x_{i0} = 1$ |
| For r = 1,…,s, | |
| $\lambda_j \geq 0$ for j = 1,…,n. | $\mu_r, v_i \geq \varepsilon > 0$ |

Table 1(continued): CCR DEA Model

| Input Oriented | |
|---|---|
| Envelopment Model | Multiplier Model |
| $\max\ \phi + \varepsilon\left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+\right)$ | $\min q = \sum_{i=1}^{m} v_i x_{i0}$ |
| subject to | subject to |
| $\sum_{j=1}^{n} x_{ij}\lambda_j + S_i^- = x_{i0}$ | $\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0$ |
| For i =1,2,…,m | |
| $\sum_{j=1}^{n} y_{rj}\lambda_j - S_r^+ = \phi y_{r0}$ | $\sum_{r=1}^{s} \mu_r y_{r0} = 1$ |
| For r = 1,…,s, | |
| $\lambda_j \geq 0$ for j = 1,…,n. | $\mu_r, v_i \geq \varepsilon > 0$ |

If the constraint $\sum_{j=1}^{n}\lambda_j$ is adjoined, they are known as BCC (Banker, Charnes, Cooper, 1984) models. This added constraint introduces an additional variable, μo, into the (dual) multiplier problems [12].
The main difference from the CCR model is the treatment of returns to scale. The CCR version bases the evaluation on constant returns to scale (CRS), whereas the BCC version is more flexible and allows variable returns to scale (VRS) [1].

### 3.2 DEA Software Tools

Although the DEA process is quite sophisticated the problem simplifies to a linear program can be solved by the simplex method. Given the large amounts of data involved, DEA is usually performed with the aid of a computer. Software such as Microsoft Excel and Lindo have itemization solvers that are capable of solving the problem. Additionally, shareware software can be found online [13, 16] to solve most DEA problems. For this research, DEA solver software was developed as an add-in for Microsoft Excel [16].Some of the DEA soft wares are given below:

- Frontier Analyst
- DEA-Solver
- OnFront
- Warwick DEA
- DEAP
- EMS
- Pioneer
- GAMS

### 3.3 Strength and Weaknesses of DEA

The advantages of DEA can be listed in as follow [1]:

- It is underpinned by economic theory and methods.
- It focuses on relative not absolute efficiency.
- It has the ability to incorporate multiple inputs and outputs simultaneously and identifying actual good practice and performance targets.
- It does not require a specific production function as opposed to typical econometric models.

DEA's main limitations include: i) the impact of omitting important variables, ii) the impact of outliers, and iii.) the impact of missing observations [30]. All three situations may cause the efficiency scores to be wrongly computed [18]. So the weaknesses of DEA can be stated as follow:

- When complex production processes are involved, specifying a model populated with good quality data can be difficult for several reasons. In particular, an unmanageable number of variables may be needed to capture the process adequately or the quality of available data may be too poor to provide accurate measurement and produce valid results [1, 31].

- DEA is a deterministic rather than statistical technique; DEA produces results that are particularly sensitive to measurement error. DEA only measures efficiency relative to best practice within the particular sample. Thus, it is not meaningful to compare the scores between two different studies.
- DEA results are insensitive to statistical noise and the measurement of comparative efficiency rests on the hypothesis that efficient units are genuinely efficient [1, 31].

## 4. DEA MODELS IN HEALTH SERVICES

Data envelopment analysis (DEA) has proven to be an effective and versatile tool for health care efficiency measurement, and its use has spread throughout the world [1].

DEA received wide acceptance by researchers and practitioners in many public and private sectors. However, it has not been used extensively in health care. The first hospital efficiency studies sought to assess hospital efficiency by concentrating on the development of single productivity indices [1, 7]. Advanced in econometric theory helped to appreciate the multi-product nature of hospital outcomes and econometric models have been developed that enable the formulation of multiple inputs and outputs in the assessment of efficiency [8, 9]. Moreover, optimization-based methodologies have emerged as tools for healthy service research including the development of diagnostic related groups and development of performance indicators [1, 10].

The first empirical test of DEA on hospitals was given by David Sherman [19] who applied DEA to a group of teaching hospitals. DEA is found to provide meaningful insights into the location and nature of hospital inefficiencies as judged by a panel of hospital experts. Grosskopf and Valdmanis [20] employed DEA to find out the effect of ownership type on hospitals efficiency. Morey *et al.* [21] compared the allocative efficiencies of 60 hospitals in the USA. Finkler and Wirtschafter [22] presented an application of DEA to a system of nine hospitals that offer obstetric services. The study confirms its robustness which justifies its inclusion in the cost managers' tool kit. Efficiency and effectiveness in general practice was measured by DEA [23]. The authors expected that DEA should prove a useful tool, offering not only a method for assessing efficiency but also the opportunity to identify practices where improvement in effectiveness may not be feasible without additional resources. Janet Lynch and Yasar Ozcan [24] used DEA to construct an efficiency index to test a hypothesis that inefficient and underutilized hospitals in competitive markets are at greater risk for closure. Chilingerian and Sherman [25] suggested the use of DEA as an evaluation method to classify physicians according to their efficiency rate.

In 1996, an issue of "*Annals of Operations Research"*; which addresses the applications of Operations Research in health care, included four out of eleven studies that addressed the application of DEA in healthcare. The first paper investigates the trends in labor efficiency in U.S. hospital markets for a five-year window using DEA [26], the second one investigates physician practice pattern to identify benchmarks for practices and to reduce costs by applying a multistage DEA [27]. Morey and Dittman [28] used DEA with non-discretionary factors and new hypothesis testing procedures to shed some light on reimbursement methods to hospitals by public insurers. The last paper used DEA to measure PHC quality in England [29]. In general, these studies not only demonstrate that DEA is an effective technique for evaluating the efficiency of Health Care organizations but also reflects the variety of problems in health care management which can be handled by DEA [9].

## 5. CONCLUSION

As DEA has the ability of accommodating multiple heterogeneous inputs and outputs to model the complex and chaotic relations it is an appropriate approach for efficiency measurements in health care. [33] And due to this ability and expediency it found a broad implementation area in this field. This basic overview study is hoped to shed light to the future works that are going to be made in the field.

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