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Second Type Chebyshev Polynomial Approximation to Linearly Anisotropic Neutron Transport Equation in Slab Geometry

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Abstract: In one-dimensional slab geometry, the neutron transport equation was solved in one-speed and linearly anisotropic scattering by implementing the method of separation of variables. The part which depended on the position selected as an exponential function on the other hand the part that was relied upon the angle was chosen as Legendre polynomials or Chebyshev polynomials. The approximation we used is called as U_N method because in the method second type Chebyshev polynomials were used. To solve these differential equations, an exponential function was suggested in both P_N and U_N method. By using the suggested function in differential equations, analytical equations in which ν eigenvalues can be calculated were obtained. These analytical equations were solved and ν eigenvalues calculated for different values ($0 \leq c_1 < 2$: $c_0 = 0$, $c_0 = 0.25$, $c_0 = 0.50$, $c_0 = 0.75$, $c_0 = 0.99$) of c_0 and c_1 (where c is the number of secondary neutrons per collision) and the results were presented in the same tables for comparison.

Key words: *S*lab geometry, neutron transport equation, Chebyshev polynomials

Slab Geometride Lineer Anizotropik Nötron Transport Denklemine İkinci Tip Chebyshev Polinom Yaklaşımı

Özet: Tek boyutlu dilim geometride, tek hızlı ve lineer anizotropik saçılmalı durumda nötron transport denklemi değişkenlere ayırma yöntemi kullanılarak çözülmüştür. Konuma bağlı kısım eksponansiyel bir fonksiyon, açıya bağlı kısım ise Legendre veya Chebyshev Polinomları olarak seçilmiştir. Chebyshev Polinomlarının II. tipi kullanıldığından U_N yaklaşımı olarak adlandırıldı. Bu diferansiyel eşitliklerin çözümü için hem P_N hem de U_N yönteminde eksponansiyel bir fonksiyon önerilmiştir. Önerilen fonksiyon diferansiyel eşitliklerde kullanılarak öz değerlerinin hesaplanabileceği birbirine bağlı analitik denklemler elde edilmiştir. Bu analitik denklemler çözülmüş c_0 ve c_1'in (çarpışma başına ortaya çıkan nötron sayıları) farklı değerler için ($0 \leq c_1 < 2$: $c_0 = 0$, $c_0 = 0.25$, $c_0 = 0.50$, $c_0 = 0.75$, $c_0 = 0.99$) ν öz değerleri hesaplanmış ve karşılaştırma yapmak için tablolar sunulmuştur.

Anahtar kelimeler: *S*lab geometri, nötron transport denklemi, Chebyshev polinomları

1. Introduction

Per each fission reaction is revealed about 2-3 neutron. These neutrons can trigger the other core to fission. For this reason it has to be controlled chain reaction. The thermal neutrons causing fission are slowed by placing into the reactor moderator. The reaction speed can be controlled by the reactor control rods suspended from [1].

The reactor must remain constant the number of neutrons in the division for the safe operation. Remain constant number of neutrons is called critical. Subcritical case the reactor damped. The neutron movement and amount successive fission reactions resulting from reaction are important for continuity.

The transport equation, under certain approaches having regard to only the position dependence of the neutron is achieved to neutron diffusion equation. Troubles design of nuclear reactors and the neutron flux distribution in the reactor to operate must be known exactly. Many methods have been developed for this purpose. The most commonly used P_N, Legendre polynomial [2] approximation, is a method. In this method, neutron transport equation is solved by opening the series in terms of Legendre polynomials. In applied science, orthogonal polynomials such as Legendre, Chebyshev etc. have a great importance. Legendary polynomials [3] have a special place in neutron transport theory.

In this study, an alternative to other methods, in order to solve neutron transport equation, angular flux function ψ (x, y) is expanded series of the second type Chebyshev polynomials. This method was called the U_N. Conkie (1959) has first time suggested [4], the neutron transport equation can be solved with Chebyshev polynomials.

To solve the resulting differential equations, exponential functions proposed a similar method to P_N. This proposed exponential function used in the differential equations and analytical equations obtain the eigenvalues can be calculated. These interconnected analytical equations are solved. The number of neutrons per collision occurring (c_0 and c_1) given the possible values and assessments were made about the υ.

2. Materials and Methods

2.1. Legendre polynomials

μ=cosɵ, k *is a constant, m is an integer,*

$$\left(1-\mu^2\right)\frac{d^2}{d\mu^2}\left[P_n^m(\mu)\right]-2\mu\frac{dP_n^m(\mu)}{d\mu}+\left(k-\frac{m^2}{1-\mu^2}\right)P_n^m(\mu)=0 \tag{1}$$

$-1 \le \mu \le 1,\ 0 \le n \le N,\ -n \le m \le n$

Equality, depending Legendre differential equation, the solution of the equation is called the Legendre polynomials connected.

2.2. Chebyshev polynomials

$$\left(1-\mu^2\right)\frac{d^2y}{d\mu^2}-\mu\frac{dy}{d\mu}+n^2y=0 \tag{2}$$

Chebyshev polynomials arise from the differential solution. This differential equation has two independent solutions. Chebyshev polynomials with two independent solutions as I (T_N) and II (U_N) are known as such I. and II. Series solutions called types are given by;

$$T_n(\mu)=\sum_{m=0}^{[n/2]}\frac{(-1)^m n!}{(n-m)!(2m)!}(1-\mu^2)^m\mu^{n-2m} \tag{3}$$

$$U_n(\mu)=\sum_{m=0}^{[n/2]}(-1)^m\frac{(n-m)!}{m!(n-2m)!}(2m)^{n-2m} \tag{4}$$

Chebyshev polynomials are special spherical polynomial [5]. II. Type Chebyshev polynomials generating function,

$$\frac{1}{1-2\mu t+t^2}=\sum_{n=0}^{\infty}t^n U_n(\mu),\ |\mu|\leq 1,\ |t|<1 \tag{5}$$

These polynomials repetition relation is given by,

$$U_{n+1}(\mu) - 2\mu\, U_n(\mu) + U_{n-1}(\mu) = 0 \tag{6}$$

$U_0(\mu)=1$ value using, for various values of $U_N(\mu)$ can be obtained polynomials. Another way to obtain the second type Chebyshev polynomials is to take the determinant of the matrix below.

$$U_n(\mu)=\begin{vmatrix} 2\mu & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 2\mu & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 2\mu & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 2\mu & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ddots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2\mu \end{vmatrix} \tag{7}$$

2.3. Neutron Transport Equation

One group and independent transport equation can be written as follows [6].

$$\Omega.\nabla\psi(r,\Omega)+\sigma_T\Psi(r,\Omega)=\int_{\Omega'}\sigma_s(\Omega.\Omega')\Psi(r,\Omega')d\Omega'+Q/2 \tag{8}$$

This equation is called the Boltzmann integro-differential equations. Always the target of shelling core cannot occur nuclear reactions. One group and one-dimensional slice of the outsourcing independent linear geometry neutron transport equation is given by,

$$\mu\frac{d\Psi(x,\mu)}{dx}+\sigma_T\Psi(\mu)=\int_{-1}^{1}\sum_{n=0}^{\infty}\frac{2n+1}{2}\sigma_{sn}P_n(\mu)P_n(\mu')\Psi(x,\mu')d\mu' \tag{9}$$

If this equality only held for n = 0,

$$\mu\frac{d\Psi(x,\mu)}{dx}+\sigma_T\Psi(\mu)=\frac{1}{2}\sigma_{so}\int_{-1}^{1}\Psi(x,\mu')d\mu' \tag{10}$$

In equation 9, unit of the σ_{s0}, σ_{s1} and σ_T are quantity cm^{-1}. In this study, all the numerical calculations were. $\sigma_T = 1\ cm^{-1}$.

2.4. Legendre polynomials Approach: P_N Method

Angular flux function $\psi(x, \mu)$ expand the series in terms of Legendre polynomials; can be expressed as follows by using matrix algebra.

$$\begin{bmatrix} \upsilon(1-c_0) & 1 & 0 & 0 & \cdots & 0 \\ 1 & 3\upsilon(1-c_1) & 2 & 0 & \cdots & 0 \\ 0 & 2 & 5\upsilon & 3 & \cdots & 0 \\ 0 & 0 & 3 & 7\upsilon & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (2n+1)\upsilon \end{bmatrix} x \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{11}$$

This (N + 1) × (N 1) square matrix determinant equalized to zero and υ values can be calculated. Giving various values to c_0 and c_1 for the P_2, P_5, P_6 and P_{10} approaches have been calculated for v values.c_0 and c_1 same values have been used to calculate for v in U_N and P_N approaches. The v values obtained from both methods for the same number of c_0 and c_1 are presented in the same table together. v values are used to calculate the flux moments.

2.5. Chebyshev Polynomial Approach: U_N Method

Angular flux function ψ(x, μ) expand the series in terms of second type Chebyshev polynomial terms, can be expressed as follows by using matrix algebra.

$$\begin{bmatrix} \upsilon(1-c_0) & 1 & 0 & 0 & \cdots & 0 \\ 1 & 2\upsilon(1-c_1) & 1 & 0 & \cdots & 0 \\ \frac{2}{2} & 1 & 2\upsilon & 1 & \cdots & 0 \\ 0 & \frac{4}{5}\upsilon c_1 & 1 & 2\upsilon & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \left[\frac{1+(-1)^n}{n+1}\right] & \frac{3}{2}\left[\frac{(n+1)\left[1-(-1)^n\right]}{n(n+2)}\right] & 0 & 0 & \cdots & 2\upsilon \end{bmatrix} x \begin{bmatrix} G_0 \\ G_1 \\ G_2 \\ G_3 \\ \vdots \\ G_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{12}$$

This (N + 1) × (N 1) square matrix determinant equalized to zero and υ values can be calculated. Giving various values to c_0 and c_1 for the U_2, U_5, U_6, and U_{10} approaches have been calculated for v values.

3. Results and Discussion

Maple9 computer program was used to calculate the value of υ in the P_N and U_N approach. Results for the calculation were rounding up to the fifth decimal place. For N even, $A_N(\upsilon_j)=0$ and $G_N(\upsilon_j)=0$ gives symmetric eigenvalues as ±υ. These values are symmetrical with respect to the origin point. For this reason positive υ_j eigenvalues are shown in Tables.

Table 1. For co =0 and 0≦c_1<1, Comparison of U_N and P_N *Approach*

		$N = 1$		$N = 4$		$N = 5$		$N = 9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_0=0.00		5.00000	5.77350	7.75164	7.75338	7.75337	7.75338	7.75338	7.75339
				0.55860	0.62933	0.69363	0.76634	0.90216	0.93437
	0.99			0.00	0.00	0.23242	0.24760	0.71583	0.73996
								0.43348	0.45845
								0.14397	0.15027
		1.11803	1.29099	1.75992	1.76942	1.76829	1.76972	1.76975	1.76976
				0.55016	0.61663	0.68553	0.75577	0.89885	0.93166
	0.80			0.00	0.00	0.23057	0.24595	0.70980	0.73422
								0.43076	0.45473
								0.14366	0.15001
		0.70710	081649	1.14663	1.16675	1.16274	1.17085	1.17213	1.17265
				0.53405	0.59144	0.66799	0.73044	0.88891	0.92253
	0.50			0.00	0.00	0.22759	0.23427	0.69578	0.72060
								0.42580	0.44787
								0.14316	0.14959
		0.50000	0.57735	0.86602	0.90617	0.90096	0.93246	0.95949	0.97390
				0.50000	0.53846	0.62348	0.66120	0.84125	0.86506
	0.00			0.00	0.00	0.22252	0.23861	0.65486	0.67941
								0.41541	0.43339
								0.14231	0.14887

As seen in Table 1 for c_0 =0 and c_1 = 0 all of eigenvalues υ is smaller than 1. One of the eigenvalues for c_0 =0 and c_1> 0, also may be larger than 1.

Table 2. For co =0.25 and 0 $\leq c_1 < 1$, Comparison of U_N and P_N *Approach*

		$N = 1$		$N = 4$		$N = 5$		$N = 9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_o=0.25		5.77350	6.66666	8.43713	8.43848	8.43848	8.43848	8.43848	8.43848
				0.57081	0.64351	0.70279	0.77393	0.90398	0.93562
	0.99			0.00	0.00	0.24337	0.26012	0.72028 0.44143 0.14877	0.74447 0.46647 0.15553
		1.29099	1.49071	1.90700	1.91423	1.91339	1.91439	1.91440 1.91441	
				0.56471	0.63433	0.69708	0.76630	0.90156	0.93362
	0.80			0.00	0.00	0.24197	0.25894	0.71585	0.74029
								0.43961 0.14855	0.46387 0.15536
		0.81649	0.94280	1.23137	1.24649	1.24323	1.24878	1.24931	1.24953
				0.55311	0.61609	0.68492	0.74855	0.89486	0.92760
	0.50			0.00	0.00	0.23971	0.25700	0.70590 0.43633 0.14819	0.73074 0.45913 0.15508
		0.57735	0.66666	0.91241 0,94414		0.93786	0.96217	0.97854	0.98827
				0.52783	0.5751	0,65260 0.69615		0.86028	0.88646
	0.00			0.00	0.00	0.23582	0.25362	0.67618 0.42949 0.14759	0.70100 0.44913 0.15461

Table 3. For co =0.50 and 0 $\leq c_1 < 1$, Comparison of U_N and P_N *Approach*

		$N=1$		$N=4$		$N=5$		$N=9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
$c_o=0.50$	0.99	7.07106	8.16496	9.66383 0.58612 0.00	9.66473 0.66128 0.00	9.66473 0.71469 0.25592	9.66473 0.78382 0.27465	9.66473 0.90651 0.72611 0.45104 0.15401	9.66473 0.93733 0.75045 0.47622 0.16132
	0.80	1.58113	1.82574	2.17424 0.58252 0.00	2.17889 0.65588 0.00	2.17838 0.71153 0.25502	2.17895 0.77934 0.27396	2.17896 0.90507 0.72342 0.45012 0.15388	2.17896 0.93613 0.74797 0.47476 0.16123
	0.50	1.00000	1.15470	1.39106 0.57584 0.00	1.40044 0.64539 0.00	1.39828 0.70511 0.25356	1.40128 0.76958 0.27285	1.40140 0.90155 0.71776 0.44848 0.15367	1.40146 0.93300 0.74269 0.47216 0.16108
	0.00	0.70710	0.81649	1.00908 0.56132 0.00	1.02887 0.62117 0.00	1.02297 0.68835 0.25104	1.03662 0.74094 0.27088	1.04104 0.88607 0.70195 0.44517 0.15331	1.04374 0.91623 0.72734 0.46685 0.16083

Table 4. For co =0.75 and 0 $\leq c_1 < 1$, Comparison of U_N and P_N *Approach*

		$N = 1$		$N = 4$		$N = 5$		$N = 9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_o=0.75		10.0000 11.5470		12.6504	12.6508	12.6508	12.6508	12.6508	12.6508
				0.60589	0.68419	0.73070	0.79720	0.91021	0.93986
	0.99			0.00	0.00	0.27044	0.29175	0.73402	0.75868
								0.46282	0.48823
								0.15976	0.16771
		2.23606 2.58198		2.83449	2.83646	2.83626	2.83646	2.83646	2.83646
				0.60466	0.68235	0.72986	0.79567	0.90973	0.93943
	0.80			0.00	0.00	0.27004	0.29152	0.73300	0.75782
								0.46261	0.48776
								0.15971	0.16768
		1.41421 1.63299		1.79916	1.80279	1.80194	1.80290	1.28091	1.80291
				0.60248	0.67899	0.72829	0.79271	0.90875	0.93852
	0.50			0.00	0.00	0.26940	0.29116	0.73110	0.75621
								0.46226	0.48696
								0.15962	0.16764
		1.00000 1.15470		1.28150	1.28829	1.28556	1.28919	1.28934	1.28946
				0.59811	0.67187	0.72480	0.78562	0.90602	0.93582
	0.00			0.00	0.00	0.26830	0.29051	0.72677	0.75244
								0.46159	0.48546
								0.15948	0.16756

Table 5. For co =0.99 and 0 $\leqq c_1 < 1$, Comparison of U_N and P_N *Approach*

		$N=1$		$N=4$		$N=5$		$N=9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_0=0.99		50.0000	57.7350	57.9655	57.9655	57.9655	57.9655	57.9655	57.9655
				0.63121	0.71349	0.75217	0.81527	0.91581	0.94369
	0.99			0.00	0.00	0.28669	0.31131	0.74461	0.76995
								0.47675	0.50252
								0.65822	0.17449
		11.1803	12.9099	12.9615	12.9615	12.9615	12.9615	12.9615	12.9615
				0.63121	0.71348	0.75220	0.81527	0.91581	0.94369
	0.80			0.00	0.00	0.28668	0.31131	0.74460	0.76995
								0.47676	0.50252
								0.16582	0.17449
		7.07106	8.16496	8.19765	8.19768	8.19767	8.19768	8.19768	8.19768
				0.63121	0.71348	0.75224	0.81526	0.91582	0.94369
	0.50			0.00	0.00	0.28666	0.31131	0.74458	0.76995
								0.47677	0.50252
								0.16581	0.17449
		5.00000	5.77350	5.79668	5.79672	5.79671	5.79672	5.79672	5.79672
				0.63120	0.71346	0.75231	0.81525	0.91583	0.94369
	0.00			0.00	0.00	0.28663	0.31131	0.74454	0.76994
								0.47680	0.50251
								0.16581	0.17449

As seen in Table 1, Table 2, Table 3, Table 4 and Table 5 for $0 < c_0 < 1$ all eigenvalues are real and one of them also may be larger than 1. Other eigenvalues (-1, 1) are in the range. According to these results, for $0 < c_0 < 1$, outside of the range (-1, 1) eigenvalues is seen to be changed at $1 < \upsilon < \infty$ intervals.

Table 6. For co =1.01 and 0 $\leq c_1 < 1$, Comparison of U_N and P_N *Approach*

		$N = 1$		$N = 4$		$N = 5$		$N = 9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_0=1.01		50.0000i	57.7350i	57.5036i	57.5036i	57.5036i	57.5036i	57.5036i	57.5036i
				0.63370	0.71636	0.75434	0.81711	0.91642	0.94411
	0.99			0.00	00	0.28816	0.31311	0.74567	0.77110
								0.47805	0.50385
								0.16635	0.17508
		11.1803i	12.9099i	12.8582i	12.8582i	12.8582i	12.8582i	12.8582i	12.8582i
				0.63370	0.71636	0.75430	0.81710	0.91641	0.94411
	0.80			0.00	0.00	0.28817	0.31311	0.74568	0.77110
								0.47804	0.50385
								0.16635	0.17508
		7.07106i	8.16496i	8.13233i	8.13236i	8.13236i	8.13236i	8.13236i	8.13236i
				0.63370	0.71635	0.75425	0.81710	0.91640	0.94411
	0.50			0.00	0.00	0.28819	0.31311	0.74570	0.77110
								0.47802	0.50385
								0.16635	0.17508
		5.00000i	5.77350i	5.75049i	5.75053i	5.75052i	5.75053i	5.75053i	5.75053i
				0.63369	0.71634	0.75416	0.81709	0.91638	0.94411
	0.00			0.00	0.00	0.28822	0.31311	0.74572	0.77109
								0.47800	0.50385
								0.16636	0.17508

Table 7. For co =1.25 and 0 ≤c_1<1, Comparison of U_N and P_N *Approach.*

	c_1	$N = 1$ U_N P_N	$N = 4$ U_N P_N	$N = 5$ U_N P_N	$N = 9$ U_N P_N
c_0=1.25	0.99	10.0000i 11.5470i	10.3303i 10.3310i 0.67008 0.75832 0.00 0.00	10.3310i 10.3310i 0.78699 0.84495 0.30748 0.33707	10.3310i 10.3310i 0.92670 0.95129 0.76136 0.78845 0.49550 0.52190 0.17304 0.18264
	0.80	2.23606i 2.58198i	2.31921i 2.32194i 0.66740 0.75445 0.00 0.00	2.32181i 2.32196i 0.78288 0.84172 0.30753 0.33664	2.32196i 2.32196i 0.92504 0.95016 0.76021 0.78669 0.49457 0.52112 0.17305 0.18260
	0.50	1.41421i 1.63299i	1.47449i 1.47790i 0.66392 0.74966 0.00 0.00	1.47751i 1.47807i 0.77786 0.83792 0.30762 0.33598	1.47806i 1.47806i 0.92321 0.94895 0.75877 0.78452 0.49325 0.52003 0.17306 0.18254
	0.00	1.00000i 1.15470i	1.04958i 1.05308i 0.65951 0.74393 0.00 0.00	1.05229i 1.05367i 0.77197 0.83365 0.30775 0.33497	1.05356i 1.05359i 0.92131 0.94772 0.75705 0.78199 0.49135 0.51849 0.17309 0.18244

Table 8. For co =1.50 and 0 $\leq c_1<1$, Comparison of U_N and P_N *Approach*. (i=$\sqrt{-1}$)

		$N = 1$		$N = 4$		$N = 5$		$N = 9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_o=1.50	0.99	7.07106i	8.16496i	6.33478i 0.72769 0.00	6.33786i 0.82442 0.00	6.33786i 0.84192 0.33129	6.33786i 0.89273 0.36772	6.33786i 0.94946 0,78513 0.51769 0.18068	6.33786i 0.96796 0.81633 0.54497 0.19136
	0.80	1.58113i	1.82574i	1.45079i 0.71049 0.00	1.45939i 0.80058 0.00	1.45959i 0.81923 0.33057	1.45982i 0.87253 0.36525	1.45978i 0.93766 0.77793 0.51377 0.18063	1.45978i 0.95911 0.80630 0.54113 0.19117
	0.50	1.00000i	1.15470i	0.93910i 0.69419 0.00	0.94729i 0.78005 0.00	0.94807i 0.80015 0.32955	0.94925i 0.85661 0.36185	0.94893i 0.92975 0.77102 0.50886 0.18055	0.94895i 0.95352 0.79720 0.53641 0.19088
	0.00	0.70710i	0.81649i	0.67895i 0.67895 0.00	0.68549i 0.76224 0.00	0.68725i 0.78401 0.32808	0.69010i 0.84395 0.35723	0.68901i 0.92435 0.76474 0.50290 0.18042	0.68913i 0.94987 0.78932 0.53082 0.19043

Table 9. For co =1.75 and 0 $\leq c_1<1$, Comparison of U_N and P_N *Approach*. (i=$\sqrt{-1}$)

		$N = 1$		$N = 4$		$N = 5$		$N = 9$	
		U_N	P_N	U_N	P_N	U_N	P_N	U_N	P_N
c_o=1.75	0.99	5.77350i	6.66666i	4.24995i 0.82726 0.00	4.25973i 0.93736 0.00	4.25979i 0.94282 0.35938	4.25974i 0.98322 0.40561	4.25974i 1.01042 0.81574 0.54356 0.18907	4.25974i 1.01880 0.85475 0.57189 0.20101
	0.80	1.29009i	1.49071i	1.03331i 0.76081 0.00	1.04952i 0.85071 0.00	1.05217i 0.86096 0.35627	1.05219i 0.90753 0.39780	1.05183i 0.95498 0.79671 0.53391 0.18883	1.05183i 0.97186 0.82785 0.56197 0.20050
	0.50	0.81649i	0.94280i	0.68955i 0.72106 0.00	0.70159i 0.80485 0.00	0.70749i 0.81909 0.35224	0.70931i 0.87195 0.38844	0.70769i 0.93558 0.78139 0.52331 0.18848	0.70774i 0.95758 0.80804 0.55133 0.19974
	0.00	0.57735i	0.66666i	0.50755i 0.69270 0.00	0.51576i 0.77417 0.00	0.52501i 0.79207 0.52501	0.52916i 0.85053 0.37745	0.52562i 0.92632 0.76995 0.51213 0.18794	0.52590i 0.95124 0.79425 0.54043 0.19859

Table 10. For co =2.00 and 0 $\leq c_1 < 1$, Comparison of U_N and P_N *Approach*. (i= $\sqrt{-1}$)

		$N=1$		$N=4$	$N=5$	$N=9$	
		U_N	P_N	U_N P_N	U_N P_N	U_N	P_N
c_0=2.00	0.99	5.00000i	5.77350i	2.71008i 2.74234i 1.03821 1.16828 0.00 0.00	2.74305i 2.74248i 1.16378 1.18945 0.39156 0.45101	2.74248i 1.19564 0.84265 0.57064 0.19819	2.74248i 1.19693 0.88651 0.59969 0.21162
	0.80	1.11803i	1.29099i	0.77246i 0.79561i 0.81447 0.90043 0.00 0.00	0.80547i 0.80521i 0.90459 0.94415 0.38360 0.43272	0.80352i 0.97695 0.81400 0.55337 0.80352	0.80349i 0.98909 0.84824 0.58184 0.21054
	0.50	0.70710i	0.81649i	0.53509i 0.54954i 0.74363 0.82448 0.00 0.00	0.56578i 0.56840i 0.83465 0.88433 0.37434 0.41392	0.56399i 0.94066 0.78987 0.53599 0.19676	0.56409i 0.96114 0.81698 0.56422 0.20898
	0.00	0.50000i	0.57735i	0.40044i 0.40949i 0.70263 0.78238 0.00 0.00	0.43065i 0.43585i 0.79775 0.85503 0.36384 0.39478	0.42876i 0.92769 0.77364 0.51944 0.19550	0.42923i 0.95218 0.79775 0.54790 0.20673

for c_0>1, as seen in the Table 6, Table 7, Table 8, table 9 and Table 10 is a pair of complex eigenvalues.

4. Conclusions

In this study; υ eigenvalues have been calculated for one-dimensional, mono energetic neutron transport equation with no external source and with anisotropic scattering, using Legendre and II. type Chebyshev polynomial approaches. The solution of the neutron transport equation with anisotropic scattering can be analyzed with II. type Chebyshev polynomials since their arguments change in the same interval range [-1,1] as the arguments of Legendre polynomials. At first, neutron transport equation opened in series in terms of Chebyshev polynomials. The eigenvalues have been calculated from the equations obtained from the opened series.

The results of the asymptotic solutions derived from U_N and P_N methods is increasingly approaching to each other starting from N = 4, as can be seen from careful examination of the tables. The coherence between the results is increasing as N increases as can be expected. The compliance of the eigenvalues calculated with both methods confirms the validity of the U_N approach.

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