On Fuzzy Ideals Of Subtraction Semigroups

Zekiye Çiloğlu[1], Yılmaz Çeven[1,*]

[1]*Department of Mathematics, Süleyman Demirel University, Isparta-TURKEY*
**Correspending author e-mail: yilmazceven@sdu.edu.tr*

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Abstract: In this paper, we introduce the notion of fuzzy interior ideal, fuzzy bi-ideal, intuitionistic fuzzy interior ideal and intuitionistic fuzzy bi-ideal of a subtraction semigroup. We characterize a non-empty subset of a subtraction semigroup X through intuitionistic fuzzy ideal, intuitionistic fuzzy bi-ideal and intuitionistic fuzzy interior ideal. We give some equivalent conditions related to these notions.

Key words: Subtraction semigroups, fuzzy interior ideal, fuzzy bi-ideal, intuitionistic fuzzy interior ideal, intuitionistic fuzzy bi-ideal.

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Çıkarma Yarıgruplarda Bulanık İdealler Üzerine

Özet: Bu makalede, çıkarma yarıgruplarında bulanık iç ideal, bulanık bi-ideal, sezgisel bulanık iç ideal ve sezgisel bulanık bi-ideal kavramlarının tanımları verilmiştir. Bir X çıkarma yarıgrubunun boş olmayan bir alt kümesinin bulanık iç ideal, bulanık bi-ideal, sezgisel bulanık iç ideal ve sezgisel bulanık bi-ideal olması için bazı karakterizasyonlar verilmiştir. Ayrıca bu kavramlarla ilgili bazı denk koşullar elde edilmiştir.

Anahtar kelimeler: çıkarma yarıgruplar, bulanık iç ideal, bulanık bi-ideal, sezgisel bulanık iç ideal, sezgisel bulanık bi-ideal

1. Introduction and Preliminaries

B. M. Schein ([11]) considered systems of the form $(\Phi;\circ,\backslash)$, where Φ is a set of functions closed under the composition "$\circ$" of functions (and hence $(\Phi;\circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$" (and hence $(\Phi;\backslash)$ is a subtraction algebra in the sense of [3]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B. Zelinka ([14])discussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y. B. Jun, H. S. Kim and E. H. Roh ([6]) introduced the notion of ideals in subtraction algebras and discussed characterizations of ideals. In [7], Y. B. Jun and H. S. Kim established the ideal generated by a set and discussed related results.

After the introduction of fuzzy sets by Zadeh ([13]), several researchers were conducted on the generalizations of the notion of fuzzy set. The concept of intuitionistic fuzzy set was introduced by Atanassov [1,2] as a generalization of the notion of fuzzy set. In [12], Williams introduce a notion of fuzzy ideals in near-subtraction semigroups, and studied their related properties. In [5], Dheena and Mohanraj introduced the notion of fuzzy ideal, fuzzy weak ideal, fuzzy weakly prime left ideal and fuzzy prime left ideal system of a near-subtraction semigroup and discussed some related results.

In this paper, we introduce the concepts of fuzzy interior ideal, fuzzy bi-ideal, intuitionistic fuzzy interior ideal and intuitionistic fuzzy bi-ideal in a subtraction semigroup and give some related properties.

Definition 1.1. An algebra (X;-) with a single binary operation "-" is called a subtraction algebra if for all $x,y,z\in X$ the following conditions hold:

(1) $x-(y-x)=x$,
(2) $x-(x-y)=y-(y-x)$,
(3) $(x-y)-z=(x-z)-y$.

The subtraction determines an order relation on X : $a\leq b\Leftrightarrow a-b=0$, where $0=a-a$ is an element that doesn't depend on the choice of $a\in X$.

In a subtraction algebra, the following are true [6,7]:

(a1) $(x-y)-y=x-y$,
(a2) $x-0=x$ and $0-x=0$,
(a3) $(x-y)-x=0$,
(a4) $(x-y)-(y-x)=x-y$,
(a5) $x\leq y$ implies $x-z\leq y-z$ and $z-y\leq z-x$ for all $z\in X$.

Definition 1.2. A non-empty subset S of a subtraction algebra X is said to be a subalgebra of X if $x-y\in S$ whenever $x,y\in S$.

Definition 1.3. Let X be a semigroup. By a subsemigroup of X, we mean a non-empty subset S of X such that $xy\in S$ for all $x,y\in S$.

Definition 1.4. A semigroup X is said to be regular if, for each $a\in X$, there exists a $x\in X$ such that $a=axa$.

Definition 1.5. A non-empty set X together with the binary operations "-" and "." is said to be a subtraction semigroup if it is satisfies the following properties:

(SS1) (X,-) is a subtraction algebra,
(SS2) (X,.) is a semigroup,
(SS3) $x(y-z)=xy-xz$ and $(x-y)z=xz-yz$
for all $x,y,z\in X$.

It is clear that, in a subtraction semigroup X, we have $0x=0$ and $x0=0$ for all $x\in X$.

Definition 1.6. (X,-,.) be a subtraction semigroup. A non-empty subset I of X is called
(I1) a left ideal if I is a subalgebra of (X,-) and $xi\in I$ for all $x\in X$ and $i\in I$,
(I2) a right ideal if I is a subalgebra of (X,-) and $ix\in I$ for all $x\in X$ and $i\in I$,
(I3) an ideal if I is both a left and a right ideal.

Example 1.7. Let (X,-,.) be a subtraction semigroup and $a\in X$.
(i) $A_a=\{x\in X: ax=0\}$ is a right ideal of X.

(ii) aX={ax: x∈X} is a right ideal of X.

Definition 1.8. Let X be a non-empty set. A mapping $\mu:X\to[0,1]$ is called a fuzzy set of X.

Definition 1.9. The level set of a fuzzy set μ of X is defined as $U=U(\mu;t)=\{x\in X : \mu(x)\geq t\}$ for all $0\leq t\leq 1$.

Definition 1.10. An intuitionistic fuzzy set (IFS) A in a non-empty set X is an object having the form

$$A=\{ (x,\mu(x),\gamma(x)) : x\in X\}$$

where the functions $\mu:X\to[0,1]$ and $\gamma:X\to[0,1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$0\leq\mu(x)+\gamma(x)\leq 1$$

for all x∈X. For the sake of simplicity, we shall use the symbol A=(μ,γ) for the IFS

$A=\{ (x,\mu(x),\gamma(x)) : x\in X\}$.

In what follows, let X be a subtraction semigroup, unless otherwise specified.

2. Fuzzy Interior Ideals and Fuzzy Bi-ideals

Definition 2.1.

(i) A subset Y of X is called an interior ideal of X if Y is a subalgebra of X and XYX⊆Y.

(ii) A subset Y of X is called a bi-ideal of X if Y is a subalgebra of X and YXY⊆Y.
Example 2.2. Let X={0,a,b,c} in which "-" and "." are defined by the following table:

−	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	a
c	0	0	a	b

Then it is easily seen that (X;-,.) is a subtraction semigroup and I={0,b} is both an interior ideal and a bi-ideal of X. But I is not an ideal of X.

Theorem 2.3.
(i) Every ideal of X is an interior ideal.
(ii) If X contains an identity element then every interior ideal of X is an ideal.
(iii) Any left (right, two-sided) ideal of X is a bi-ideal.

Proof. It is clear by the definitions.

If I is a single side ideal of a subtraction semigroup X then I is not an interior ideal of X. For example, the sets A_a and aX in Example 1.7 are not interior ideals generally.

Definition 2.4. For a fuzzy set μ in X, consider the following axioms:

(i) $\mu(x-y)\geq\min\{\mu(x),\mu(y)\}$,
(ii) $\mu(xy)\geq\min\{\mu(x),\mu(y)\}$

for all x,y∈X. Then μ is called a fuzzy subalgebra of X if it satisfies (i), and μ is called a fuzzy subsemigroup of X if it satisfies (ii).

Let A be a non-empty subset of X and μ_A be a fuzzy set in X defined by

$$\mu_A(x)=\begin{cases} s, & \text{if } x\in A \\ t, & \text{if } x\notin A \end{cases} \tag{2.1}$$

for all x∈X and s,t∈[0,1] with s>t.

Theorem 2.5. Let A be a non-empty subset of X and μ_A be a fuzzy set in X defined in (2.1). Then

(i) A is a subalgebra of X if and only if μ_A is a fuzzy subalgebra of X
(ii) A is a subsemigroup of X if and only if μ_A is a fuzzy subsemigroup of X.

Proof. (i) Let x,y∈X. If x,y∈A, then $\mu_A(x-y)=s=\mu_A(x)=\mu_A(y)$ since x-y∈A. If x∉A or y∉A, then $\mu_A(x)=t$ or $\mu_A(y)=t$. Hence $\mu_A(x-y)\geq t=\min\{\mu_A(x),\mu_A(y)\}$ by the definition of μ_A. Hence μ_A is a fuzzy subalgebra of X. Conversely, let x,y∈A. Then since $\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y)\}=s$, we have $\mu_A(x-y)=s$ and so x-y∈A.

(ii) Similar to the proof of (i).

Definition 2.6. [12] A fuzzy set μ in X is called a fuzzy ideal of X if it satisfies the following axioms:

(FI1) $\mu(x-y)\geq\min\{\mu(x),\mu(y)\}$,
(FI2) $\mu(xy)\geq\mu(y)$,
(FI3) $\mu(xy)\geq\mu(x)$
for all x, y ∈X.

Note that μ is called a fuzzy left ideal of X if it satisfies (FI1) and (FI2), and μ is called a fuzzy right ideal of X if it satisfies (FI1) and (FI3).

Example 2.7. Let X={0,a,b,c} in which "-" and "." are defined by the following table:

–	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then (X;-,.) is a subtraction semigroup ([4]). Let μ be a fuzzy set on X defined by μ(0)=0.8, μ(a)=0.5, μ(b)=0.3, μ(c)=0.1. Then it is easy to see that μ is a fuzzy ideal of X.

Proposition 2.8. Let μ be a fuzzy ideal in X. Then μ(0)≥μ(x) for all x∈X.

Proof. Using (FI2), we have μ(0)=μ(0x)≥μ(x) for all x∈X.

For the sake of completeness, we give the following theorems which are special cases of the Theorem 3.3 and Theorem 3.4 in [12].

Theorem 2.9. Let μ be a fuzzy left (right) ideal in X. Then the set I_μ *={ x∈X : μ(x)=μ(0)} is a left (right) ideal of X.*

Theorem 2.10. The subset A of X is an ideal of X if and only if μ_A *defined by in (2.1), is a fuzzy ideal of X. Moreover* I_{μ_A} *=A.*

Corollary 2.11. Let μ *be a fuzzy subset in X. If a non-empty level subset U of* μ *is an ideal of X, then* μ_U *is a fuzzy ideal of X.*

Theorem 2.12. [5] *Let μ be a fuzzy subset in X. μ is a fuzzy ideal of X if and only if any non-empty level subset U of μ is an ideal of X.*

Definition 2.13. For a fuzzy set μ in X, consider the following axiom:

(FII2) μ(xay)≥μ(a)

for all x,y,a∈X. μ is called a fuzzy interior ideal (FII) of X if it satisfies FI1 and FII2.

Proposition 2.14. Every FI of X is a FII.

Proof. By (FI2) and (FI3), we have μ(xay)=μ((xa)y)≥μ(xa)≥μ(a) for all x,y,a∈X.

Theorem 2.15. A is an interior ideal of X if and only if μ_A *is a fuzzy interior ideal of X.*

Proof. Let A be an interior ideal of X and denote μ_A= μ. Let x, a, y∈X. If a∈A, we have xay∈A since A is interior ideal of X. So we get μ(xay)=s=μ(a). If a∉A, then by the definition of μ, μ(xay)≥t=μ(a). Since A is also a subalgebra of X, by Theorem 2.5 (i), μ satisfies (FI1). Conversely, let μ is a FII of X. By Theorem 2.5 (i), A is a subalgebra. Now let xay be any element of XAX. Then since a∈A and μ is FII of X, we have μ(xay)≥μ(a)=s and hence we get μ(xay)=s. So it is obtained that xay∈A.

Theorem 2.16. Let μ be a fuzzy set of X. If μ is a FII of X, then each non-empty level set U=U(μ;t) (0≤t≤1) of X is an interior ideal of X.

Proof. Let μ be a fuzzy interior ideal of X and U=U(μ;t)={x∈X : μ(x) ≥t } for any t where 0≤t≤1 be a level set of μ. For x, y ∈U, since μ(x) ≥t and μ(y) ≥t and μ satisfies (FI1), we get μ(x-y) ≥ min{ μ(x), μ(y)} ≥t. Hence U is a subalgebra of X. For all x, z∈X and y∈U we have μ(xyz)≥μ(y)≥t. So we obtain xyz∈U, that is, XUX⊆U.

Definition 2.17. A fuzzy set μ in X is called a fuzzy bi-ideal (FBI) of X if it satisfies (FI1) and the following condition:

(BI2) μ(xyz)≥min{μ(x),μ(z)}
for all x,y,z∈X.

Proposition 2.18. Every FI of X is an FBI of X.

Proof. Let μ be a FI of X. Then we get μ(xyz)=μ((xy)z)≥μ(z) and μ(xyz)=μ(x(yz))≥μ(x). Hence we can write μ(xyz)≥min{μ(x),μ(z)} for all x,y, z∈X.

Theorem 2.19. Let Y be a non-empty subset of X and μ_Y be a fuzzy set of X defined by (2.1). Then Y is a bi-ideal of X if and only if μ_Y is a FBI of X.

Proof. Denote μ_Y=μ. Let x,w,y∈X. If x,y∈Y, we have xwy∈Y since Y is a bi-ideal. So we get μ(xwy)=s=μ(x)=μ(y)=min{μ(x),μ(y)}. If x∉Y or y∉Y, then by the definition of μ, we have μ(xwy)≥t=μ(x)=min{μ(x),μ(y)} (or μ(xwy)≥t=μ(y)=min{μ(x),μ(y)}). Since Y is also a subalgebra of X, by Theorem 2.5 (i), μ satisfies (FI1). Conversely, let μ is a FBI of X. By Theorem 2.5 (i), Y is a subalgebra. For all x,y∈Y and w∈X, we have μ(xwy)≥min{μ(x),μ(y)}=s and hence we get μ(xwy)=s. So it is obtained that xwy∈Y, that is, YXY⊆Y.

Theorem 2.20. Let μ be a fuzzy set of X. If μ is a FBI of X, then each non-empty level set U =U(μ;t) *(0≤t≤1) of X is a bi-ideal of X.*

Proof. Let μ be a FBI of X and U=U(μ;t)={x∈X : μ(x)≥t} for any t where 0≤t≤1 be a level set of μ. We have that U is a subalgebra of X as in the proof of Theorem 2.16. For all x,z∈U and y∈X, we have μ(xyz)≥min{μ(x),μ(z)}≥t. So we obtain xyz∈U, that is, UXU⊆U.

3. Intuitionistic Fuzzy Interior Ideals and Intuitionistic Fuzzy Bi-ideals

Definition 3.1. For an IFS A=(μ,γ) in X, consider the following axioms:

(S1) μ(x-y)≥min{μ(x),μ(y)},

(S2) $\gamma(x-y)\leq\max\{\gamma(x),\gamma(y)\}$,
(S3) $\mu(xy)\geq\min\{\mu(x),\mu(y)\}$,
(S4) $\gamma(xy)\leq\max\{\gamma(x),\gamma(y)\}$.

Then A is called intuitionistic fuzzy subalgebra (IFSA) of X if it satisfies (S1) and (S2) and A is called intuitionistic fuzzy subsemigroup (IFSS) of X if it satisfies (S3) and (S4).

Theorem 3.2. Let Y be a non-empty subset of X and A=(μ,γ) be an IFS of X defined by, for all x∈X,

$$\mu(x)=\begin{cases} s, & \text{if } x\in Y \\ t, & \text{if } x\notin Y \end{cases}, \quad \gamma(x)=\begin{cases} s, & \text{if } x\notin Y \\ t, & \text{if } x\in Y \end{cases} \tag{3.1}$$

where s,t∈[0,1], 0≤s+t≤1 and s>t. Then

(i) If Y is a subalgebra of X if and only if A is an IFSA of X,
(ii) If Y is a subsemigroup of X if and only if A is an IFSS of X.

Proof. (i) Let Y be a subalgebra of X and x,y ∈X. If x,y∈Y, then since x-y∈Y, we have $\mu(x-y)=s=\min\{\mu(x),\mu(y)\}$ and $\gamma(x-y)=t=\max\{\gamma(x),\gamma(y)\}$. If at least one of x and y does not belong to Y, then by the definition of μ and γ, we have $\mu(x-y)\geq t=\min\{\mu(x),\mu(y)\}$ and $\gamma(x-y)\leq s=\max\{\gamma(x),\gamma(y)\}$. So (S1) and (S2) are satisfied. Conversely, let x,y∈Y. Since A satisfies (S2) and $\gamma(x-y)\leq\max\{\gamma(x),\gamma(y)\}=t$, we have $\gamma(x-y)=t$ by the definition of γ. Hence we have x-y∈Y.

(ii) Let Y be a subsemigroup of X and x,y ∈X. If x,y∈Y, then since xy∈Y, we have $\mu(xy)=s=\min\{\mu(x),\mu(y)\}$ and $\gamma(xy)=t=\max\{\gamma(x),\gamma(y)\}$. If at least one of x and y does not belong to Y, then by the definition of μ and γ, we have $\mu(xy)\geq t=\min\{\mu(x),\mu(y)\}$ and $\gamma(xy)\leq s=\max\{\gamma(x),\gamma(y)\}$. So (S3) and (S4) are satisfied. Conversely, let x,y∈Y. Since A satisfies (S3) and $\mu(xy)\geq\min\{\mu(x),\mu(y)\}=s$, we have $\mu(xy)=s$ by the definition of μ. Hence we have xy∈Y.

Definition 3.3. An IFS A=(μ,γ) in X is called an intuitionistic fuzzy ideal (IFI) of X if it satisfies (S1), (S2) and the following conditions:

(S5) $\mu(xy)\geq\mu(y)$,
(S6) $\gamma(xy)\leq\gamma(y)$,
(S7)) $\mu(xy)\geq\mu(x)$,
(S8) $\gamma(xy)\leq\gamma(x)$
for all x,y∈X.

Note that A is an intuitionistic fuzzy left ideal (IFLI) of X if it satisfies (S1), (S2), (S5), (S6) and A is an intuitionistic fuzzy right ideal (IFRI) of X if it satisfies (S1), (S2), (S7), (S8).

Theorem 3.4. Let A=(μ,γ) be an IFLI of X. Then the set

$X_A=\{x\in X : \mu(x)=\mu(0),\ \gamma(x)=\gamma(0)\}$

is a left ideal of X.

Proof. Let x, y∈ X_A. Then since μ(x-y)≥min{μ(x),μ(y)}=μ(0) and μ(0)=μ(0(x-y))≥μ(x-y), we get μ(x-y)=μ(0). Also, since γ(x-y)≤max{γ(x),γ(y)}=γ(0) and γ(0)=γ(0(x-y))≤γ(x-y), we have γ(x-y)=γ(0). Hence x-y∈ X_A.

For all a∈X and x∈ X_A, since μ(ax)≥μ(x)=μ(0), μ(0)=μ(0(ax))≥μ(ax), γ(ax)≤γ(x)=γ(0) and γ(0)=γ(0(ax))≤γ(ax), we have μ(ax)=μ(0) and γ(ax)=γ(0). Hence ax∈ X_A. So X_A is a left ideal of X.

Similarly, the Theorem 3.4 can be proved for the right case.

Theorem 3.5. Let Y be a non-empty subset of X and A=(μ,γ) be an IFS of X defined by (3.1). Then A is an IFI of X if and only if Y is an ideal of X.

Proof. Suppose A is an IFI of X. By Theorem 3.2 (i), Y is a subalgebra of X. For all y∈Y and x∈X, since γ(xy)≤γ(y)=t, we have γ(xy)=t by the definition of γ. Hence we have xy∈Y. Similarly, it is obtained that yx∈Y for all y∈Y and x∈X. So Y is an ideal of X. Conversely, let Y be an ideal of X and x,y ∈X. By Theorem 3.2, (S1) and (S2) are satisfied. Also, if at least one of x and y belong to Y, since Y is an ideal of X, we have xy∈Y and so we get μ(xy)=s≥μ(x) (or μ(y)) and γ(xy)=t≤γ(x) (or γ(y)). If both x and y does not belong to Y, then we obtain μ(xy)≥t=μ(x)= μ(y) and γ(xy)≤s=γ(x)= γ(y). Hence (S5)-(S8) are satisfied, and A is an IFI of X.

Definition 3.6. An IFS A=(μ,γ) in X is called an intuitionistic fuzzy interior ideal (IFII) of X if it satisfies (S1), (S2) and the following conditions:

(II1) μ(xay)≥μ(a),
(II2) γ(xay)≤γ(a)
for all x, a, y∈X.

Theorem 3.7. Every IFI of X is an IFII.

Proof. Let A=(μ,γ) be an IFI of X. Then A satisfies (S1), (S2) and (S5)-(S8). In (S5) and (S6), if we write ay instead of y and use (S7) and (S8), we have μ(xay)≥μ(ay)≥μ(a) and γ(xay)≤γ(ay)≤γ(a).

Theorem 3.8. If X is a regular subtraction semigroup, then every IFII of X is an IFI.
Proof. It is clear by (S1), (S2) and Theorem 3.10 in [8].

Theorem 3.9. Let Y be a non-empty subset of X and A=(μ,γ) be an IFS of X defined by (3.1). Then Y is an interior ideal of X if and only if A=(μ,γ) is an IFII of X.

Proof. Let x, a, y∈X. If a∈Y, we have xay∈Y since Y is an interior ideal. So we get μ(xay)=s=μ(a) and γ(xay)=t=γ(a). If a∉Y, then, by the definition of μ and γ,

$\mu(xay)\geq t=\mu(a)$ and $\gamma(xay)\leq s=\gamma(a)$. Since Y is also a subalgebra of X, by Theorem 3.2 (i), A satisfies (S1) and (S2). Conversely, let $A=(\mu,\gamma)$ is an IFII of X. By Theorem 3.2 (i), Y is a subalgebra of X. Now let xay be any element of XYX. Then since $a\in Y$ and A is an IFII of X, we have $\mu(xay)\geq\mu(a)=s$ and hence we get $\mu(xay)=s$. So it is obtained that $xay\in Y$.

Definition 3.10. An IFS $A=(\mu,\gamma)$ in X is called an intuitionistic fuzzy bi-ideal (IFBI) of X if it satisfies (S1), (S2) and the following conditions:

(BI1) $\mu(xwy)\geq\min\{\mu(x),\mu(y)\}$,
(BI2) $\gamma(xwy)\leq\max\{\gamma(x),\gamma(y)\}$
for all x, w, $y\in X$.

Theorem 3.11. Let Y be a non-empty subset of X and $A=(\mu,\gamma)$ be an IFS of X defined by (3.1). Then Y is a bi-ideal of X if and only if $A=(\mu,\gamma)$ is an IFBI of X.

Proof. Let x, w, $y\in X$. If x, $y\in Y$, we have $xwy\in Y$ since Y is a bi-ideal. So we get $\mu(xwy)=s=\mu(x)=\mu(y)=\min\{\mu(x),\mu(y)\}$ and $\gamma(xwy)=t=\gamma(x)=\gamma(y)=\max\{\gamma(x),\gamma(y)\}$. If $x\notin Y$ or $y\notin Y$, then by the definition of μ and γ, we obtain $\mu(xwy)\geq t=\mu(x)=\min\{\mu(x),\mu(y)\}$ (or $\mu(xwy)\geq t=\mu(y)=\min\{\mu(x),\mu(y)\}$) and $\gamma(xwy)\leq s=\gamma(x)=\max\{\gamma(x),\gamma(y)\}$ (or $\gamma(y)=\max\{\gamma(x),\gamma(y)\}$). Since Y is also subalgebra of X, by Theorem 3.2 (i), A satisfies (S1) and (S2). Conversely, let $A=(\mu,\gamma)$ is an IFBI of X. By Theorem 3.2 (i), Y is a subalgebra. Now let xwy be any element of YXY. Then since $x,y\in Y$ and A is an IFBI of X, we have $\mu(xwy)\geq\min\{\mu(x),\mu(y)\}=s$ and hence we get $\mu(xwy)=s$. So it is obtained that $xwy\in Y$.

Let Y be a non-empty subset of X and χ be the characteristic function of Y. Then $A=(\chi,\chi')$ where $\chi'=1-\chi$ is special case of A defined by (3.1). Therefore $A=(\chi,\chi')$ satisfies all proved Theorems about A defined by (3.1).

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Zekiye Çiloğlu e-mail: zekiyeciloglu@sdu.edu.tr