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CAN GEOGEBRA MAKE EASIER THE UNDERSTANDING OF CARTESIAN CO-ORDINATES? A QUANTITATIVE STUDY IN TURKEY

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ABSTRACT

A complex number z is a number of the form a + bi where a and b are real numbers. A complex number z in complex plane can be represented by Cartesian co-ordinates, its real and imaginary parts, but equally useful is the representation of z by polar co-ordinates. Generally, students cannot easily visualize them by Cartesian co-ordinates and that makes the subject difficult to be understood for them. The aim of this study is to evaluate quantitatively and qualitatively the students' opinions about GeoGebra and to contribute to Turkish GeoGebra Library. First we have translated "Polar Grapher" (created by Jerel L. Welker) into Turkish and developed a new worksheet with GeoGebra about "Visualizing Complex Numbers on Cartesian Co-ordinates". After presenting material to 4th grade students from high schools located in Istanbul and Sakarya cities (Turkey) respectively, a questionnaire has been conducted. The data collected through questionnaires have been analyzed and the views of students and their teacher's on the use of GeoGebra have been reported.

Key Words: Geogebra, cartesian co-ordinates, Turkey.

INTRODUCTION

Review of Literature

Mathematics is one of the oldest sciences. According to some researchers, mathematics is an art, sometimes a language that we really need to know to understand the universe, sometimes a game such as chess (Ülger, 2003). As Tepedelenlioğlu (2007) stated, it is an abstract state of the methods that we use in our life. Mathematics, is designated as a compulsory subject, in the historical process in each country and each school, arithmetic, geometry and astronomy lectures are given in schools and universities even in the Middle Ages (Ersoy, 2003). Unfortunately as a tool, only blackboard and chalk have been used in math teaching and education. Development of IT has provided a different perspective on this issue (Ersoy, 2003). Today, face to face education by traditional teaching tools and models is leaving its place to multiple learning environments which are created by IT technologies (Karadağ, Sağlam & Baloğlu, 2008).

According to Yuyucu & Ayvaz Reis (2010), today the main goal of mathematics education is to gain the ability to think mathematically and they have also expressed those mathematical concepts, symbols or graphics as well as, multiple representations can be understood more quickly and efficiently thanks to appropriate computer software.

GeoGebra is dynamic mathematics software that combines Geometry, Algebra and Calculus with the advanced interaction of spreadsheets and graphics (Taş, 2010). GeoGebra, is free and available for everyone who wants

to use it that the mathematical issues like secondary school, university geometry and integral also derivative based on geometry can easily be applied to (GeoGebra Official Web Site in Taş, 2010).
GeoGebra which is actively involved in the areas of mathematics education and learning activities, is used for developing creative and interactive visual applications with dynamic worksheets, in algebra and geometry fields by which students can understand some complex mathematical topics easily (Taş, 2010; Taş, Gülseçen & Kabaca, 2010).

In this study GeoGebra was introduced to 217 fourth grade students from 4 different high schools in Istanbul and Sakarya cities in Turkey respectively. Course material titled "Polar Grapher" prepared by Walker (2008), has been translated into Turkish and presented to students.

On the basis of Polar Grapher, a different course material has been developed in order to show complex numbers and their various operations (for example, calculation of nth power or presence of conjugate of a complex number) in polar coordinates. Some of screenshots of the material are presented in below.

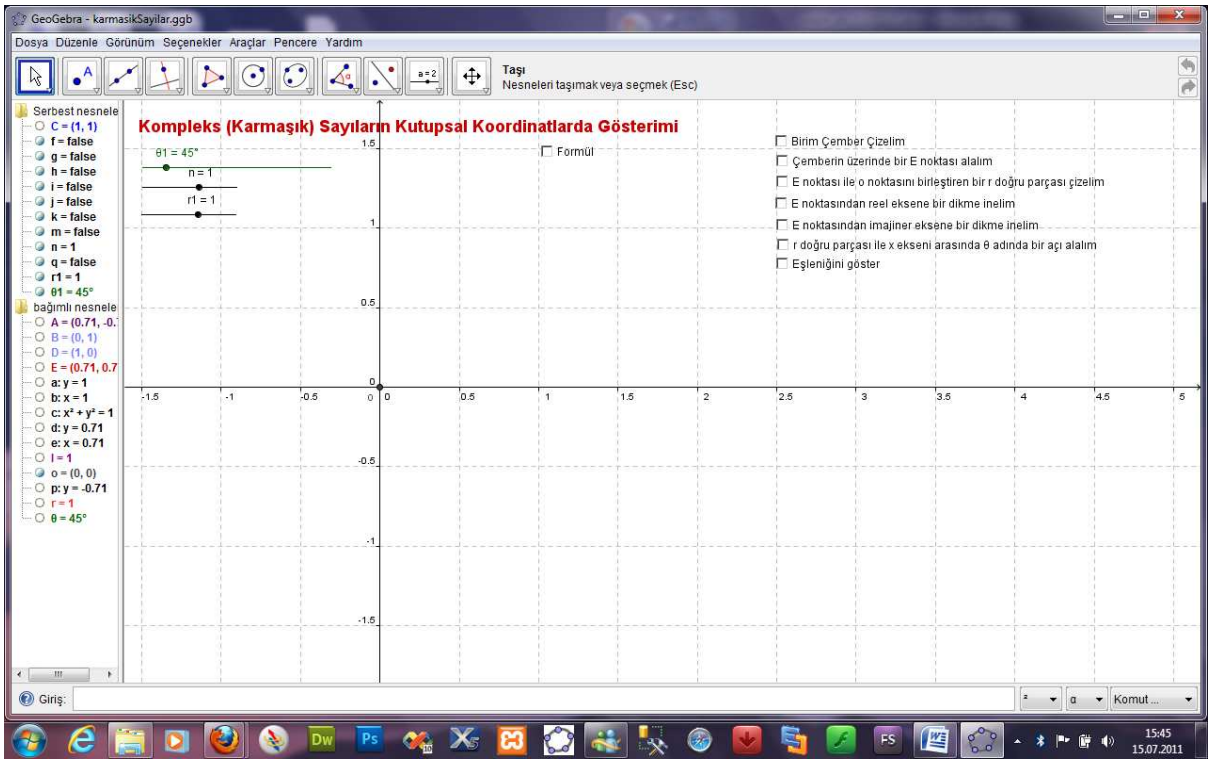

Figure 1: A screenshot from the GeoGebra Material

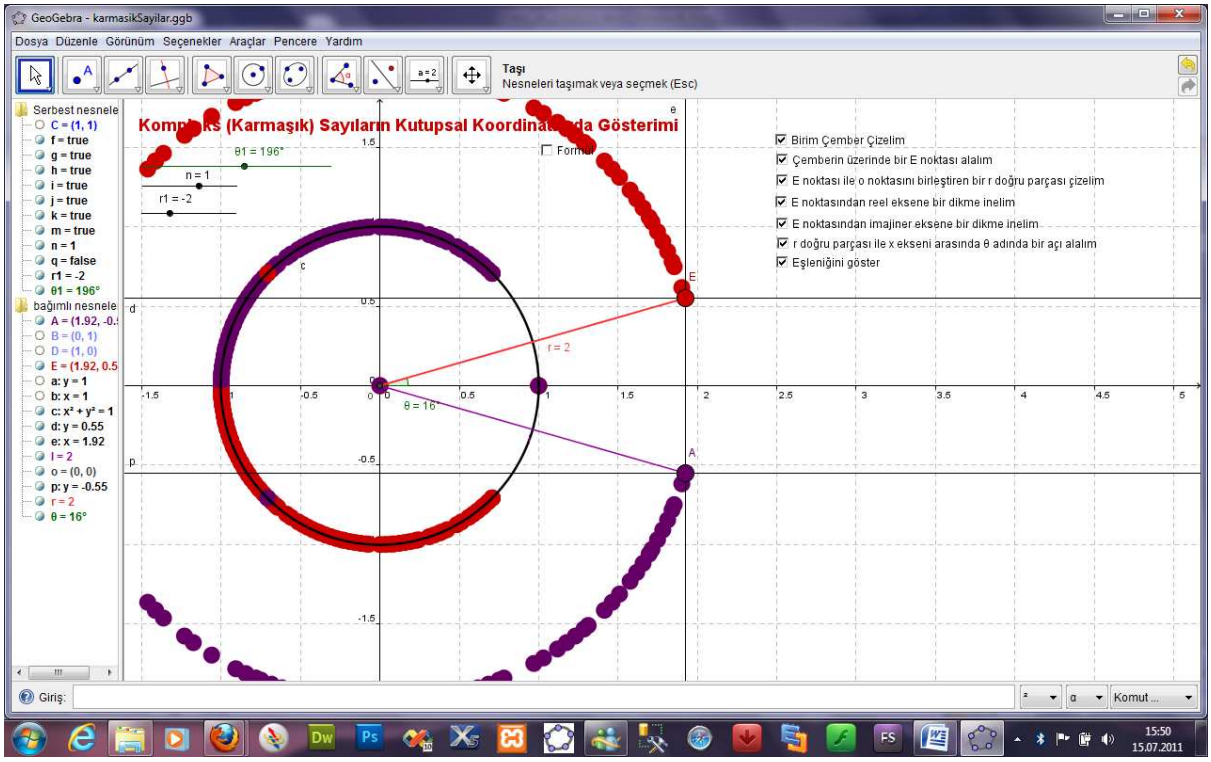

Figure 2: Illustration of Complex Numbers with GeoGebra

This material aims to provide “the visualization of complex numbers on polar co-ordinates” in the curriculum of high school math for students. Also, “Expression of Complex Numbers on Polar Coordinates" Assessment of Student Opinion Survey was applied to students in order to collect their views on GeoGebra.

In the first section “Mathematics Education in Historical Process” and in the second section “Expression of Complex Numbers on Polar Coordinates" will be discussed. In the sections following these two, more detailed information is given about the study.

Mathematics Education in Secondary Schools in Turkey

In historical process, mathematics has affected informatics and informatics has affected technology, too (Yazıcı, 2008). Mathematics education is a concept which goes back to early years of mathematics: such that Pythagoras’ secret societies, agreed on the idea of reducing the universe to rational numbers, was a kind of mathematics school that Plato doesn’t let people who don’t know geometry, join to his academy, during the Middle Ages that with music, logic and rhetoric (eloquence) has kept the location with Mathematics in curriculums (Yıldırım, 2004).

Many developments (Abacus, Napier’s bones, Pascaline, Hollerith’s punched card device, ENIAC ..., etc.) which may be revealed as a success on behalf of informatics can be seen as a supporter of this idea.

As it was in past century, nowadays in Turkey, great importance is given to math education and innovation in math education. Mustafa Kemal Ataturk himself wrote a book called "Geometry" immediately after the 3rd Turkish Language General Assembly (August 24-31, 1936) (Özata, 2007). Thus, translating the reasonably hard mathematical terms to Turkish has provided an easy way to take part in mathematics and geometry education.

The use of IT tools in the field of education in our country began with the Computer Education Expertise Commission in Secondary Education which was initiated by the Ministry of National Education (MNE) (Karadağ et al., 2008). The use of computers by teachers and an in-service training program on BASIC programming language was designed in 1985, total of 225 teachers were trained in this program (Keser, 1989). Year of 1987 has great importance for Computer-Aided Instruction and was brought in 1987-1988 academic year, as "Computer-Aided Education Project" (Karadağ et al., 2008). Ministry of Education dealt with nine companies in 1989-1990 academic year. In such a collaborative way, the continuous training of teachers and production of new educational software have been possible (Akkoyunlu & İmer, 2009). All of these developments in our country have prepared infrastructure for computer-aided mathematics education.

Middle East Technical University, on the other hand, supports research and initiatives including a calculator in mathematics education based on collaboration with other universities in Turkey (Ersoy, 2003). Also Istanbul University, Department of Informatics supports activities related with mathematical computer-aided software GeoGebra which utilized the present study and adds new ones. "First Eurasia Meeting of GeoGebra" meeting was held in the scope of "3rd International Future-Learning Conference On Innovations in Learning for the Future 2010: e-Learning" which was organized by Istanbul University, Turkish Informatics Foundation, Istanbul Kultur University and Informatics Association of Turkey on May 10-14, 2010. GeoGebra sessions have been performed and GeoGebra workshop have been practiced with participation of approximately 100 mathematics teachers from all over Turkey (Istanbul University Department of Informatics Official Web Site).

Understanding Cartesian Coordinates Through GeoGebra: A Quantitative Study

Demonstration of Complex Numbers in Polar Coordinates

Despite infinity of real numbers and all the wealth of its structures that it contained, -1 is not a square number in real numbers cluster (King, 2004). (x, y) pairs are used to improve these numbers which we need. In 1660ties Rene Descartes described the square root of a negative number as "virtual". The new cluster that has occurred with the help of appropriate definitions and proofs has been called "complex numbers" (King, 2004). Earl (2004: 2, 6) expressed Complex Numbers as follows:

"A complex number 3 is a number of the form a + bi where a and b are real numbers. If z = a + bi then a is known as the real part of z and b as the imaginary part. We write a = Rez and b = Imz. Note that real numbers are complex – a real number is simply a complex number with zero imaginary part.

A complex number z in complex plane can be represented by Cartesian co-ordinates, its real and imaginary parts, but equally useful is the representation of z by polar co-ordinates. If we let r be the distance of z from the origin, and if, for z 6= 0, we define ϑ to be the angle that the line connecting the origin to z makes with the positive real axis, then we can write

$$z = x + iy = r \cos \vartheta + ir \sin \vartheta."$$

Gaussian revealed ideas that provided to develop "Complex Analysis" subfield by thinking complex numbers as points on a plane (King, 2004).

Routinely Complex Analysis performs to find a correlation between math topics which don't seem related with each other at first glance. Euler is proved, for all real θ (theta)'s that (King, 2004),

$$e^{i\theta} = \cos \theta + i \sin \theta$$

it was found that it was a connection between complex valued exponent on the left side of the equation and real valued sin and cos functions on the right side of the equation.

Global Outcomes

This study aims to explore if there is a significant impact on students' math attitudes after the GeoGebra material that was prepared and tries to investigate the following in the four high schools in Istanbul and Sakarya cities respectively.

- "Can GeoGebra simplify visualization of Complex Numbers on Cartesian co-ordinates for students?"
- "What are the students' approaches towards computer-aided mathematics instruction materials?"

- "Can students integrate computer aided tools to mathematics subjects which they learned before?"
- "Is there any visualization problem about Complex Numbers for students?" is sub-problem in our study.

METHOD

Subjects

The scope of the research has been 4th Grade students studying at Istanbul and Sakarya Anatolian High Schools and their mathematics teachers. The sample of the research is high schools which have suitable educational conditions on computer-aided education and training, in the below-mentioned four Anatolian High Schools and 217 4th Grade high school students who have already known "Complex Numbers", participated in our research. These figures are summarized in Table 1 below:

Table 1: Participants

School Name	Participant Number
Sakarya Anatolian High School	56
TES-İŞ Adapazarı Anatolian High School	47
Vefa High School	50
Fatih Pertevniyal High School	64

Limitations

This research is limited with:

1. The use of "Polar Grapher" that helps to visualize complex numbers on Cartesian Co-ordinates and to develop a different material with GeoGebra about complex numbers,
2. 4th grade high school students from four high schools. The schools have enough computer based equipment and use it in their classes who have already knew "Complex Numbers",
3. The questionnaire that was conducted according to the rules and policies of National Ministry of Education of Turkey.

Instruments

In this study, qualitative and quantitative research techniques have been used. And the survey results have been interpreted according to these techniques.

The data were collected through a questionnaire. The necessary information was given to the sample group before conducting the questionnaire. There are 20 questions in questionnaire. Question types in questionnaire are Closed-ended (questions with answers like Yes/No, Very Difficult/Difficult/Moderate/Easy/Very Easy) and open-ended (questions has answers not given in questionnaire and students can present their opinions).

Questions about sex, age, accessing/not accessing internet/a computer, student opinions about mathematics, computer-aided mathematics education and GeoGebra in students' eye; are used in the questionnaire.

In this survey, there aren't any questions contrary to national and spiritual values with the constitution and the basic law of the national education, that infringing human rights, feeding religious racial and gender discrimination, providing certain political approaches, including crime matters accepted by the universal declaration of human rights, disclosing personality and family privacy.

Necessary permissions were obtained from Turkish Republic Ministry of National Education Educational Research and Development Directorate (ERDD) as study includes more than one city. We contacted with school administrators about the survey and made appointment for study.

The data, obtained from survey, will not be shared with any institution or person except researchers and Turkish Republic Ministry of National Education, obtained results will be presented at the conference for the first time.

Also interview method is used to take mathematics teachers' opinions.

Data Analysis

Data of this research have been collected by means of a questionnaire and interviews. Later the collected data have been typed into Microsoft Excell Worksheet then all of these raw data have been transferred to SPSS 18 and have been processed. Frequency and percentage tables have been obtained. For the sake of understanding most of the data have been visualized by pie graphics.

FINDINGS

Findings Obtained from the Students

The number of 217 forth Grade, 16-20 aged students have participated our research. 46% of student group is female, 54% of the group is male. The obtained results are given below:
Approximately,

- 87% of the group has their own computer.
- 61% of the group spends time at the computer every day.
- 82% of the group can access internet at their home.
- 61% of the group use internet every day. The way that they choose to access internet is given below in Figure 1.

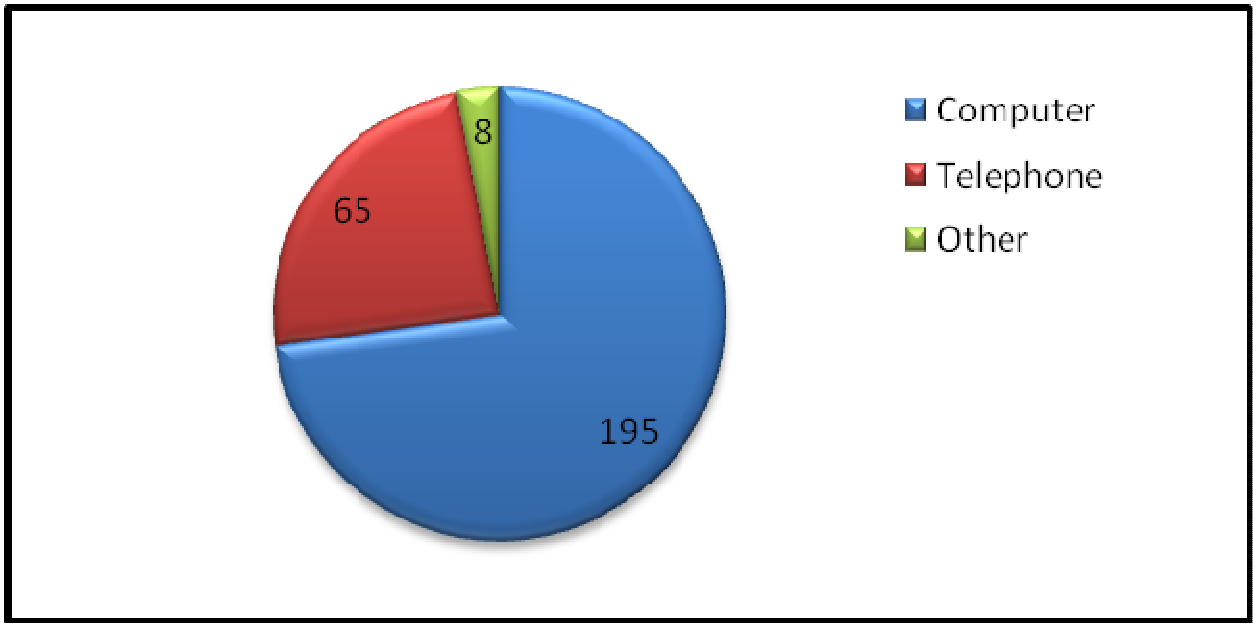

Figure 3: The way that students choose to access internet

- 69% of the group uses internet for their studies.
- 86% of the group has taken no computer-aided education before.
- Only 12% of the group mentions that they don't love mathematics.

- Only 3% of the group has heard GeoGebra software before.
- The opinions about mathematics lesson in Figure 2 and "Complex Numbers" in Figure3, are given in below.

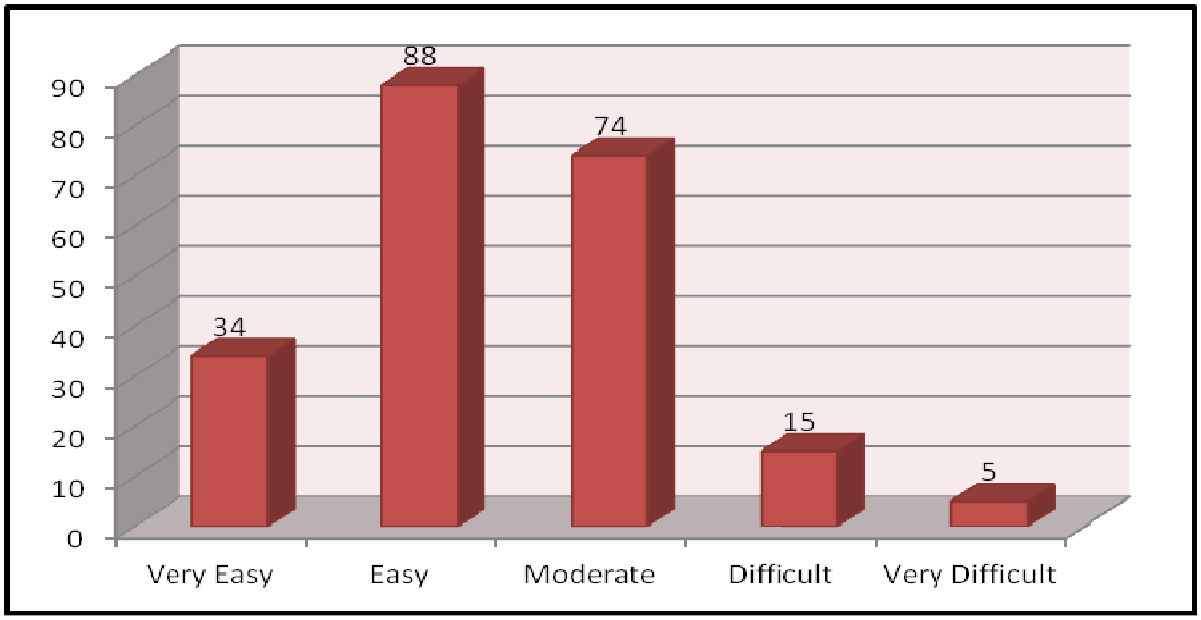

Figure 4: The opinions about mathematics lesson

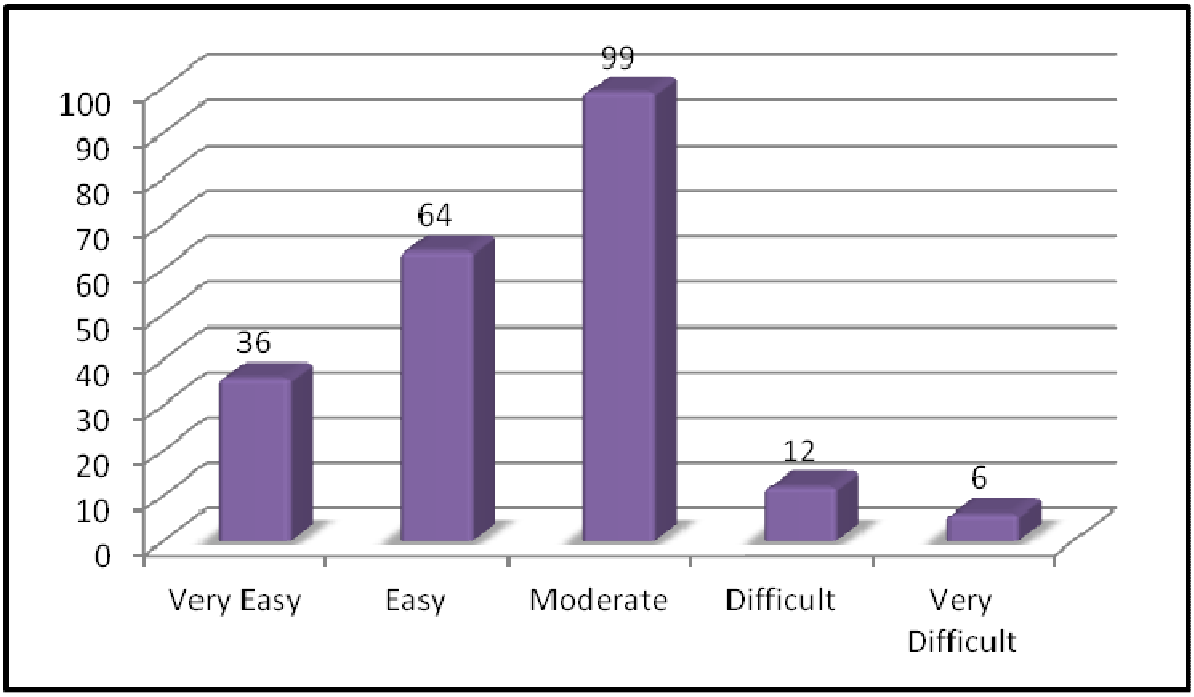

Figure 5: The opinions about "Complex Numbers"

- After presenting the materials that are prepared with GeoGebra before, 41% of the group indicated that GeoGebra makes the subject "clear and understandable".
- Students indicated the subjects that they want to visualize with their causes. Especially parabola, trigonometry, logarithms, limits, solid bodies and derivative seems like subjects that students need visual support. In this point, some remarkable comments belongs to students is given below:

"Parabola is based on most of formula and I can't understand it."
"Analytic Geometry; because it is more clear."
"Astrometry; because 3 dimensional presentation makes the subject more clear."
"Derivative; because it's said that its proof is too hard and demanding, but it would be more clear with GeoGebra"
"Representation of functions with graphics; because it's easier to keep in mind.

- The student opinions related to the most remarkable features about GeoGebra can be summarized as in below:

1. Have different features and ease of use,
2. The opportunity to provide a better understanding of visual sense, its colors,
3. Making the subject concrete,
4. Saving in time,
5. Eliminating the possibility of the failure because of more specific details,
6. Everything can be seen more easily through shapes,
7. Formulas can be registered so providing to think more abstract beyond memorising formulas
8. Available on Internet

- In students' opinion, Education with using GeoGebra will be more different than face to face education with the visual features that GeoGebra has. Also, because of the same reason, students want teachers to use GeoGebra in math lessons.

Findings Obtained from the Teachers

Below are some findings based on the opinions obtained from teachers who participated in our study:

- They prefer computer-aided mathematics education.
- They usually use mathematics software such as GeoGebra and Cabri 3D when preparing exam questions and when teaching in classroom.
- They think that GeoGebra makes math course easier and provides time saving.
- Because of the current university entrance exam system, they believe that students in Turkey wants to solve as much as possible test questions in order to be ready for that exam, instead of using such tools as the math software GeoGebra.

DISCUSSION AND CONCLUSION

Present study includes only two relatively high success leveled schools (If we take the university entrance examination as an objective criterion). Despite of this, laptop and projector devices were brought to the classrooms in order to present the slides prepared for the study. Also there were no smart boards in some classes that constitute negation in terms of computer-aided instruction.

Significant proportion of students, on the other hand, expressed their opinions verbally that they liked the diynanism and colors of materials prepared with GeoGebra. Also they expressed that it is more easy to understand topics taught through such materials.

In conclusion, present study has shown that computer-aided instruction by using GeoGebra, is an excellent way to focus students' understanding in math topics. So that, new dynamic GeoGebra worksheets can be created in the direction of student opinions.

We hope that our study might influence other schools' administrators to increase the number of technological devices used in classrooms, e.g. smart boards and multimedia projectors, as well as, to encourage the use of GeoGebra in teaching math.

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REFERENCES

Akkoyunlu, B., & İmer, G. (1998). Türkiyede Eğitim Teknolojisinin Görünümü, Çağdaş Eğitimde Yeni Teknolojiler Ünite-10. In Özer, B. (Ed.) *Çağdaş Eğitimde Yeni Teknolojiler Ünite 1-10* (pp. 159-176). Anadolu University.

Earl, R. (2004). *Complex Numbers*. Retrieved January 31, 2010, from http://www.maths.ox.ac.uk/files/imported/prospectivestudents/undergraduate/background/complex/complex.pdf.

Ersoy, Y. (2003). Teknoloji Destekli Matematik Eğitimi- 1: Gelişmeler, Politikalar ve Stratejiler. *İlköğretim Online, 2 (1)*. Retrieved September 10, 2009, from http://ilkogretim-online.org.tr/vol2say2/v02s02e.pdf.

GeoGebra Official Web Site. Retrieved 2010, from http://www.geogebra.org/cms/.

Istanbul University Informatics Department Official Web Site. Retrieved January 15, 2011, from enformatik.istanbul.edu.tr/tr/duyurular/2010/d18061004.htm.

Karadağ, E., Sağlam, K., & Baloğlu, N. (2008). Bilgisayar Destekli Eğitim [Bde]: İlköğretim Okulu Yöneticilerinin Tutumlarına İlişkin Bir Araştırma. *Uluslararası Sosyal Araştırmalar Dergisi. 1(3)*, Retrieved September 3, 2009, from http://www.sosyalarastirmalar.com/cilt1/sayi3/sayi3_pdf/karadag_saglam_baloglu.pdf.

Kaya, R. (2002). *Analitik Geometri*. İstanbul:Bilim Teknik Yayınevi.

Keser, H. (1989). Türk Okul Sisteminde Bilgisayarların Kullanılması. *A.Ü.Eğitim Bilimleri Fakültesi Dergisi.* 22(1). Retrieved September 5, 2009, from http://dergiler.ankara.edu.tr/dergiler/40/511/6291.pdf.

King, J. P. (1992). *Matematik Sanatı*. Tübitak Popüler Bilim Kitapları.

Özata, M. (2007). *Atatürk Bilim ve Üniversite*. TÜBİTAK Kitapları/Popüler Bilim Kitapları.

Taş, M. (2010). *Visualization of curvilinear integrals using the dynamic mathematics software geogebra*. Unpublished master dissertation, Istanbul University, Istanbul.

Taş, M., Gülseçen, S., & Kabaca, T. (2010). Çeşitli Parametrik Eğrilerin İnşaasının Görselleştirilmesi. *The International Conference on Innovations in Learning for the Future 2010: e-Learning First Eurasia Meeting of GeoGebra Proceedings Book*, *126* (pp.56-67). TC. İstanbul Kültür University Publication.

Tepedelenlioğlu, N. (2007). *Kim Korkar Matematikten*. Retrieved September 10, 2009, from http://www.matematikdunyasi.org/arsiv/makalelert/kimkorkar_kimkorkar.pdf.

Ülger, A. (2003). Matematiğin Kısa Bir Tarihi- I Mısır ve Mezopotamya Matematiği. *Matematik Dünyası*, 2003-Kış, 42-45.

Welker, J. L. (2008). Polar Grapher. Retrieved 2010, from http://www.geogebra.org/en/upload/files/english/jwelker/polar_grapher.html.

Yazıcı, A. *Matematik ve Bilişim*. Retrieved September 7, 2009, from http://aliyazici.atilim.edu.tr/bilisim-matematik.ppt.

Yıldırım, C. (2004). *Matematiksel Düşünme*. Remzi Kitabevi.

Yuyucu, M., & Ayvaz Reis, Z. (2010). Türev Konusunun GeoGebra Yardımı ile Anlatımı. *The International Conference on Innovations in Learning for the Future 2010: e-Learning First Eurasia Meeting of GeoGebra Proceedings Book*, *126* (pp.69-77). T.C. İstanbul Kültür University Publication.