SEIGNIORAGE REVENUE AND TURKISH ECONOMY

Cem SAATÇİOĞLU* ve H. Levent KORAP**

Abstract

In our paper, we try to investigate the courses of inflation tax and seigniorage revenue for policy makers of the Turkish economy. For this purpose, we first construct an *ex-ante* seigniorage revenue maximizing inflation model, and then calculate annual inflation tax and seigniorage revenues for the post-1980 period Turkish economy. Following these theoretical issues, an empirical model is constructed upon Turkish economy, and our *ex-post* estimation results reveal that inside the period considered, the Turkish economy lies on the correct or efficient side of the seigniorage maximizing Laffer curve.

Key Words: Seigniorage, Inflation Tax, Laffer Curve, Turkish Economy

Senyoraj Geliri ve Türkiye Ekonomisi

Özet

Çalışmamızda, Türkiye ekonomisi için politika yapılcılar açısından enflasyon vergisi ve senyoraj gelirlerinin nasıl bir gelişim gösterdiği incelenmeye çalışılmaktayız. Bu amaçla ilk olarak kuramsal beklentilerimiz doğrultusunda senyoraj gelirlerini encoğraflıran enflasyon oranı modeli oluşturulmaka ve 1980 sonrası Türkiye ekonomisi verileri dikkate alınarak yıllık elde edilen enflasyon vergisi ve senyoraj gelirleri hesaplanarak bu gelirleri encoğraflıran enflasyon oranı tahmin edilmektedir. Elde ettiği tahmin sonuçları Türkiye ekonomisinin inceleme dönemi içerisinde senyoraj gelirlerini encoğraflıran Laffer eğrisinin doğru ya da etkin tarafında bulunduğu göstermektedir.

Anahtar Kelimeler: Senyoraj, Enflasyon Vergisi, Laffer Eğrisi, Türkiye Ekonomisi

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I. INTRODUCTION

Inflation had been one of the most important issues affecting the courses of Turkish business cycles since the 1970s and 1980s. Having a two digits characteristics of chronic inflationary framework, as can be seen in Table 1 below, the Turkish experience constituted a privileged position in the world economy, which was neither hyperinflation of the two digits price increases in a monthly basis nor a moderate inflation of the single digits in an annual basis. Saatçioğlu and Korap (2006) examine various potential reasons leading the Turkish economy into such a process of inflation,

TABLE 1: ANNUAL PER CENT CHANGE in CONSUMER PRICES of SOME DEVELOPING COUNTRIES  
10 year average
Turkey 70.9 85.0 83.6 63.5 54.3 53.9 44.8 25.3 10.6
South Africa 12.1 8.6 6.9 5.2 5.4 5.7 9.2 5.8 1.4
Hungary 21.8 18.3 14.3 10.0 9.8 9.2 5.3 4.7 6.8
Chile 15.3 6.1 5.1 3.3 3.8 3.6 2.5 2.8 1.1
Brazil 36.7 20.6 15.9 16.6 9.5 6.4 5.0 4.5 4.7
Bulgaria 63.2 1061.2 18.8 10.0 7.3 5.3 4.7 3.6 2.5
Poland 78.2 14.9 11.8 7.3 10.1 5.5 1.9 0.8 3.5
Argentina 76.8 154.8 59.1 45.8 45.7 34.5 22.5 15.3 11.9
Russia 656.6 6.9 3.2 4.9 7.1 6.8 8.4 14.8 6.6
Peru 287.4 8.5 7.3 3.5 3.8 2.0 0.2 2.3 3.7

As a developing country, that the central government resorts to seigniorage revenues so as to finance its expenditures or to domestic borrowing possibilities or the case of taxation may, to the great extent, affect the course of domestic inflation, and this case can easily be related for empirical purposes to the question of whether the governments succeed in collecting the maximum possible seigniorage revenue due to the monopoly of printing money. Thus, examining this issue may help us to obtain some clues or foresight for the privileged position of the Turkish economy inside the world economy.

The outline of the paper is as follows. Section two interests in theoretical relationships between the seigniorage and inflation tax, while section three gives some stylized facts in the Turkish economy following the definitions presented in the prior section. In line of these considerations, section four tries to estimate an empirical model à la Cagan (1956: 25-117) and examines whether the seigniorage
maximizing inflation values are attained in the Turkish economy. And the final section concludes.

II. SEIGNIORAGE AND INFLATION TAX

From a theoretical perspective, some distinction would probably be necessary to be made between seigniorage revenue collected from printing money and inflation tax. Seigniorage can be defined as the value of real resources acquired by the government through its ability to print money (Begg, Fischer and Dornbusch, 1994: 491).¹ Let SE represent the real seigniorage revenue, M nominal money balances or the non-interest bearing high powered money and P price level. So we can construct the real seigniorage relationship such as,

\[
SE = \left( \frac{\Delta M}{P} \right) = \left( \frac{\Delta M}{M} \right) \left( \frac{M}{P} \right) = \mu m \tag{1}
\]

where \( \mu \) is the change in nominal money balances, \( m \) the real money balances and \( \Delta \) the difference operator. Following Blanchard (1997: 430), the larger the real money balances held in the economy, the larger the amount of seigniorage corresponding to a given rate of money growth. On the other side, the inflation tax refers to the increase in nominal money balances which individuals have to accumulate to keep their real balances constant in an inflationary framework. Let IT be the inflation tax,

\[
IT = \left( \frac{\Delta P}{P} \right) \left( \frac{M}{P} \right) = \pi m \tag{2}
\]

where \( \pi \) is the inflation rate. Equation 2 provides that government can reduce the real value of the non-interest-bearing part of the government debt by using inflation (Begg, Fischer and Dornbusch, 1994: 491). In this sense, we can interpret \( \pi \) as the inflation tax rate and \( m \) as the tax base. When the inflation rate is zero, the government gets no revenue from inflation, while the amount of inflation tax received by the government would increase as the inflation rate rises. But as the inflation rate rises, people would reduce their holdings of the money base due to the fact that monetary base is now more costly to hold. Thus, individuals hold less currency, and banks hold as little excess reserves as possible, and eventually the real monetary base falls so much that the total amount of inflation tax revenue received by the government falls (Dornbusch and Fischer, 1994: 555-556).

The difference between seigniorage and inflation tax arises from changes in real money demand, which in turn may be the consequence of financial liberalization or changes in the inflation rate, real income, and interest rates. This difference is sometimes referred to as the non-inflationary component of seigniorage, as it is the increase in money demand that is consistent with a zero real interest rate.

¹ The term seigniorage comes from the French word seigneur which represents the feudal lord of the Middle Age who had monopoly power on his land to coin money (Blanchard, 1997: 431).
inflation rate (Rodrik, 1990). Besides, as the economy grows the government can obtain some revenue from seigniorage even if there is no inflation. That is because when the demand for real monetary base is growing, the government can create some base without producing inflation (Dornbusch and Fischer, 1994: 555; Paya, 1997: 377; Sönmez, 1998: 364).

If we consider a Laffer curve to represent the seigniorage revenue against inflationary framework,

**FIGURE 1: SEIGNIORAGE LAFFER CURVE**

![Diagram](image)

Source: Şiklar (1998: 8)

where $S$ represents the seigniorage revenue as a proportion of the GDP and $\pi$ the domestic inflation rate. In Figure 1 above, we see that the seigniorage maximizing inflation rate is B with an inflation rate of $\pi_0$. Before this point the higher the inflation rate the larger the seigniorage revenue by means of an increase in the base money, and to the right of the point B the higher the domestic inflation the lower the seigniorage revenue, since economic agents would try to avoid holding base money balances so that they can protect themselves from incurring inflation tax by reducing real monetary balances in their hands. We can also notice in Figure 1 that the same seigniorage revenue can be collected by imposing different inflation rates such as $\pi_2$ and $\pi_1$, where the tax rate is higher but the tax base is lower, that is the wrong side of the seigniorage maximizing Laffer curve in the latter case with respect to the former. In this line, the former coincides with the correct or efficient side of the Laffer curve in which there is still opportunity for a higher seigniorage at higher inflation rates (Kiguel and Neumeyer, 1995: 672), and there is an implicit
loss of seigniorage revenue if the economy moves to a lower level of inflation (Soydan, 2003).²

III. THE COURSE OF SEIGNIORAGE AND INFLATION TAX IN THE TURKISH ECONOMY

If we now follow the definitions given above, we can calculate the seigniorage revenue and inflation tax incurred by the Turkish economy as of the beginning of 1980 in an annual basis such as Anand and van Wijnbergen (1988), Rodrik (1990) and Altunkemer (1994). Following Rodrik (1990) in Table 2 below, seigniorage in our paper refers to the revenue raised by the monetary authority by issuing non-interest-bearing liabilities, i.e. base or reserve money (MB), and seigniorage revenue as a share of gross national product (GNP) is given by the increase in MB in a given year divided by the last year’s GNP, while inflation tax is obtained by multiplying annual CPI inflation with preceding year’s MB over the GNP.³ All the calculations are based on the data taken from the electronic data delivery system of the Central Bank of Republic of Turkey (CBRT),

<table>
<thead>
<tr>
<th>YEAR</th>
<th>SEIGNIORAGE (ΔMB/GNP)</th>
<th>INFLATION TAX (π*MB(-1)/GNP)</th>
<th>INFLATION (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>3.51</td>
<td>2.64</td>
<td>41.88</td>
</tr>
<tr>
<td>1982</td>
<td>3.95</td>
<td>2.04</td>
<td>27.49</td>
</tr>
<tr>
<td>1983</td>
<td>2.28</td>
<td>2.72</td>
<td>31.39</td>
</tr>
<tr>
<td>1984</td>
<td>3.06</td>
<td>2.24</td>
<td>32.62</td>
</tr>
<tr>
<td>1985</td>
<td>1.55</td>
<td>2.80</td>
<td>44.98</td>
</tr>
<tr>
<td>1986</td>
<td>1.56</td>
<td>1.86</td>
<td>34.59</td>
</tr>
<tr>
<td>1987</td>
<td>1.81</td>
<td>1.84</td>
<td>38.91</td>
</tr>
<tr>
<td>1988</td>
<td>2.32</td>
<td>2.95</td>
<td>77.63</td>
</tr>
<tr>
<td>1989</td>
<td>3.47</td>
<td>2.17</td>
<td>63.16</td>
</tr>
<tr>
<td>1990</td>
<td>1.11</td>
<td>2.41</td>
<td>60.26</td>
</tr>
<tr>
<td>1991</td>
<td>1.54</td>
<td>2.11</td>
<td>66.08</td>
</tr>
<tr>
<td>1992</td>
<td>1.71</td>
<td>1.91</td>
<td>70.10</td>
</tr>
</tbody>
</table>

² Bruno and Fischer (1990: 353-374) relate analytically such a dual equilibrium obtained at either a low or high inflation rate as a reflection of Laffer curve to expectations of economic agents and indicate that under rational expectations the high inflation equilibrium is stable and the low inflation equilibrium is unstable while under adaptive expectations or lagged adjustment of money balances with rational expectations the low inflation equilibrium may be stable. They also emphasize that a unique equilibrium is attained when the government sets a nominal anchor for the economy.

³ For 1980, 1981 and 1982 the GNP deflator is used in constructing the annual inflation series.
Consistent with the findings of Rodrik (1990), we see in Table 2 that public sector seems to rely on seigniorage and inflation tax revenue in the range of %1.5 and %3.0 of the GNP along the whole 1980s, but beginning from the early 1990s and in line of the findings of Özmen (1998: 545-553) and Koru and Özmen (2003: 591-597) both inflation tax and the seigniorage revenue are tried to be kept relatively constant around only %1 of the GNP in the face of fluctuating and accelerating inflation inside the period, except the pre-1994 crisis period which points out applying to inflation tax for the policy makers. Besides, relatively high and accelerating periods of inflation do not necessarily require for larger seigniorage, as the higher levels of inflation result in substantial erosion in the real demand for money and reduce the base of the tax (Rodrik, 1990).4

IV. ESTIMATION OF THE CAGAN MODEL FOR TURKISH ECONOMY

To examine the issues of seigniorage revenue and inflation tax expressed so far, a standard way used for empirical purposes is of the form in Cagan’s (1956: 25-117) semi-logarithmic money demand function relating real balances demanded as the outside money under the control of monetary authority or the narrowly defined money balances to the measure of inflation. Expressing analytically and using \( (M/P)^d \) for real monetary balances demanded and \( \pi \) for the domestic inflation in exponential form,

\[ (M/P)^d = a \cdot e^{b \pi} \]

4 De Haan, Zelhorst and Roukens (1993: 313-314) present inflation tax data of 42 developing countries including Turkey for the period 1962-1985 in which Turkey is inside the first 12 countries obtaining maximum inflation tax revenue as percentage of the GNP. Similarly, Adam, Ndulu and Sowa (1996: 536) give similar calculations of seigniorage revenue and inflation tax over twenty years for some African countries consisting of Kenya, Tanzania and Ghana.
(M/P)^d = Ae^{-\alpha \pi} \quad (3)

or explicitly in natural logarithms,

\log_e(M/P) = a - \alpha \pi + \varepsilon \quad (4)

where \alpha is the semi-elasticity of money demand with respect to inflation as the opportunity cost to holding money. Taking the partial derivative of real monetary balances with respect to the domestic inflation below,

\pi^* = \frac{1}{\alpha} \quad (5)

we obtain the inflation rate (\pi^*) that maximizes the seigniorage revenue. Using the model constructed so far, we now apply to the Turkish data and try to estimate a Cagan type model of money demand for the period of 1987.1-2004.4 using quarterly observations. We use a variety of econometric procedures available in the program Eviews 5.0. All the data we use are taken from the electronic data delivery system of the Central Bank of Republic of Turkey (CBRT) and indicate seasonally unadjusted values. The monetary variable we consider (LNREALM1) is the narrowly defined monetary balances in natural logarithms which is the sum of currency in circulation and demand deposits in the banking system. Under the assumption of no money illusion which is quite reasonable for a chronic-high inflation country, we can suppose that the demand for money is a demand for real money balances. In our case, we use the consumer price index to deflate the narrow money supply. The variable representing alternative cost to hold money in our paper is the semi-elasticity of annualized monthly domestic inflation rate (INFLATION) based on consumer price index (CPI) with the base year 1987: 100 which is calculated as (CPI – CPI(-12)) / CPI(-12). Two impulse-dummy variables

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5 Soydan (2003) express that if the source of seigniorage revenue is the base money creation, monetary base needs to be used in the analysis. On the other hand, holding demand deposits encounters a loss in the purchasing power, therefore is subject to inflation tax with the underlying implicit assumption of the direct relationship between M1 and base money, which is based on the constant money multiplier. Phylaktis and Taylor (1993: 35) touch on a similar issue emphasizing stationarity of the relationship of the ratio of narrow money to high-powered money.

6 Following Calvo and Leiderman (1992: 182), Easterly, Mauro and Schmidt-Hebbel (1995: 590), Selçuk (2001: 43) and Soydan (2003), an alternative representation of the opportunity cost of holding money in relation with inflation can be considered such as \[ \frac{\pi}{1+\pi} \] rather than \pi or ln(P_t – lnP_{t-1}) where P is the price level and ln the natural logarithm operator, since the latter cases can be considered as better for the continuous time series and the former for discrete time series. Thus, as Soydan (2003) expresses, if the sequential values are not very close
which take on values of unity for the years 1994 and 2001 concerning the financial crises occured in 1994 and 2001 are considered as exogeneous variables. Below we give in Figure 1 the time series representation of the variables used in this paper,

FIGURE 1: TIME SERIES USED IN THE PAPER

As a next step for our econometric analysis, we investigate the time series properties of the variables. Granger and Newbold (1974: 111-120) indicate the occurance of the spurious regression problem in the case of using non-stationary time series causing unreliable correlations within the regression analysis. At first, by using the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests, we check for the stationarity condition of our variables assuming constant and trend terms in the regressions. Thus for the ADF and PP tests, we compare the ADF and PP statistics obtained with the MacKinnon (1996: 601-618) critical values also possible in Eviews 5.0, and for the case of stationarity we expect that these statistics are larger than the MacKinnon critical values in absolute value and that they have a minus sign. Although differencing eliminates trend, we report the results of the unit root tests for the first differences of the variables with a linear time trend in the test regression. The results are shown in Table 3 below.7

7 For the MacKinnon critical values, we consider %1 and %5 level critical values for the null hypothesis of a unit root. The numbers in parantheses are the lags used for the ADF stationary test and augmented up to a maximum of 10 lags, and we add a number of lags sufficient to remove serial correlation in the residuals, while the Newey-West bandwidths are used for the PP test. The choice of the optimum lag for the ADF test was decided on the basis of minimizing the Schwarz Information Criterion (SC). The test statistics and the critical values are from the ADF or UNITROOT procedures in Eviews 5.0. '*' indicate the rejection of a unit root for the %1 level.

to each other the cost of holding money can be considered such as $\pi / (1+\pi)$. But in our paper we apply to the semi-elasticity of original inflation series.
TABLE 3: UNIT ROOT TESTS (assuming constant & trend in levels)

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test (in levels)</th>
<th>PP test (in levels)</th>
<th>ADF test (in first differences)</th>
<th>PP test (in first differences)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNREALM1</td>
<td>-1.359490(0)</td>
<td>-0.986744(8)</td>
<td>-8.271087(1)*</td>
<td>-10.4519(19)*</td>
</tr>
<tr>
<td>INFLATION</td>
<td>-1.045391(4)</td>
<td>-2.000710(3)</td>
<td>-6.349821(3)*</td>
<td>-7.964732(2)*</td>
</tr>
</tbody>
</table>

Test Critical Values (ADF and PP)

%1 level: -4.096614
%5 level: -3.476275

When we examine the results of the unit root tests, we see that the null hypothesis that there is a unit root cannot be rejected for both variables using constant & trend terms in the test equation in the level form. But inversely, for the first differences of the variables the null hypothesis of a unit root is strongly rejected. So we accept that both variables contain a unit root, that is, non-stationary in their level forms, but stationary in their first differenced forms, thus enable us testing for cointegration.

We now examine whether the variables used are cointegrated with each other. Engle and Granger (1987: 251-276) indicate that even though economic time series may be non-stationary in their level forms, there may exist some linear combination of these variables that converge to a long run relationship over time. If the series are individually stationary after differencing but a linear combination of their levels is stationary then the series are said to be cointegrated. That is, they cannot move too far away from each other in a theoretical sense (Dickey, Jansen and Thornton, 1991: 58). For this purpose, we estimate a VAR-based cointegration relationship using the methodology developed in Johansen (1995) in order to specify the long run relationships between the variables considered making use of Eviews 5 User’s Guide by QMS (2004: 735-748). Let us assume a VAR of order p,

\[ y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + B x_t + \varepsilon_t \]  

where \( y_t \) is a k-vector of non-stationarity I(1) variables, \( x_t \) is a d-vector of deterministic variables such as constant term, linear trend, seasonal dummies, and crisis variables and \( \varepsilon \) is a vector of innovations, i.e. independent Gaussian variables with mean zero and variance \( \Omega \). Such kind of exogeneous variables are often included to take account of short-run shocks to the system, such as policy interventions and shocks or crises which have an important effect on macroeconomic conditions. It is worth noting that including any other dummy or dummy-type variable will affect the underlying distribution of test statistics, so that the critical values for these tests are different depending on the number of dummies included (Harris, 1995: 81). We can rewrite this VAR as,

\[ \Delta y_t = \Pi y_{t-1} + \sum \Gamma_i \Delta y_{t-i} + B x_t + \varepsilon_t \]

\( i = 1 \)
where

\[ \Pi = \sum_{i=1}^{p} A_i I \quad \Gamma_i = \sum_{j=i+1}^{p} A_j \]  

Granger representation theorem asserts that if the coefficient matrix \( \Pi \) has reduced rank \( r < k \), then there exist \( k \times r \) matrices \( \alpha \) and \( \beta \) each with rank \( r \) such that \( \Pi = \alpha \beta' \) and \( \beta' y_t \) is I(0). \( r \) is the number of cointegrating relations (the rank) and each column of \( \beta \) is the cointegrating vector. The elements of \( \alpha \) are known as the adjustment parameters in the vector error correction (VEC) model and measure the speed of adjustment of particular variables with respect to a disturbance in the equilibrium relationship. Johansen’s method is to estimate the \( \Pi \) matrix from an unrestricted VAR and to test whether we can reject the restrictions implied by the reduced rank of \( \Pi \). Also we can express that this method performs better than other estimation methods even when the errors are non-normal distributed or when the dynamics are unknown and the model is over-parametrized by including additional lags in the error correction model (Gonzalo, 1994: 225). We thus first determine the lag length of our unrestricted VAR model for which the maximum lag number selected is 8 considering five lag order selection criterions, that is, sequential modified LR statistic, final prediction error criterion (FPE), Akaike information criterion (AIC), Schwarz information criterion (SC) and Hannan-Quinn information criterion (HQ). As the lag order selected in Table 4 below, LR, FPE, AIC and HQ statistics suggest 5, and SC statistics suggest 1 lag orders. We choose the lag order selected by minimized AIC and sequential LR statistics, that is, lag order 5. \(^8\) We also add eleven centered seasonal dummies which sum to zero over a year as exogeneous variable. In this way, the linear term from the dummies disappears and is taken over completely by the constant term, and only the seasonally varying means remain (Johansen, 1995: 84),

\(^8\) For the appropriate lag length to ensure that the residuals are Gaussian, i.e., they do not suffer from autocorrelation, non-normality, etc., considering the presence of cointegrating relationships, Cheung and Lai (1993: 313-328) find that Monte Carlo experience carried out using data generating processes (DGP) suggests that tests of cointegration rank are relatively robust to over-parametrizing, while setting too small a value of lag length –such as lag length one or two generally suggested by SC statistics also producing serial correlation problem- severely distorts the size of the maximum likelihood tests (Cheung and Lai, 1993: 319-322: Harris, 1995: 121 footnote 12). Gonzalo (1994: 220-221) also reveals that the cost of over-parametrizing by including more lags in the maximum likelihood based error correction model (ECM) is small in terms of efficiency lost, but this is not the case if the ECM is underparametrized.
TABLE 4: VAR LAG ORDER SELECTION CRITERIA

<table>
<thead>
<tr>
<th>Lag</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NA</td>
<td>0.0008</td>
<td>-1.4203</td>
<td>-1.0014</td>
<td>-1.2564</td>
</tr>
<tr>
<td>1</td>
<td>168.65</td>
<td>3.71E-05</td>
<td>-4.5303</td>
<td>-3.9718*</td>
<td>-4.3118</td>
</tr>
<tr>
<td>2</td>
<td>8.1280</td>
<td>3.61E-05</td>
<td>-4.5595</td>
<td>-3.8614</td>
<td>-4.2864</td>
</tr>
<tr>
<td>3</td>
<td>3.8710</td>
<td>3.82E-05</td>
<td>-4.5068</td>
<td>-3.6691</td>
<td>-4.1791</td>
</tr>
<tr>
<td>4</td>
<td>8.7792</td>
<td>3.63E-05</td>
<td>-4.5643</td>
<td>-3.5869</td>
<td>-4.1820</td>
</tr>
<tr>
<td>5</td>
<td>17.178*</td>
<td>2.84E-05*</td>
<td>-4.8214*</td>
<td>-3.7044</td>
<td>-4.3845*</td>
</tr>
<tr>
<td>6</td>
<td>4.4650</td>
<td>2.95E-05</td>
<td>-4.7944</td>
<td>-3.5377</td>
<td>-4.3028</td>
</tr>
<tr>
<td>7</td>
<td>5.4327</td>
<td>2.98E-05</td>
<td>-4.7968</td>
<td>-3.4006</td>
<td>-4.2507</td>
</tr>
<tr>
<td>8</td>
<td>3.1116</td>
<td>3.20E-05</td>
<td>-4.7454</td>
<td>-3.2095</td>
<td>-4.1446</td>
</tr>
</tbody>
</table>

* indicates lag order selected by the criterion

As a next step, we estimate the long run cointegrating relationship(s) between the variables by using two likelihood test statistics offered by Johansen and Juselius (1990: 169-210) known as maximum eigenvalue for the null hypothesis of r versus the alternative of r+1 cointegrating relationships and trace for the null hypothesis of r cointegrating relations against the alternative of k cointegrating relations, for r = 0,1, ... ,k-1 where k is the number of endogeneous variables. Following Harris (1995: 87-88) briefly to say, to test the null hypothesis that there are at most r cointegrating vectors and thus k-r unit roots amounts to,

\[ H_0: \lambda_i = 0, \quad i = r+1, \ldots, k \] (9)

where only the first r eigenvalues are non-zero. This restriction can be imposed for different values of r and then the log of the maximised likelihood function for the restricted model is compared to the log of the maximised likelihood function of the unrestricted model and a standard likelihood ratio test computed. That is, it is possible to test the null hypothesis using the trace statistic,

\[ \lambda_{\text{trace}} = -2 \log (Q) = -T \sum_{i=r+1}^{k} \log (1-\lambda_i), \quad r = 0, 1, 2, \ldots, k-2, k-1 \] (10)

where \( Q = \frac{\text{maximised likelihood}}{\text{unrestricted maximised likelihood}} \), T is the sample size. Asymptotic critical and their probability values are provided in Osterwald-Lenum (1992: 461-472) and in MacKinnon, Haug and Michelis (1999: 563-577). Another test of the significance of the largest \( \lambda_i \) is the maximal-eigenvalue statistic,

\[ \lambda_{\text{max}} = -T \log (1-\lambda_{r+1}), \quad r = 0, 1, 2, \ldots, k-2, k-1 \] (11)
which tests that there are r cointegration vectors against the alternative that r+1 exist. Table 5 below reports the results of Johansen Cointegration Test using max-eigen and trace tests based on critical values taken from Osterwald-Lenum (1992: 461-472) also available from the VAR and COINT procedures in Eviews 5.0. For the cointegration test, we restrict intercept and the trend factor into our long run variable space, so assume that the trend factor can include the effects of other factors which are not considered in our cointegrating analysis.9 From the Table 5 in which we normalize the cointegrating vector estimated upon real monetary balances, both trace and max-eigen statistics indicate jointly 1 potential cointegrating vector in the long run variable space considering %5 level critical values,

<table>
<thead>
<tr>
<th>TABLE 5: COINTEGRATION RANK TEST ASSUMING LINEAR DETERMINISTIC TREND RESTRICTED IN COINTEGRATING EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null hypothesis</td>
</tr>
<tr>
<td>Eigenvalue</td>
</tr>
<tr>
<td>λ trace</td>
</tr>
<tr>
<td>%5 Critical Value</td>
</tr>
</tbody>
</table>

9 We follow here the so-called Pantula principle. Johansen (1992: 383-397) and Harris (1995: 97) suggest the need to test the joint hypothesis of both the rank order and the deterministic components, and the former tries to demonstrate how to use the tables in Johansen and Juselius (1990: 169-210) for conducting inference about the cointegration rank. The reason that inference is difficult is that the asymptotic distribution under the null of the test statistic depends on which parameter value is considered under the null. In the case of a cointegration analysis, the limit distribution depends on the actual (true) number of the cointegrating relations and also on the presence of a linear trend. Following Pantula (1989: 256-271), they propose to identify the sub-hypotheses, which give different limit distributions, and construct a test statistic and a critical region for each of these sub-hypotheses. The critical region for the test of the original null hypothesis is then the intersection of the critical regions constructed for each of the sub-hypotheses or, in other words, the hypothesis in question is only rejected if all sub-hypotheses are rejected. Following Harris (1995: 97), the test procedure is to move through from the most restrictive model and at each stage to compare the trace or max-eigen test statistics to its critical value and only stop the first time the null hypothesis is not rejected. However, a critical point to be considered here may be that assuming quadratic deterministic trends in cointegrating space allowing for also linear trends in the short run VEC model may be economically difficult to justify especially if the variables are entered in log-linear form or in linear growth rates, since this would imply an implausible ever-increasing or decreasing rate of change (Harris, 1995: 96).
Below, we give the normalized cointegrating vector upon real money balances in a parsimonious vector error correction form including both an F- and LR-test for the reduction of insignificant variables in our model. Using QMS (2004: 563), these tests are for whether a subset of variables in an equation all have zero coefficients and might thus be deleted from the equation.

**TABLE 6: VECTOR ERROR CORRECTION ESTIMATES**

Redundant Variables: DUMMY2 DLNREALM1(-1) DLNREALM1(-2) DLNREALM1(-4) DLNREALM1(-5) DINFLATION(-1) DINFLATION(-2) DINFLATION(-3) DINFLATION(-4) DINFLATION(-5)

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>Log likelihood ratio</th>
</tr>
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<tbody>
<tr>
<td>0.313699</td>
<td>2.555136</td>
</tr>
</tbody>
</table>

Log likelihood ratio

| 0.018 - 0.0945 DUMMY1 - 0.043 D_Q2 + 0.060 D_Q3 - 0.075 D_Q4 + 0.360 DLNREALM1(-3) + 0.202 DINFLATION(-2) + 0.338 DINFLATION(-3) + 0.282 DINFLATION(-4) + 0.192 DINFLATION(-5) - 0.725 (3.584) (2.856) (2.350) (-4.850) [LNREALM1(-1) + 0.786522 INFLATION(-1) + 0.004203 TREND - 4.440591]
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>14.527</td>
<td>(14.527)</td>
</tr>
</tbody>
</table>

where the latter ‘D’ beginning of a variable indicates first difference operator, [ ] the normalized cointegrating vector on real money balances and t-statistics are in parentheses.

In Table 6, we see that normalized semi-elasticity of domestic inflation is -0.786522 and this leads us to the seigniorage revenue maximizing annualized quarterly inflation rate of %127. In our sample period, the mean of actual annualized quarterly inflation rate is %65, and thus we can conclude that Turkish
economy lies on the correct side of the Laffer curve with respect to seigniorage revenue inside the estimation period considered. These estimation results are consistent with empirical papers on this issue upon Turkish economy, such as Akçay (1995: 210-229), Kural (1997: 45-57), Özmen (1998: 545-553), to some extent Selçuk (2001: 41-50), Soydan (2003) and Özdemir and Turner (2004), while Metin and Muslu (1999: 415-426) estimate that monetary authorities expanded the money supply in order to maximize the inflation tax revenue in Turkey and find also that the Cagan model cannot be linked with the rational expectations for the Turkish case contradicting the estimation results of Tunay (2003: 65-83).

Özmen (1998: 545-553) expresses that quantity theoretical hypotheses state that the long run money demand variable space can be postulated to explain inflation with money (and also real income) being weakly exogeneous in the system, while Keynesian theory alleges that the long run money demand variable space can be postulated to explain money conditioned upon the demand arguments such as real income and inflation. Adjustment coefficients estimated in Table 5 above indicate that our results give support to the Keynesian view rather than that of Quantity Theory. In this line and following Özmen, these findings do not necessarily imply that, had there been an active seigniorage policy instead of accommodating the demand, the government would have obtained a higher inflation tax revenue. This is because the real tax revenue may not be invariant to the way the public sector borrowing requirement is financed, and also attempts to increase seigniorage revenue might push the economy in a hyperinflationary path (Soydan, 2003).

For the diagnostics given below, we find that our model estimation procedure is a good approximation of the actual data generating process, maybe except possible structural break for the post-1994 period indicating changes in the monetary policy for policy makers. Besides, we can see that the vector diagnostics do not indicate any problem of autocorrelated residuals, but some non-normality due to excess kurtosis, no problem in our model through Gonzalo (1994: 203-233).\textsuperscript{10}

\textsuperscript{10} We have estimated the same real money demand function with a real income variable in natural logarithms in addition to the inflation variable in semi-logarithmic form. Friedman (1971: 847) expresses that an inflation tax or seigniorage revenue maximizing analysis such as in our paper is entirely correct for a stationary economy with fixed real income. But this is seriously misleading for a growing economy in which the issuer of money obtains a yield from two sources, i.e., a tax on existing real cash balances and provision of the additional real cash balances that are demanded as income rises. In this case, the rate of price rise that will give maximum total yield would be lower for a growing economy. In terms of Equation (4), if we include real income variable in natural logarithms ($y'$) into the
money demand specification, and then construct the inflation rate maximizing seigniorage revenue by taking partial derivative of money demand function,

$$\log_e(M/P) = b - \alpha \pi + \delta y + \varepsilon$$

(12)

we find that seigniorage maximising inflation rate would now be,

$$\pi^* = \frac{1}{\alpha} - \beta g$$

(13)

where $\beta$ represents the income elasticity of demand for real money balances and $g_t$ the growth rate of real income (Friedman, 1971: 850; Bruno and Fischer, 1990: 356). Using a trivariate same order integrated variables, the VAR system such as estimated in this paper with three centered seasonal dummies and two impulse crisis dummies and a restricted trend in the long run variable space have produced a unique cointegrating vector resulting in both an approximately same and statistically significant semi-elasticity of inflation and insignificant real income elasticity. These results are available upon request. So we consider in our paper a bivariate stationary relationship between real monetary balances in natural logarithms and inflation in semi-elasticities such as original Cagan (1956: 25-117) type money demand function.
VEC Residual Serial Correlation LM Test (Probs. from chi-square with 4 df.)
H0: no serial correlation at lag order h

<table>
<thead>
<tr>
<th>Lags</th>
<th>LM-Stat</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5.133632</td>
<td>0.2739</td>
</tr>
</tbody>
</table>

VEC Residual Normality Test
H0: residuals are multivariate normal

<table>
<thead>
<tr>
<th>Skewness</th>
<th>$\chi^2(2)=0.187777$ Prob. 0.6648</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>$\chi^2(2)=9.160030$ Prob. 0.0103</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>$\chi^2(4)=9.420369$ Prob. 0.0514</td>
</tr>
</tbody>
</table>

V. CONCLUDING REMARKS

In our paper, we try to investigate the course of inflation tax and seigniorage revenue for policy makers of the Turkish economy. For this purpose, we first construct an ex-ante seigniorage revenue maximizing inflation model à la Cagan (1956: 25-117), and then calculate annual inflation tax and seigniorage revenues for the post-1980 period Turkish economy. Following these theoretical issues, an empirical model is constructed upon Turkish economy, and our ex-post estimation results reveal that inside the period considered, the Turkish economy lies on the correct or efficient side of the seigniorage maximizing Laffer curve.

A complementary analysis to our paper can be implemented by including the currency substitution phenomenon into the real money demand equation producing seigniorage maximizing inflation rate for a future work such as Phylaktis and Taylor (1993: 32-37) and Şiklar (1998: 3-14) so as to see whether those are in line with our ex-post estimation results in this paper. Fischer (1982: 306-307) expresses that currency substitution or dollarization may bring out substantial seigniorage costs that would be paid to a foreign country to import high-powered money, plus the excess welfare burden incurred by giving up independent control over the domestic rate of high-powered money creation for the domestic economy. So such an issue should be dealt with by the researchers for the case of Turkish economy.

Of course, our estimation results in this paper should be taken into account cautiously considering an economics policy perspective, since policy implementations in favor of larger seigniorage revenue due to the estimation results giving support to that the Turkish economy lies inside the correct or efficient side of the seigniorage maximizing Laffer curve can take the economy into a hyperinflationary path.

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