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A hybrid iterated local search-firefly algorithm for solving discrete p-center problem

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Abstract

P-center problem is one of the most well-known location problems which is classified as NP-Hard. The purpose of this problem is to locate P new facilities among customers such that the maximum distance between customers and their nearest facility is minimized. Firefly method is a new meta-heuristic method that is used for optimization of NP-Hard and combinatorial problems which were mainly developed for optimization of continuous problems inspired by social behavior of fireflies in producing light for the mating. In this study, a hybrid iterated local search-firefly algorithm (HIF) approach was proposed to solve discrete p-center problem which was achieved by combining iterated local search (ILS) and firefly methods. The proposed algorithm was used for solving the OR-LIB problems and the results of method implementing were compared to obtained results from greedy harmony search (GHS) method. It was found the better performance of the proposed HIF algorithm than the GHS method. According to the results, the proposed HIF method on average has more than 60 percent lower error than GHS method and the time to yield the solution decreased about 32 percent.

Keywords: P-center problem, hybrid iterated local search-firefly algorithm, Firefly algorithm, Iterated local search algorithm

1. INTODUCTION

P-center problem is one of the most well-known facility location problems. The purpose of solving this problem is locating facilities, or centers among demand points, in such a way that the maximum distance between the demand points and their nearest facility is minimized. Applications of this problem include location of facilities, emergency and critical centers within transportation networks such as locating ambulance stations, firefighting centers, police stations, database centers etc. In emergency services, locating facilities or service centers should be done in a way that the farthest distance from these centers or facilities that should be sent to the required place after receiving an emergency call is minimized [1 and 2] Figure (1) shows a number of supply and demand points. Smaller circles are the demand points (n), while hollow larger circles are the facilities or supply points (P). In this figure, the demand points are connected to their nearest facility and the maximum distance is specified by a dashed line. Purpose of a p-center problem is to determine the location of P supply points so that the maximum distance is minimized.

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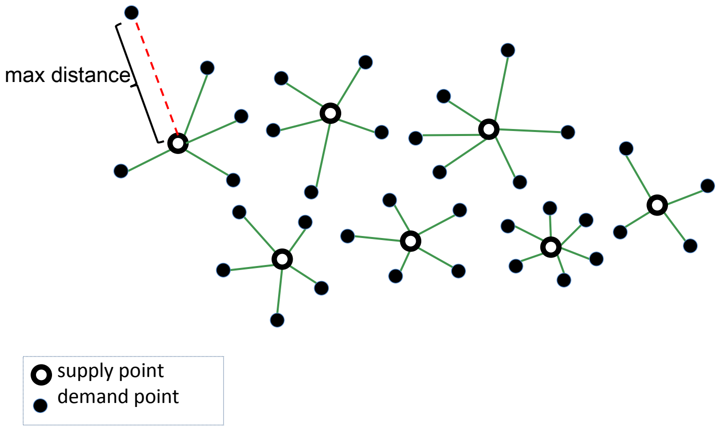

Figure 1. Schematic p-center problem.

Nowadays, meta-heuristic methods have become very powerful tools for solving NP-Hard and combinatorial optimization problems. Meta-heuristic algorithms present a computational method that repeatedly tries to improve a candidate solution by measuring quality of the solution. Although the meta-heuristic methods do not guarantee an optimal solution, they are able to produce quick, approximate and sometimes optimum solutions. Among these methods, there are some algorithms which are inspired by the nature and knowledge of biology [3].

Firefly meta-heuristic method was inspired for the first time by Yang in 2008 from social behavior of the fireflies in emitting light for the mating purpose which can be used to optimize NP-Hard problems [4] In this study, ILS and firefly methods were combined as a hybrid method and named HIF to solve the discrete p-center problem.

P-center problem concepts were first introduced by Hakimi in a study in which these concepts were used to find an optimal location for switching centers in a telecommunication network and also to locate the best place to build a police station in a highway system [1]. Cariv and Hakimi, proved in their study that p-center problem is NP-hard. By designing algorithms for solving Absolute p-center and Vertex p-center problems, when the number of centers on the network is $1<p<n$, they have shown that P-center problem is still NP-hard, even if network has a simple structure [2] Drezner *proposed two* heuristic algorithms and an exact algorithm for optimization of p-center problem with Euclidean distance [5] In a study aimed to solve large P-median problems, Beasley proposed a set of large sample problems for solving these problems as a set called OR-Library [6] Later, these kinds of sample problems were used and solved by researchers in p-center problem literature. Mladenovic et al. for the first time, used a meta-heuristic variable neighborhood search (VNS) algorithm and two Tabu search (TS) algorithms to solve p-center problems. Since no satisfactory exact and heuristic method has been proposed for solving large problems, meta-heuristic methods were proposed for the first time for solving multi-facilities center problems [7] Kaveh and Nasr in a study modified harmony search meta-heuristic method, which was originally introduced by Geem et al. in 2001 for solving optimization problems, to solve the discrete p-center problem. They called this method GHS and proposed it for solving the discrete p-center problem on the network. In this study, a number of experimental pmed problems of OR-LIB set were solved to evaluate performance of the proposed GHS method. In the conducted tests, results of GHS algorithm implementation were compared with other approximation methods in the research literature, including VNS, TS and Scatter search methods as well as classic harmony search algorithm. Finally, it was shown that the proposed GHS method is able to find the optimum solution better than any other methods, in terms of time [8 and 9] Yang in 2008, presented the firefly meta-heuristic algorithm to optimize continuous problems which was inspired by social behavior of the fireflies in emitting light for the mating purpose.

The proposed HIF algorithm to solve a discrete p-center problem

P-center problem is also known as minimax problem because of its objective function. In p-center problem on a network, a set of nodes (N) is specified as the customers and these demand nodes are connected to each other by edges (E). Graph G is shown as G=(N,E). This graph contains N nodes and E edges where the nodes are connected to each other by edges. This set forms a network.

The distance between nodes in the network is measured based on the shortest path [2] The distance between two points on the network is defined as the shortest path connecting these two points on the network. There are algorithms that specify the shortest path between two nodes, such as Dijkstra's algorithm. In this study, Dijkstra's algorithm will be used to calculate the shortest distance of demand nodes from the center ones.

P-center problems are either continuous or discrete. In continuous problems, p-center points can be located at any place on the network space including nodes and edges (Absolute P-Center), while in discrete problems (Vertex P-Center), each new facility can only be located on the network nodes [2] In this study, a discrete p-center problem will be studied.

Fireflies are among the most interesting examples of insects in the nature that inspired many poets and scientists in their works and researches. Fireflies are known for the bio-fluorescent light that is produced through a chemical process. Emitting this light is considered as a sign for mating.

The first light signals are produced by male fireflies to attract female fireflies which are on the ground without light. In response to these signs, female fireflies produce continuous or flashing lights. Both male and female fireflies emit lights with specific patterns with precisely scheduled radiation so that information about the species and gender is encoded in the light. Female fireflies usually prefer males with higher light radiation. The light intensity varies with distance to the light source. This algorithm inspired from variation of light intensity was first developed by Yang [4] According to his findings, the firefly algorithm performs effectively with high success rate in finding global optimum. Two main issues in the firefly algorithm include light intensity variation and formulation of attractiveness.

In this algorithm, some of the properties of flashing light of fireflies were used as inspiration for solving NP-Hard optimization problems. To simplify the firefly meta-heuristic algorithm, the following four laws are founded:

A. All fireflies are categorized in one gender, such that each firefly is attracted to other fireflies regardless of the gender.
B. Attractiveness is proportional to the light intensity. Therefore, for two flashing fireflies, the firefly with lower light moves towards the firefly with higher light.
C. Attractiveness is proportional to the brightness and both of them decrease as the distance increases.
D. If there is no other firefly with higher light than a specific firefly, the firefly will move randomly in the space.

Light intensity for each firefly is obtained by calculating the objective function value based on equation (1). Thus, this value is defined for problems, considering whether there is maximization or minimization problem.

$$I(x) \;\alpha\; f(x) \tag{1}$$

While the attractiveness β is relative and this value can be judged from the perspective of an observer or other fireflies. Hence, level of the attraction varies based on the distance r_{ij} between fireflies i and j. Light intensity decreases as distance from the source increases. Thus, attractiveness (β) is calculated based on equation (2). In this equation I_s is the light intensity value in the source and γ is the light absorption coefficient which is a constant value .

$$\beta = I_s / r^{\gamma} \quad (2)$$

ILS is a meta-heuristic method that deals with application of repeating process in a heuristic local search method. Instead of randomly generated initial solution for each iteration, ILS perturb current solution and use it as a starting solution for next iteration. Instead of making a new initial solution, the perturbation mechanism provides an initial promising solution by maintaining a part of the previous solution structure which has turned it into a good solution. In local search methods, there is a chance of being trapped in a local optimum. ILS is the idea of perturbing the solution in order to avoid being trapped in a local optimum and to search within the interesting areas of problem space. The assumption of this method is based on the fact that obtaining the better solution when a local search begins from a good minimum point is easier than when it begins from a completely random point [10]

Despite simplicity of this method, studies have shown that ILS has been a successful approach for solving optimization problems. Meta-heuristic algorithms that rely on a single solution like ILS method are more inclined to extract solution, while meta-heuristic methods based on population solution, such as firefly method, are more exploration oriented and conduct a more extensive search within the search space [11]

In the proposed HIF hybrid method, the following symbols are used:

N	Number of fireflies
M	Max firefly generation
T	Generation counter
I_n	Light intensity of firefly n
γ	Light absorption coefficient (0< γ<1)
r_{ij}	Distance between firefly i,j
Itr	Iteration counter

Proposed HIF method consists of two main steps:

1- Generating the initial solution: since ILS method is based on the improvement of produced initial single solution, first an initial random solution is generated based on number of centers (p) which must be locating. Then, value of the objective function would be calculated for the solution. The objective function value in p-center problem is as follows:

$$d(x, P_i) = \min \{d(x_j, P_i)\} : \quad j=1,\dots,n , i=1,\dots, p \quad (3)$$

$$f(x) = \max \{d(x, Pi)\} : \quad i=1,\dots, p \quad (4)$$

Equation (3) is related to the minimum distance of each node to its closest facility which is calculated by Dijkstra's *algorithm.* Equation (4) is related to the maximum distance among the distances calculated by equation (3), which should be minimized based on the objective function of the problem.

2- Local search: the improvements are made on the initial solution produced in the previous step with ILS solutions using the concept of the firefly optimization method. According to Figure (2), populations of n fireflies are generated for first center of the initial solution.

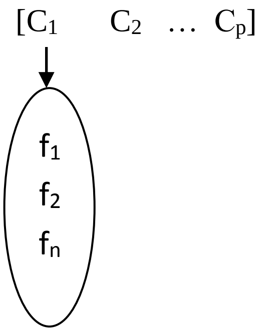

Figure 2. Generating of n explorer fireflies for first center.

In order to use firefly method for solving discrete p-center problem, some changes were made to the original algorithm. In the firefly search method, the light intensity value and the attractiveness must be specified. The light intensity value for each firefly is calculated by calculating its objective function value based on equation (5).

$$I_j = f(x_j) \tag{5}$$

Based on the classic firefly algorithm, the light intensity value with specific distance r from the source is inversely related to the distance. This means that by increasing the distance r, light intensity value I decreases. In the proposed method, the attractiveness is obtained by equation (6):

$$\beta j = e^{\frac{m-t}{r^{\gamma}}} * \left(\frac{1}{r^{\gamma}\left(\frac{m-t}{i}\right)}\right) \tag{6}$$

In equation (6) *r* is the distance between two fireflies and since the problem space in this study is a discrete network, the closest distance between them is calculated by Dijkstra's *algorithm.* Also γ is the light absorption coefficient which is a constant value ($0 \leq \gamma \leq 1$).

Extent of the search process (exploration) will increase, by increasing the size of firefly’s population (n). On the other hand, by increasing the number of firefly, CPU usage time also increases, thus an appropriate size for the fireflies’ population should be specified considering size of the problems.

In following, light intensity value would be calculated for each firefly based on equation (5). Considering type of objective function of the problem which is of a minimization type, a firefly with less light attracts the firefly with higher light.

The proper movement is not only based on the amount of light intensity and should be conducted by calculating the attractiveness based on equation (6). If the objective function value for only one firefly is less than the current objective function value, then the current center is replaced by this firefly in the initial solution. If the amount of light intensity of more than one firefly is less than the objective function value for the current center, value of *GA* is calculated for those fireflies whose amount of light is less than the objective function value using equation (7).

$$GA_j = I_j * \beta_j \qquad (7)$$

A firefly whose *GA* value is less than others, would be replaced with the current center. Next, the same process will repeat for the next centers. In equation (6), *m* is the number of produced firefly generations, *t* is the generation counter and *i* is the order of studied centers in the solution. In this equation, $(m-t)$ causes that the fireflies (nodes) near the studied center to be selected in the initial generations and the fireflies that produce the most minimization in the objective function to be selected in the last generations. $\frac{m-t}{i}$ also causes that the fireflies (nodes) near the studied area to be selected in the initial generations and fireflies that created the highest minimization in objective function to be selected in the last generations and centers. This would maintain power of the algorithm in searching for the global optimum (intensification). Local search process will continue until the desired termination criterion is met. The Pseudo code of proposed HIF hybrid algorithm is shown in Figure (3).

```
Begin
Objective function f(x);     x = (x1,…, xp)
Define n, m, γ, Maxiteration,
While  itr < Maxiteration
     S← generate initial solution
     While (t < m)
        for i=1:p  all centers
       Generate initial population of fireflies xj (j = 1,2,…,n)
       Light intensity Ij  at  xj  is determined by f(xj)
             for j =1:n   all n fireflies
                          if (Ii > Ij)
               Move firefly i towards j via GAj
                  Attractiveness varies with distance rij via
                  βj = e^((m−t)/r^γ) * (1/(r^γ((m−t)/i)))
              end if
             Evaluate new solutions and update light intensity
           end for j
          Rank the fireflies and find the current best
     end for i
   End While
End Wile
Postprocess results and visualization
End
```

Figure 3. Pseudo code of proposed HIF Algorithm.

Numerical results

In the research literature related to p-center problem, researchers have used pmed test samples of OR-LIB set, which are the examples of p-median problem, to evaluate their proposed methods and to compare it with other provided solution methods in research literature of p-center problem.

In this study, in order to evaluate performance of two methods, first algorithm of GHS and HIF methods were coded in Matlab. Then, pmed test samples of OR-LIB set that was solved in Kaveh and Nasr study in 2011 were solved using the two above-mentioned methods and the experimental results were compared.

As previously mentioned, it was shown in previous studies that exact solution of p-center problem is a NP-hard. Also, results of the studies have shown that the approximate solution of the problem, when the deviation error is small, is also NP-hard [2] Error level in the approximate algorithms can be considered as an approximation factor. These methods are able to find approximate, quick and sometimes optimum solutions [12]

Thus, in this study to compare the proposed HIF method with GHS method for solving discrete p-center problem, first each problem was solved 15 times by each of the algorithms. Finally, the best result obtained from15 times running was recorded for each method.

Termination criterion was based on achieving the optimum solution and maximum allowable time for CPU usage which varies between 50 and 250 seconds for different problems based on difficulty of the problem. To solve the problems, a computer with an Intel Core i3-2120, 3.30 GHz CPU and 4GB RAM was used.

Results of implementing HIF algorithm in comparison with GHS results are shown in table (1). In this table, in the fifth column from left, the optimum solution of the problem which was extracted from the literature is shown.

Table 1. Results of solving OR-LIB tests using GHS and HIF algorithms.

Test No.	Nodes (n)	Centers (p)	Edges	Opt.	GHS Results				HIF Results			
					Best of HIF	Dif. to Opt.	Deviation Percent	CPU Time (s) for HIF	Best of GHS	Dif. to Opt.	Deviation Percent	CPU Time (s) for GHS
#1	100	5	200	127	**<u>127</u>**	0	%0	34	**<u>127</u>**	0	%0	**5**
#2	100	10	200	98	112	14	%12.5	48	**<u>98</u>**	0	%0	**26**
#3	100	10	200	93	106	13	%12.3	80	**95**	2	%2.1	**62**
#6	200	5	800	84	88	4	%4.5	64	**85**	1	%1.2	**20**
#7	200	10	800	64	75	11	%14.7	50	**67**	3	%4.5	**37**
#11	300	5	1800	59	61	2	%3.3	124	**<u>59</u>**	0	%0	**92**
#12	300	10	1800	51	60	9	%15	160	**54**	3	%5.6	**100**
#16	400	5	3200	47	50	3	%6	147	**<u>47</u>**	0	%0	**98**
#17	400	10	3200	39	46	7	%15.2	172	**41**	2	%4.9	**95**
#21	500	5	5000	40	45	5	%11.1	81	**41**	1	%2.4	**60**
#22	500	10	5000	38	50	12	%24	112	**42**	4	%9.5	**102**
#26	600	5	7200	38	41	3	%7.3	98	**<u>38</u>**	0	%0	**74**
#27	600	10	7200	32	37	5	%13.5	207	**35**	3	%8.6	**183**
#31	700	5	9800	30	33	3	%9.1	136	**32**	2	%6.3	**104**
#32	700	10	9800	29	40	11	%27.5	193	**33**	4	%12.1	**168**
#35	800	5	12800	30	**32**	2	%6.3	182	**32**	2	%6.3	**114**
#36	800	10	12800	27	36	9	%25	209	**32**	5	%15.6	**159**
#38	900	5	16200	29	33	4	%12.1	236	**30**	1	%3.3	**162**
#39	900	10	16200	23	29	6	%20.7	250	**26**	3	%11.5	**219**
Bold numbers are better results and underlined numbers indicate that optimum solution is met												

y investigating table (1), it is noted that the proposed HIF algorithm in all the problems had better performance than the GHS algorithm in terms of both the solution quality and the time required to obtain the solution (CPU time).

Results indicated that HIF yielded the optimum solution in 5 problems among 19 problems, while GHS method achieved the optimum solution only in one of the problems. Also, results related to CPU time for the two algorithms showed that in all the problems, the proposed HIF method had significantly better time efficiency compared to GHS method. The reason for this result is due to the high convergence rate of firefly method in achieving the solution.

In Figure (4), the best results by GHS and HIF methods are compared with the optimum solutions of 19 problems tested in this study.

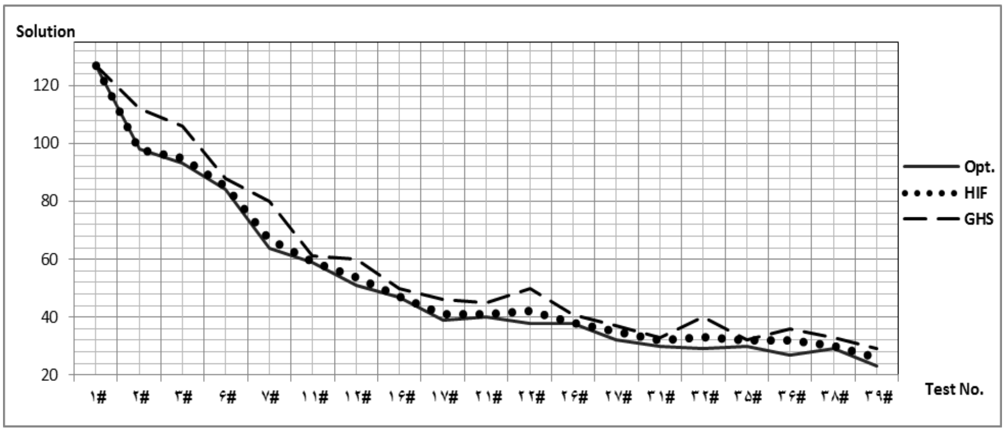

Figure 4. The best results obtained by GHS and HIF method compared with optimum solution.

In table (1), difference of the best results obtained by GHS and HIF methods with the optimum solution and also the deviation error percentage from the optimum solution related to 19 problems are specified. It is observed that on an average basis, the proposed HIF method produced 60 percent lower error than GHS method and the time required to achieve the solution decreased by about 32%.

Discussion and Conclusion

Results of solving OR-LIB test problems with HIF and GHS methods and comparing them indicated significantly better performance of the proposed HIF method, both in terms of the solution quality (average error of GHS and the proposed HIF methods are equal to 12.64% and 4.94% respectively) and also the time required to obtain the solution (the average time to achieve solution by GHS and the proposed HIF methods are equal to 136 seconds and 99 seconds, respectively) compared to the GHS. The reason for success of the proposed HIF method is the fast convergence of this optimization method. We can conclude that this method provides a very fine and targeted balance between diversification by the firefly method and power of the algorithm in converging to the optimum point (intensification) by ILS method. In another word, it uses the randomization factor to extent the search space and to prevent being trapped in local optimum solution, also benefits from targeted movement of the fireflies towards the global optimum with appropriate speed; unlike the GHS that only use randomization in all the steps of algorithm implementation.

Diversification in the firefly algorithm is controlled by characteristic of this method which relies on fireflies' population and power of algorithm in convergence to the optimum solution is accomplished by the movement of fireflies towards fireflies that provide better estimate of the objective function. In addition, capability to adjust these two parameters, i.e. initial population of fireflies and the rate of attraction of fireflies' movement towards fireflies with higher level of light, plays an important role in the success of this meta-heuristic method depending on size and difficulty of the problem.

In the proposed HIF method, ILS was used as a framework for firefly method. ILS is a meta-heuristic method that relies on a single solution and the methods based on single solution have the approach to extract the solution. Hence, combination of this method with firefly method reinforced the capability of firefly method to achieve solution. In fact, in HIF method, an initial single solution is produced and then improved by capabilities of extent of search and fast convergence in the firefly method and the capability of perturbation in the ILS. We can say that by combining capabilities of these two methods, power of the algorithm in avoiding getting trapped in local optimum and providing appropriate solution in low time termination criteria significantly increased.

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