Abstract

The nuclear fragmentation in the liquid-gas phase transition region is studied on the basis of Statistical Multifragmentation Model (SMM). We demonstrate the evolution of fragment mass and charge distributions for Au nucleus in the excitation energy ranges 3-8 MeV/nucleon. It is observed that variation of temperature with excitation energy exhibits a plateau-like behavior, which can be interpreted as a sign of liquid-gas phase transition in the system.

Keywords: Nuclear multifragmentation, phase transition

1. Introduction

The nuclear multifragmentation is one of the most interesting phenomena in heavy ion collisions, for studying the behavior of nuclear matter under extreme conditions of temperature and pressure, and possible phase transitions at subnuclear densities. Phase transitions have attracted a strong interest from various scientific communities. The search of cloud and galaxy formations, aerosol, growth in alloys, quark gluon plasma in...
Nuclear Fragmentation in the Liquid-Gas Phase Transition Region

high energy nuclear collisions, Bose condensates with a small number of particles, crystal phases, melting of solid atomic clusters and vaporization of atomic nuclei are examples of attempts to study phase transitions in finite systems. The problem usually encountered experimentally is how to control the equilibrium and how to extract the thermodynamic variables from observable quantities in order to identify the possible phase transition in the system under consideration.

When the nuclei are excited by light particles or heavy ions, a compressed and hot blob of nuclear matter is formed. Assuming this blob to be close to thermodynamic equilibrium it will expand and enter the region of subnuclear densities, where it becomes unstable to density inhomogeneities. As a result of these density fluctuations nuclear matter will break up into several intermediate hot fragments. This may be related to the liquid-gas type phase transition on the basis of the similarity between van der Waals and nuclear matter equations of state. In recent years, nuclear fragmentations have extensively been investigated to realize whether the break up of nuclear matter into several intermediate hot droplets is a liquid-gas-type phase transition or not [1-11].

2. Nuclear Fragmentation

It is pointed out by numerous studies that the mass and charge distributions of fragments produced in the disintegration of nuclei in the liquid-gas type phase transition region evolve with the excitation energy [12,13]. We have carried out the calculations within Statistical Multifragmentation Model (SMM) for Au$^{197}$, which has been widely used in experiments. We demonstrate the evolution of fragment mass distributions for this nucleus, in the excitation energy range 3-8 MeV/nucleon. We also show the evolution of the critical exponent (parameter in this energy range [12]).

In fact, a model used for extracting the properties of nuclear matter, should be capable of reproducing the observed fragmentation properties. Having in mind, the shortcomings of existing models, SMM [14,15] has provided a good reproduction of experimental data, so far. According to SMM, we assume a micro-canonical ensemble of breakup channels, and the system should obey the laws of conservation of energy $E^*$, mass number $A$ and charge number $Z$. The probability of generating any breakup channels is assumed to be proportional to its statistical weight as

$$W_j \propto \exp(S_j(E^*, A, Z)) \tag{1}$$

where $S_j$ denotes the entropy of a multifragment state of the breakup channel $j$. The breakup channels are generated by Monte Carlo method according to their weights. The total excitation energy $E^*$ is measured with respect to the ground state energy $E^*_{A_0, Z_0}$ of a compound system consisting of $A_0$ nucleons and $Z_0$ protons. Then the law of the energy conservation at fragmentation may be written as

$$E_j = E^* + E^*_{A_0, Z_0} = E. \tag{2}$$

Left-hand side of Eq. (2) depends on a specific break-up configuration. A break-up partition can be defined as
\[ j : \{ N_{AZ}, 1 \leq A \leq A_0, 0 \leq Z \leq Z_0 \} \] (3)

where the number of fragments with mass \( A \) and charge \( Z \) is denoted by \( N_{AZ} \). It can take the values 0,1,2,... All final states could be classified according to the set of fragment multiplicities. This set is a matrix with \( A_0 \) elements in rows and \( Z_0 + 1 \) elements in columns. The elements in rows and columns are ordered in accordance with increasing \( A \) and \( Z \). All partitions \( j \) which satisfy the constraints on the total mass and charge of the system, are possible. As a result one obtains the equation constraining only the average energy associated with a partition \( j \)

\[ E_j(T, V) = E, \] (4)

where the left-hand side is given by

\[ E_j(T, V) = E^+ (T, V) + \sum_{(A,Z)} E_{AZ}(T, V) N_{AZ} + E^C(V). \] (5)

The first term is the total translational motion energy, \( E_{AZ}(T, V) \) is the average energy of an individual fragment of \((A,Z)\) type including the internal energy and Coulomb clusterization energy. The last term is the Coulomb energy of the uniformly charged sphere with the charge \( Z_0e \). Eq. (4) should be regarded as the equation for the equilibrium temperature \( T_j \) characterizing a partition \( j \). Thus, under given and the break-up temperature \( T_j \) turns out to be the function, or, more precisely, the functional of the set of fragment multiplicities \( \{N_{AZ}\} \) forming the partitions. There is no restriction on the temperature of partitions. The temperature of fragments in partitions is introduced in standard way for nuclear matter, therefore, it is equivalent to the temperature commonly used in nuclear physics. It is well known that excitation energy of isolated nuclei can be presented as \( E^* = aT^2 \) (where \( a \) is level density parameter), one can also directly deduce from our formulae. The physical quantities such as average mass, charge fragment yields, temperature and its variance are calculated by running the summations over all members of the ensemble according to the standard definition of the ensemble.

Light fragments with mass number \( A \leq 4 \) and charge number \( Z \leq 2 \) are considered as stable particles (nuclear gas) with masses and spins taken from the nuclear tables. Only translational degrees of freedom of these particles contribute to the entropy of the system. Fragments with \( A > 4 \) are treated as heated nuclear liquid drops, and their individual free energies \( F_{AZ} \) are parameterized as a sum of the bulk, surface, Coulomb and symmetry energy contributions

\[ F_{AZ} = F_{AZ}^B + F_{AZ}^S + E_{AZ}^C + E_{AZ}^{sym} \] (6)

The bulk contribution is given by \( F_{AZ}^B = (-W_0 - T^2/\epsilon_g)A \) where \( T \) is the temperature, the parameter \( \epsilon_g \) is related to the level density, and \( W_0 = 16 \text{ MeV} \) is the binding energy of infinite nuclear matter. Contribution of the surface energy is \( F_{AZ}^S = B_0 A^{2/3} \left( \frac{T_c^2 - T^4}{T_c^2 + T^4} \right)^{5/4} \), where \( B_0 = 18 \text{ MeV} \) is the surface coefficient, and \( T_c = 18 \text{ MeV} \) is the critical temperature
of the infinite nuclear matter. Coulomb energy contribution is 
\[ E_{AZ}^{*m} = \gamma (A - 2Z)^2 / A \],
where \( c \) is the Coulomb parameter obtained in the Wigner-Seitz approximation,
\[ c = (3/5)(e^2/r_0)(1 - (\rho/\rho_0)^{1/3}) \] the charge unit \( e \), \( r_0 = 1.17 \text{ fm} \), and \( \rho_0 \) is the normal nuclear matter density (0.15 \text{ fm}^{-3}). And finally, the symmetry term is 
\[ E_{AZ}^{*m} = \gamma (A - 2Z)^2 / A \],
where \( \gamma = 25 \text{ MeV} \) is the symmetry energy parameter. All the parameters given above are taken
from the Bethe-Weizsäcker formula and correspond to the assumption of isolated fragments with normal density in the freeze-out configuration. This assumption has been found to be quite successful in many applications. However, for a more realistic treatment one should consider the expansion of the primary fragments in addition to their excitations. The residual interactions among them should be considered as well. These effects can be taken into account in the fragment free energies by changing the corresponding liquid-drop parameters.

We have applied the SMM to the present study by taking also an overall excitation energy
in the range of \( E^*/A = 2-20 \text{ MeV} \) and a freeze-out volume \( V = 3V_0 \), where
\[ V_0 = 4\pi R^3 / 3 \] Ris the volume of a ground state nucleus with . However, the initial source expands while
fragments are forming, so when they are well defined (because they do not feel anymore the nuclear interaction), the volume inside which the intermediate mass fragments are located is larger than the initial volume of the unstable source. In this model, the fragment formation is described at a low-density freeze-out \( (\rho = \rho_0 / 3, \rho_0 \) is the normal nuclear matter density 0.15 \text{ fm}^{-3}) \), where the nuclear liquid and gas phases coexist. In this study the relation between surface tension and the ratio of \( T/T_c \) is taken the same as that of Ref.[12].

In order to obtain the values of critical exponents \( \tau \) (for mass distribution) and \( \tau_z \) (for charge distribution), we applied a power-law fitting, \( N(A) \propto A^{-\tau} \) and \( N(Z) \propto Z^{-\tau_z} \), where
\( N(A) \) and \( N(Z) \) denote the multifragmentation mass yield and charge yield as a function of fragment mass number \( A \) and charge number \( Z \), respectively. In all calculations, we consider the multifragments in the range \( 6 \leq A \leq 40 \) for mass and \( 3 \leq Z \leq 18 \) for charge numbers. The lighter fragments are considered as a nuclear gas.

3. Results and Conclusion

In Fig.1 we have shown typical mass and charge distributions for Au nucleus, at an excitation energy \( E^* = 8 \text{ MeV/nucleon} \). It is seen from this figure that the extracted \( \tau \) and \( \tau_z \) values are very close to each other. This is because the neutron-to-proton ratio of produced IMFs change very little within their narrow charge range [13,16]. In Fig.2, we show the values of extracted temperatures as a function of excitation energies. This figure exhibits a plateau-like behavior that can be interpreted as a sign of liquid-gas phase transition in the system. The mass distribution of fragments produced in the disintegration of various nuclei evolves with the excitation energy. We presented the results somewhere else [17]. In this paper it was shown that at low temperatures \( (T \leq 5 \text{ MeV}) \), there is a U-shape distribution corresponding to partitions with few small fragments and one big residual fragment. At high temperatures \( (T \geq 6 \text{ MeV}) \), the big fragments disappear and an exponential-like fall-off is
observed. In the transition region (T ≈ 5-6 MeV), however, one observes a smooth transformation. All these results are in good agreement with experimental data [6-11]. To show the effect of the size of a nucleus, we have taken into account the disintegration of Au\(^{197}\), Sn\(^{124}\) and Kr\(^{84}\), the dispersion of temperature of these nuclei is given in Fig. 3. The maximum dispersion values of the temperature are seen in the transition region (5.5-6 MeV/nucleon). In view of these theoretical results it is instructive to demonstrate the possibility of the application of such approaches for the analysis of experimental data for nuclear reactions and astrophysical studies [6,18].
REFERENCES

Figure 1. Average fragment mass and charge distributions of Au nucleus at excitation energy of 8 MeV/nucleon. Solid lines denote $\tau$ and $\tau_z$ fits of IMF yields.

Figure 2. Variation of values of temperature with excitation energy per nucleon.
Figure 3. Variance of extracted temperature for the fragmentation of Au$^{197}$, Sn$^{124}$ and Kr$^{84}$. 