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ADMITTANCE DETERMINATION FOR MODE TRANSFORMATION IN CYLINDRICAL SOFT AND HARD SURFACE CHIROWAVEGUIDES

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ABSTRACT

The admittance dyadics of the cylindrical corrugated chirowaveguides are calculated and analyzed for mode transformation. It is shown that by choosing proper lengths of chirowaveguides TM admittance can be transformed to TE admittance and vice versa. It is assumed that the depth of the corrugation for axial and transverse corrugations, forming hard and soft surfaces respectively, is a quarter wavelength. The results of this study point to the possibility of making mode transformers and phase shifters by using the cylindrical corrugated chirowaveguides. In the analysis of the admittance equations, it is observed that the length of the waveguide is of critical importance.
Keywords: Soft and hard surface, chirowaveguides, admittance, mode.

SİLİNDİRİK YUMUŞAK VE SERT YÜZEYLİ BAKIŞIMSIZ DALGA KILAVUZLARINDA MOD DEĞİŞİMİ İÇİN ADMİTANS TAYİNİ

ÖZET

Bu çalışmada silindirik oluklu bakışımsız dalga kılavuzunda admitans diyadik hesaplanıp mod değişimi için analiz edilmiştir. Uygun bakışımsız dalga kılavuzu uzunluğu seçilerek TM admitansın TE admitansa, TE admitansın da TM admitansa çevrilebileceği gösterilmiştir. Yumuşak ve Sert Yüzeyleri oluşturan enine ve boyuna oluk derinliğinin çeyrek dalga boyu olduğu kabul edilmiştir. Sonuç olarak silindirik oluklu bakışımsız dalga kılavuzları kullanarak mod değiştiricileri ve faz kaydırıcılarının yapılmasının mümkün olduğu görülmüştür.
Anahtar Sözcükler: Yumuşak ve Sert Yüzey, bakışımsız dalga kılavuzu, admitans, mod.

1. INTRODUCTION

During the past decade, considerable attention has been given to the design of periodic structures that control the propagation of electromagnetic waves or the boundary conditions of electromagnetic fields in the desired direction. These kinds of structures are known as electromagnetic bandgap (EBG) structures. Soft and hard surfaces are also related to the EBG structures. These (SHS) boundaries are well known from acoustics. They have also been defined for dually polarized electromagnetic waves. Kildal [1] explained the concept of SHS in detail by considering different geometries. Loading a conducting surface with longitudinal and transverse corrugations can form these surfaces. Corrugation in hard surface waveguide is in the axial direction, whereas corrugation in soft surface is in the transverse direction [2]. Chiral media have

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also drawn attention in the last two decades due to its potential applications in the fields of electromagnetic, optics, microwave and millimeter wave frequencies. The lack of geometric symmetry between an object and its mirror image is referred to as chirality [3]. An artificial chiral medium for microwave frequency can be constructed by embedding such chiral objects as a wire helix, a möbius strip and an irregular tetrahedron in a non-chiral host medium [4]. Attention has focused on the properties of chiral media because the developments in constructing artificial chiral materials provide the additional degree of freedom that these materials offer for design processes via the chirality parameter. Several studies [5-6] have investigated extensively the characteristics of chirowaveguides and electromagnetic wave propagation in guided-wave structures containing chiral materials, which have been named as *chirowaveguides* by Pelet and Engheta [7]. Viitanen [8-9] considered the wave propagation in corrugated cylindrical waveguide filled with chiral material. He derived impedance dyadic and used it to analyze the mode transforming and phase shifting properties of the waveguide. The subject of SHS waveguides and chiral materials continue to be of great interest and practical importance owing to a variety of potential applications, as also by the special issue recently published on the subject [10]. Among these applications are mode transformers, phase shifters, filters and polarizers.

In this study, unlike the previous studies the admittance dyadics of the cylindrical corrugated chirowaveguides are calculated and analyzed for mode transformation. It is shown that by choosing proper lengths of chirowaveguides TM admittance can be transformed to TE admittance and vice versa. It is assumed that the depth of the corrugation for axial and transverse corrugations, forming hard and soft surfaces respectively, is a quarter wavelength. In these types of cylindrical waveguides, it is known that there exists a weak coupling between eigenfields for small chirality parameters, which produces a change in polarization of the propagating field. The results of this study point to the possibility of making mode transformers and phase shifters by using the cylindrical corrugated chirowaveguides.

2. THEORY

Fig.1 presents the geometry of the cylindrical corrugated chirowaveguide. The surface S, assumed to be corrugated either axially or transversely, defines an infinite cylinder. The cylinder is filled with a chiral medium with the constitutive relations

$$\mathbf{D} = \varepsilon \mathbf{E} - j\kappa\sqrt{\mu_0\varepsilon_0}\,\mathbf{H}, \qquad \mathbf{B} = \mu \mathbf{H} + j\kappa\sqrt{\mu_0\varepsilon_0}\,\mathbf{E} \tag{1}$$

(assuming the e^{jwt} time dependence), where ε , μ and κ are permittivity, permeability and chirality admittance of the medium, respectively. If we assume a lossless chiral medium, all of the parameters are real numbers.

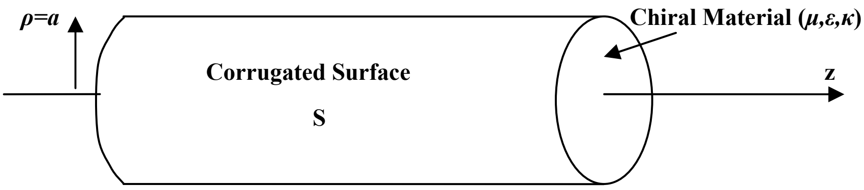

Figure 1. The geometry of a cylindrical corrugated chirowaveguide.

The electromagnetic fields considered inside the chirowaveguide propagate along the z-axis, $e^{-j\beta z}$, where β is the propagation factor. It is well known that inside the chiral medium there exist two propagating waves, i.e., right and left circularly polarized waves, denoted by + and

– waves, respectively. The total fields in the waveguide structure can be written with transverse and axial partial parts as

$$\mathbf{E} = \mathbf{e} + E_z\,\mathbf{u}_z, \qquad \mathbf{H} = \mathbf{h} + H_z\,\mathbf{u}_z. \tag{2}$$

Inserting the constitutive relations into the sourceless Maxwell equations and reducing it to the Helmholtz equation for the axial field components one will obtain the general solution in cylindrical coordinates with Bessel functions of the first kind, as

$$E_{z\pm}(\rho,\varphi) = A_{n\pm} J_n(k_{c\pm}\rho) e^{jn\varphi} \tag{3}$$

where $k_{c\pm} = \sqrt{k^2{}_{\pm} - \beta^2}$ and $A_{n\pm}$ are constants to be determined by initial conditions. Then, one can obtain the expression for the partial transverse fields $\mathbf{e}_{\pm}$ which can be completely determined by the axial parts $E_{z\pm}$ as

$$e_{\pm} = \left[-\frac{j\beta}{k_{c\pm}^{\ 2}} \nabla_t \mp \frac{k_{\pm}}{k_{c\pm}^{\ 2}} u_z \times \nabla_t \right] E_{z\pm} \tag{4}$$

where $E_{z\pm} = \frac{1}{2}(E_z \mp j\eta H_z)$, $k_{\pm} = k \pm k_0\kappa$, and $\nabla_t = \nabla - u_z \frac{\partial}{\partial z}$. Equation number (2) can be rewritten as the summation of transverse and axial partial fields as

$$\mathbf{E} = e_+ + e_- + (E_{z+} + E_{z-})\mathbf{u}_z, \qquad \mathbf{H} = \frac{j}{\eta}\left[e_+ - e_- + (E_{z+} - E_{z-})\mathbf{u}_z \right] \quad . \tag{5}$$

The parameters $k_{c\pm}$ can be determined by the boundary condition for the hard and soft surface at $\rho = a$, as

$$\mathbf{u} \cdot \mathbf{E} = 0, \qquad \mathbf{u} \cdot \mathbf{H} = 0 \tag{6}$$

where $\mathbf{u} = \mathbf{u}_z$ for hard surface and $\mathbf{u} = \mathbf{u}_\varphi$ for soft surface boundary. These equations lead to the eigenvalue equation $J_n(k_{c+}a) J_n(k_{c-}a) = 0$. The solution to this equation is $k_{c\pm} = \frac{p_{ns}}{a}$, where p_{ns} are zeros of the Bessel functions. For hard surface waveguide all index n exist but for soft surface waveguide only index $n = 0$ exists, as given in [8-9]. Different propagation factors for + and – waves can be obtained by $\beta_{\pm} = \sqrt{k_{\pm}^{\ 2} - \left(\frac{p_{ns}}{a}\right)^2}$. It means that inside the corrugated chirowaveguide the eigenmodes with the same index n are always propagating separately. For nonchiral case $k_{\pm} = k$ and $\beta_{\pm} = \beta$. For the small chirality parameter, $\beta_{\pm} \approx \beta \pm \frac{k^2}{\beta}\kappa_r$, where $\kappa_r = \sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}$; and assuming $|\kappa_r| << 1$, equation (4) is reduced to the form

$$e_{\pm} \approx \left[-\frac{j\beta}{k_c^{\ 2}} \nabla_t \mp \frac{k}{k_c^{\ 2}} u_z \times \nabla_t \right] E_{z\pm}(\rho,\varphi,z)\,. \tag{7}$$

Here $E_{z\pm} \approx A_{n\pm} J_n(k_c\rho) e^{jn\varphi} e^{-j\beta z} e^{\mp j(k^2/\beta)\kappa_r z}$, which is the solution to the Helmholtz equation, and β and k_c are the same as in nonchiral waveguide [8-9]. In order to find

the admittance values of the corrugated chirowaveguides the general field inside the waveguide can be presented as a combination of TE and TM fields in the following forms:
The total axial electric field

$$E_z(z) = \left[E_n \cos(\frac{k^2}{\beta}\kappa_r z) - \eta H_n \sin(\frac{k^2}{\beta}\kappa_r z) \right] \cdot J_n(k_c \rho) e^{jn\varphi} e^{-j\beta z} \tag{8}$$

and the total axial magnetic field

$$H_z(z) = \left[H_n \cos(\frac{k^2}{\beta}\kappa_r z) + \frac{E_n}{\eta} \sin(\frac{k^2}{\beta}\kappa_r z) \right] \cdot J_n(k_c \rho) e^{jn\varphi} e^{-j\beta z} , \tag{9}$$

denoting at z = 0 the axial field components as $E_z(0) = E_n = A_{n+} + A_{n-}$ and $H_z(0) = H_n = \frac{j}{\eta}\left[A_{n+} - A_{n-}\right]$. These can be determined by the initial conditions. Similarly, the total transverse electric field

$$\mathbf{e} = -\frac{j\beta}{k_c^2} \left[E_n \cos(\frac{k^2}{\beta}\kappa_r z) - \eta H_n \sin(\frac{k^2}{\beta}\kappa_r z) \right] \times \nabla_t (J_n(k_c \rho) e^{jn\varphi}) e^{-j\beta z}$$
$$+ \frac{jk}{k_c^2} \left[E_n \sin(\frac{k^2}{\beta}\kappa_r z) + \eta H_n \cos(\frac{k^2}{\beta}\kappa_r z) \right] \mathbf{u}_z \times \nabla_t (J_n(k_c \rho) e^{jn\varphi}) e^{-j\beta z} \tag{10}$$

and the total transverse magnetic field

$$\mathbf{h} = -\frac{j\beta}{k_c^2} \left[H_n \cos(\frac{k^2}{\beta}\kappa_r z) + \frac{E_n}{\eta} \sin(\frac{k^2}{\beta}\kappa_r z) \right] \times \nabla_t (J_n(k_c \rho) e^{jn\varphi}) e^{-j\beta z}$$
$$+ \frac{jk}{k_c^2} \left[H_n \sin(\frac{k^2}{\beta}\kappa_r z) - E_n \cos(\frac{k^2}{\beta}\kappa_r z) \right] \mathbf{u}_z \times \nabla_t (J_n(k_c \rho) e^{jn\varphi}) e^{-j\beta z} . \tag{11}$$

Equations (8)-(11) are valid for hard surface and soft surface but it is known that n is equal to zero for soft surface. The reason for n being zero is that the eigenwaves are coupled, and for other values of n there is no coupling effect between them.

3. ADMITTANCE DETERMINATION FOR HARD SURFACE WAVEGUIDE

Wave admittance is defined in terms of the transverse fields (10) and (11) as

$$\mathrm{h} = -\bar{\bar{Y}} \cdot \mathrm{e} \tag{12}$$

The generalized admittance dyadic, when at z = 0 and the coefficient being $H_n = 0$, can be obtained from Equation (12) as

$$\bar{\bar{Y}} = \frac{1}{\eta} \frac{\frac{k^2 - \beta^2}{2} \sin(2\frac{k^2}{\beta}\kappa_r z) \bar{\bar{I}}_t - \beta k \bar{\bar{J}}}{\beta^2 \cos^2(\frac{k^2}{\beta}\kappa_r z) + k^2 \sin^2(\frac{k^2}{\beta}\kappa_r z)} \tag{13}$$

where $\overline{\overline{I}}_t$ is a transverse unit dyadic and $\overline{\overline{J}} = u_z \times \overline{\overline{I}}$ is a 90^0 rotator. If $H_n = 0$ at z = 0 from Equations (10) and (11), the corresponding TM admittance is $\overline{\overline{Y}}(0) = -\frac{1}{\eta}\frac{k}{\beta}\overline{\overline{J}}$. After the distance $z = \frac{\pi\beta}{2k^2\kappa_r} = \frac{\lambda_p}{4}$, where λ_p is the wavelength, the dyadic admittance is $\overline{\overline{Y}}(\lambda_p/4) = -\frac{1}{\eta}\frac{\beta}{k}\overline{\overline{J}}$,which is changed to TE admittance.

Similarly, when at z = 0, the coefficient being $E_n = 0,$ the generalized expression for the dyadic admittance is

$$\overline{\overline{Y}} = \frac{1}{\eta}\frac{\frac{\beta^2 - k^2}{2}\sin(2\frac{k^2}{\beta}\kappa_r z)\,\overline{\overline{I}}_t - \beta k\,\overline{\overline{J}}}{\beta^2 \sin^2(\frac{k^2}{\beta}\kappa_r z) + k^2 \cos^2(\frac{k^2}{\beta}\kappa_r z)}\,. \tag{14}$$

At points z = 0 and $z = \frac{\lambda_p}{4}$, the dyadic admittance is equal to $\overline{\overline{Y}}(0) = -\frac{1}{\eta}\frac{\beta}{k}\overline{\overline{J}}$ and $\overline{\overline{Y}}(\frac{\lambda_p}{4}) = -\frac{1}{\eta}\frac{k}{\beta}\overline{\overline{J}}$, respectively. It is clearly seen that at point z = 0 TE admittance is changed to TM admittance at $z = \frac{\lambda_p}{4}$.

4. ADMITTANCE DETERMINATION FOR SOFT SURFACE WAVEGUIDE

The wave admittance for chiral soft surface waveguide can be derived from the equation

$$\mathbf{h} = -\,\overline{\overline{Y}}\cdot\,(\mathbf{u}_z \times \mathbf{e}). \tag{15}$$

It is possible to obtain the dyadic admittance from Equation (15) as

$$\overline{\overline{Y}} = \frac{1}{\eta}\frac{\frac{\beta^2 - k^2}{2}\sin(2\frac{k^2}{\beta}\kappa_r z)\,\overline{\overline{J}} - \beta k\,\overline{\overline{I}}_t}{\beta^2 \cos^2(\frac{k^2}{\beta}\kappa_r z) + k^2 \sin^2(\frac{k^2}{\beta}\kappa_r z)}\,. \tag{16}$$

In the case when at z = 0 $H_0 = 0$ (that is, we have TM fields), from Equations (10) and (11) the wave admittance is $\overline{\overline{Y}}(0) = -\frac{1}{\eta}\frac{k}{\beta}\overline{\overline{I}}_t$. After the distance $z = \frac{\lambda_p}{4}$, $\overline{\overline{Y}}(\frac{\lambda_p}{4}) = -\frac{1}{\eta}\frac{\beta}{k}\overline{\overline{I}}_t$, at which point dyadic admittance is changed to TE admittance. Similarly, when at z = 0 $E_0 = 0$, the generalized expression for the dyadic admittance at point z is

$$\bar{\bar{Y}} = \frac{1}{\eta} \frac{\frac{k^2 - \beta^2}{2} \sin(2 \frac{k^2}{\beta} \kappa_r z) \bar{\bar{J}} - \beta k \bar{\bar{I}}_t}{\beta^2 \sin^2(\frac{k^2}{\beta} \kappa_r z) + k^2 \cos^2(\frac{k^2}{\beta} \kappa_r z)} . \quad (17)$$

If at z = 0 $E_0 = 0$ (that is, we have TE fields), one obtains the expression $\bar{\bar{Y}}(0) = -\frac{1}{\eta} \frac{\beta}{k} \bar{\bar{I}}_t$ for the TE admittance and $\bar{\bar{Y}}(\frac{\lambda_p}{4}) = -\frac{1}{\eta} \frac{k}{\beta} \bar{\bar{I}}_t$ for the TM admittance.

We follow a procedure similar to Vitanen's [8-9] and it is not surprising to see the admittance dyadic for the propagating hybrid mode (i.e., TE and TM modes) transformed to an admittance dyadic of another hybrid mode with a proper length of the chiral SHS waveguides.

5. CONCLUSION

The admittance dyadics of the cylindrical corrugated chirowaveguides are calculated and analyzed for mode transformation. It is assumed that the depth of the corrugation in axial or transverse direction is a quarter wavelength. Due to small chirality parameters, there exists a weak coupling between eigenfields. It is shown that the propagating hybrid mode can be changed into another hybrid mode by using a chiral SHS waveguide of a proper length.

In conclusion, the generalized dyadic admittance presented in this study for chiral SHS waveguides is obtained in a functional form. In doing so, it has been illustrated that the admittance can be used to describe the mode transforming effect of these kinds of waveguides in agreement with the impedance dyadic.

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