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Intuitionistic fuzzy soft semigroups

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Abstract

Maji et al. introduced the concept of intuitionistic fuzzy soft sets, which is an extension to the soft set and intuitionistic fuzzy set. In this paper, we apply the concept of intuitionistic fuzzy soft sets to semigroup theory. The notion of intuitionistic fuzzy soft ideals over a semigroup is introduced and their basic properties are investigated. Some lattice structures of the set of all intuitionistic fuzzy soft ideals of a semigroup are derived.

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1 Introduction

Uncertain or imprecise data are inherent and pervasive in many important applications in the areas such as economics, engineering, environment, social science, medical science and business management. Uncertain data in those applications could be caused by data randomness, information incompleteness, limitations of measuring instruments, delayed data updates, etc. Due to the importance of those applications and the rapidly increasing amount of uncertain data collected and accumulated, research on effective and efficient techniques that are dedicated to modeling uncertain data and tackling uncertainties has attracted much interest in recent years and yet remained challenging at large. There have been a great amount of research and applications in the literature concerning some special tools like probability theory, (intuitionistic) fuzzy set theory, rough set theory, vague set theory and interval mathematics. However, all of these have theirs advantages as well as inherent limitations in dealing with uncertainties. One major problem shared by those theories is their incompatibility with the parameterization tools. To overcome these difficulties, Molodtsov [15] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets.

At present, works on the soft set theory are progressing rapidly. Maji et al. [10] described the application of soft set theory to a decision making problem. Chen et al. [5] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attribute reduction in rough set theory. In theoretical aspects, Maji et al. [11] defined and studied several operations on soft sets. Maji et al. [12] and Majumdar and Samanta [14] extended (classical) soft sets to fuzzy soft sets, respectively. Maji et al. [13] extended (classical) soft sets to intuitionistic fuzzy soft sets. Aktaş and Çağman [1] compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups, and derived some related properties. Feng et al. [6] investigated soft semirings by using the soft set theory. Jun [7] introduced and investigated the notion of soft BCK/BCI-algebras. Jun and Park [8] and Jun et al. [9] discussed the applications of soft sets in ideal theory of BCK/BCI-algebras and in d-algebras, respectively. Aygünoğlu and Aygün [4] introduced and studied (normal) fuzzy soft groups. Zhan and Jun [17] studied soft BL-algebras based on fuzzy sets. Xu et al. [16] introduced the notion of vague soft set which is an extension to the soft set and vague set, and discussed the basic properties of vague soft sets.

The purpose of this paper is to deal with the algebraic structure of semigroups by applying intuitionstic fuzzy soft set theory. The concept of intuitionstic fuzzy soft ideals over a semigroup is introduced, their characterization and algebraic properties are investigated. The rest of this paper is organized as follows. In Section 2, we summarize some basic concepts which will be used throughout the paper. In Section 3, we introduce the concept of intuitionstic fuzzy soft ideals over a semigroup and investigate some of their basic properties. Some conclusions are given in the last Section.

2 Preliminary

An intuitionistic fuzzy set A in a non-empty set X is an object having the form

$$A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. The set of all intuitionistic fuzzy sets in X is denoted as $\mathscr{I}\mathscr{F}(X)$.

For the sake of simplicity, we use $A = (\mu_A, \lambda_A)$ to denote the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$. For any intuitionistic fuzzy sets A and B in X, by $A \subseteq B$ we mean that $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$ for all $x \in X$. The intersection and union of A and B, denoted as $A \cap B$ and $A \cup B$, are defined as $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$ and $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$, respectively. For any $r, t \in [0,1]$, denote $A^{(r,t)} = \{x \in X | \mu_A(x) \geq r \text{ and } \lambda_A(x) \leq t\}$, which is called the (r,t)-*level cut* of A.

Lemma 2.1 [2, 3] *Let $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$ be intuitionistic fuzzy set in X and let $r, t \in [0,1]$ such that $r + t \leq 1$. Then*
(1) $\Box A = \{(x, \mu_A(x), \mu_A^c(x)) | x \in X\}$,
(2) $\Diamond A = \{(x, \lambda_A^c(x), \lambda_A(x)) | x \in X\}$,
(3) $P_{r,t}(A) = \{(x, \max\{r, \mu_A(x)\}, \min\{t, \lambda_A(x)\}) | x \in X\}$,
(4) $Q_{r,t}(A) = \{(x, \min\{r, \mu_A(x)\}, \max\{t, \lambda_A(x)\}) | x \in X\}$.
Then $\Box A, \Diamond A, P_{r,t}(A)$ and $Q_{r,t}(A)$ are intuitionistic fuzzy sets in X.

Let U be an initial universe, E a set of parameters, $\mathscr{P}(U)$ the power set of U, and $X \subseteq E$. Maji et al. [13] introduced the concept of intuitionistic fuzzy soft set as follows.

Definition 2.2 A pair $(\tilde{F}, X)$ is called an *intuitionistic fuzzy soft set* over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : X \mapsto \mathscr{I}\mathscr{F}(U)$.

Definition 2.3 For two intuitionistic fuzzy soft sets $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ over a universe U, we say that $(\tilde{F}, X)$ is an *intuitionistic fuzzy soft subset* in $(\tilde{G}, Y)$, if

$$X \subseteq Y \text{ and } \tilde{F}(\alpha) \subseteq \tilde{G}(\alpha)$$

for all $\alpha \in X$. This relationship is denoted by $(\tilde{F}, X) \tilde{\subseteq} (\tilde{G}, Y)$. Similarly, $(\tilde{F}, X)$ is said to be an *intuitionistic fuzzy soft superset* in $(\tilde{G}, Y)$, if $(\tilde{G}, Y)$ is an intuitionistic fuzzy subset in $(\tilde{F}, X)$. We denote it by $(\tilde{F}, X) \tilde{\supseteq} (\tilde{G}, Y)$. $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ over a universe U are said to intuitionistic fuzzy soft equal if $(\tilde{F}, X) \tilde{\subseteq} (\tilde{G}, Y)$ and $(\tilde{G}, Y) \tilde{\subseteq} (\tilde{F}, X)$.

Definition 2.4 Let $(\tilde{F}, X)$ be an intuitionistic fuzzy soft set over a universe U and let $r, t \in [0,1]$ such that $r+t \leq 1$. Define the intuitionistic fuzzy soft sets $(\Box\tilde{F}, X)$, $(\Diamond\tilde{F}, X)$, $(P_{r,t}(\tilde{F}), X)$, $(Q_{r,t}(\tilde{F}), X)$ over universe U by $(\Box\tilde{F})(\alpha) = \Box(\tilde{F}(\alpha))$, $(\Diamond\tilde{F})(\alpha) = \Diamond(\tilde{F}(\alpha))$, $(P_{r,t}(\tilde{F}))(\alpha) = P_{r,t}(\tilde{F}(\alpha))$ and $(Q_{r,t}(\tilde{F}))(\alpha) = Q_{r,t}(\tilde{F}(\alpha))$ respectively for all $\alpha \in X$.

Definition 2.5 If $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ are two intuitionistic fuzzy soft sets over a universe U. "$(\tilde{F}, X)$ AND $(\tilde{G}, Y)$", denoted by $(\tilde{F}, X)\tilde{\wedge}(\tilde{G}, Y)$, is defined by $(\tilde{F}, X)\tilde{\wedge}(\tilde{G}, Y) = (\tilde{H}, X \times Y)$, where $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cap \tilde{G}(\beta)$ for all $(\alpha, \beta) \in X \times Y$.

Definition 2.6 If $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ are two intuitionistic fuzzy soft sets over a universe U. "$(\tilde{F}, X)$ OR $(\tilde{G}, Y)$", denoted by $(\tilde{F}, X)\tilde{\vee}(\tilde{G}, Y)$, is defined by $(\tilde{F}, X)\tilde{\vee}(\tilde{G}, Y) = (\tilde{O}, X \times Y)$, where $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cup \tilde{G}(\beta)$ for all $(\alpha, \beta) \in X \times Y$.

Definition 2.7 The *union* of two intuitionistic fuzzy soft sets $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ over a universe U is an intuitionistic fuzzy soft set denoted by $(\tilde{H}, Z)$, where $Z = X \cup Y$ and

$$\tilde{H}(\alpha) = \begin{cases} \tilde{F}(\alpha) & \text{if } \alpha \in X - Y, \\ \tilde{G}(\alpha) & \text{if } \alpha \in Y - X, \\ \max\{\tilde{F}(\alpha), \tilde{G}(\alpha)\} & \text{if } \alpha \in X \cap Y, \end{cases}$$

for all $\alpha \in Z$. This is denoted by $(\tilde{H}, Z) = (\tilde{F}, X)\tilde{\cup}(\tilde{G}, Y)$.

Definition 2.8 The *intersection* of two intuitionistic fuzzy soft sets $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ over a universe U is an intuitionistic fuzzy soft set denoted by $(\tilde{H}, Z)$, where $Z = X \cup Y$ and

$$\tilde{H}(\alpha) = \begin{cases} \tilde{F}(\alpha) & \text{if } \alpha \in X - Y, \\ \tilde{G}(\alpha) & \text{if } \alpha \in Y - X, \\ \min\{\tilde{F}(\alpha), \tilde{G}(\alpha)\} & \text{if } \alpha \in X \cap Y, \end{cases}$$

for all $\alpha \in Z$. This is denoted by $(\tilde{H}, Z) = (\tilde{F}, X)\tilde{\cap}(\tilde{G}, Y)$.

In contrast with the above definitions of intuitionistic fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

Definition 2.9 Let $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ be two intuitionistic fuzzy soft sets over a universe U such that $X \cap Y \neq \emptyset$. The *bi-union* of $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ is defined to be the intuitionistic fuzzy soft set $(\tilde{H}, Z)$, where $Z = X \cap Y$ and $\tilde{H}(\alpha) = \tilde{F}(\alpha) \cup \tilde{G}(\alpha)$ for all $\alpha \in Z$. This is denoted by $(\tilde{H}, Z) = (\tilde{F}, X)\tilde{\sqcup}(\tilde{G}, Y)$.

Definition 2.10 Let $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ be two intuitionistic fuzzy soft sets over a universe U such that $X \cap Y \neq \emptyset$. The *bi-intersection* of $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ is defined to be the intuitionistic fuzzy soft set $(\tilde{H}, Z)$, where $Z = X \cap Y$ and $\tilde{H}(\alpha) = \tilde{F}(\alpha) \cap \tilde{G}(\alpha)$ for all $\alpha \in Z$. This is denoted by $(\tilde{H}, Z) = (\tilde{F}, X)\tilde{\sqcap}(\tilde{G}, Y)$.

3 Intuitionistic fuzzy soft ideals of a semigroup

Definition 3.1 An intuitionistic fuzzy soft set $(\tilde{F}, X)$ over a semigroup S is called an *intuitionistic fuzzy soft ideal* over S if it satisfies:

(F1) $\mu_{\tilde{F}(\alpha)}(xy) \geq \max\{\mu_{\tilde{F}(\alpha)}(x), \mu_{\tilde{F}(\alpha)}(y)\}$ and $\lambda_{\tilde{F}(\alpha)}(xy) \leq \min\{\lambda_{\tilde{F}(\alpha)}(x), \lambda_{\tilde{F}(\alpha)}(y)\}$, for all $x, y \in S$ *and* $\alpha \in X$.

Example 3.2 The set $\mathbb{N}_0$ of all non-negative integers with usual addition is a semigroup. Define an intuitionistic fuzzy soft set $(\tilde{F}, X)$ over $\mathbb{N}_0$, where $X = \mathbb{N}_0$, by

$$\mu_{\tilde{F}(\alpha)}(x) = \begin{cases} 1 & \text{if } x = 0, \\ \frac{1}{\alpha} & \text{if } x \in (\alpha) - \{0\}, \\ 0 & \text{otherwise,} \end{cases} \qquad \lambda_{\tilde{F}(\alpha)}(x) = \begin{cases} 0 & \text{if } x = 0, \\ 1 - \frac{1}{\alpha} & \text{if } x \in (\alpha) - \{0\}, \\ 1 & \text{otherwise,} \end{cases}$$

for all $\alpha, x \in \mathbb{N}_0$. Then $(\tilde{F}, X)$ is an intuitionistic fuzzy soft ideal over $\mathbb{N}_0$.

The following theorem presents the relationship between intuitionistic fuzzy soft ideals and crisp ideals of a semigroup S.

Theorem 3.3 *Let $(\tilde{F}, X)$ be an intuitionistic fuzzy soft set over a semigroup S. Then $(\tilde{F}, X)$ is an intuitionistic fuzzy soft ideal over S if and only if $\tilde{F}(\alpha)^{(r,t)}$ is an ideal of S for all $r \in (0, 1], t \in [0, 1)$ and $\alpha \in X$.*

Proof. It is straightforward.

Proposition 3.4 *Let $(\tilde{F}, X)$ be an intuitionistic fuzzy soft set over a semigroup S and let $Y \subset X$. If $(\tilde{F}, X)$ is an intuitionistic fuzzy soft ideal over S, then so is $(\tilde{F}, Y)$ whenever it is non-null.*

Proof. It is straightforward.

Next let's consider the intuitionistic fuzzy soft ideals over a semigroup S induced by an intuitionistic fuzzy soft ideal over S.

Proposition 3.5 *If $(\tilde{F}, X)$ is an intuitionistic fuzzy soft ideal over a universe U, then so are (1) $(\Box\tilde{F}, X)$, (2) $(\Diamond\tilde{F}, X)$, (3) $(P_{r,t}(\tilde{F}), X)$, (4) $(Q_{r,t}(\tilde{F}), X)$, where $r, t \in [0, 1]$ and $r + t \leq 1$.*

Proof. It is straightforward.

In view of Proposition 3.5, it is easy to verify that the following theorem is valid.

Theorem 3.6 *An intuitionistic fuzzy soft set* $(\tilde{F}, X)$ *over a semigroup* S *is an intuitionistic fuzzy soft ideal over* S *if and only if* $(\Box\tilde{F}, X)$ *and* $(\Diamond\tilde{F}, X)$ *are intuitionistic fuzzy soft ideals over* S.

Theorem 3.7 *If* $(\tilde{F}, X)$ *and* $(\tilde{G}, Y)$ *are two intuitionistic fuzzy soft ideals over a semigroup* S*, then so are* $(\tilde{F}, X)\tilde{\wedge}(\tilde{G}, Y)$ *and* $(\tilde{F}, X)\tilde{\sqcap}(\tilde{G}, Y)$.

Proof. By Definition 2.5, we can write $(\tilde{F}, X)\tilde{\wedge}(\tilde{G}, Y) = (\tilde{H}, Z)$, where $Z = X \times Y$ and $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cap \tilde{G}(\beta)$ for all $(\alpha, \beta) \in Z$. Now for any $(\alpha, \beta) \in Z$, since $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ are intuitionistic fuzzy soft ideals over S, we have

$$\begin{aligned}\mu_{\tilde{H}(\alpha,\beta)}(xy) &= \min\{\mu_{\tilde{F}(\alpha)}(xy), \mu_{\tilde{G}(\beta)}(xy)\}\\ &\geq \min\{\max\{\mu_{\tilde{F}(\alpha)}(x), \mu_{\tilde{F}(\alpha)}(y)\}, \max\{\mu_{\tilde{G}(\beta)}(x), \mu_{\tilde{G}(\beta)}(y)\}\}\\ &= \max\{\min\{\mu_{\tilde{F}(\alpha)}(x), \mu_{\tilde{G}(\beta)}(x)\}, \min\{\mu_{\tilde{F}(\alpha)}(y), \mu_{\tilde{G}(\beta)}(y)\}\}\\ &= \max\{\mu_{\tilde{H}(\alpha,\beta)}(x), \mu_{\tilde{H}(\alpha,\beta)}(y)\}\end{aligned}$$

and

$$\begin{aligned}\lambda_{\tilde{H}(\alpha,\beta)}(xy) &= \max\{\lambda_{\tilde{F}(\alpha)}(xy), \lambda_{\tilde{G}(\beta)}(xy)\}\\ &\leq \max\{\min\{\lambda_{\tilde{F}(\alpha)}(x), \lambda_{\tilde{F}(\alpha)}(y)\}, \min\{\lambda_{\tilde{G}(\beta)}(x), \lambda_{\tilde{G}(\beta)}(y)\}\}\\ &\leq \min\{\max\{\lambda_{\tilde{F}(\alpha)}(x), \lambda_{\tilde{G}(\beta)}(x)\}, \max\{\lambda_{\tilde{F}(\alpha)}(y), \lambda_{\tilde{G}(\beta)}(y)\}\}\\ &= \min\{\lambda_{\tilde{H}(\alpha,\beta)}(x), \lambda_{\tilde{H}(\alpha,\beta)}(y)\}\end{aligned}$$

for all $x, y \in S$ and $(\alpha, \beta) \in Z$. It follows that $(\tilde{F}, X)\tilde{\wedge}(\tilde{G}, Y)$ is an intuitionistic fuzzy soft ideal over S.

The case for $(\tilde{F}, X)\tilde{\sqcap}(\tilde{G}, Y)$ can be similarly proved.

Theorem 3.8 *If* $(\tilde{F}, X)$ *and* $(\tilde{G}, Y)$ *are two intuitionistic fuzzy soft ideals over a semigroup* S*, then so are* $(\tilde{F}, X)\tilde{\vee}(\tilde{G}, Y)$ *and* $(\tilde{F}, X)\tilde{\sqcup}(\tilde{G}, Y)$.

Proof. By Definition 2.6, we can write $(\tilde{F}, X)\tilde{\vee}(\tilde{G}, Y) = (\tilde{O}, Z)$, where $Z = X \times Y$ and $\tilde{O}(\alpha, \beta) = \tilde{F}(\alpha) \cup \tilde{G}(\beta)$ for all $(\alpha, \beta) \in Z$. Now, for any $x, y \in S$ and $(\alpha, \beta) \in Z$, we have

$$\begin{aligned}\mu_{\tilde{O}(\alpha,\beta)}(xy) &= (\mu_{\tilde{F}(\alpha)} \cup \mu_{\tilde{G}(\beta)})(xy) = \max\{\mu_{\tilde{F}(\alpha)}(xy), \mu_{\tilde{G}(\beta)}(xy)\}\\ &\geq \max\{\max\{\mu_{\tilde{F}(\alpha)}(x), \mu_{\tilde{F}(\alpha)}(y)\}, \max\{\mu_{\tilde{G}(\beta)}(x), \mu_{\tilde{G}(\beta)}(y)\}\}\\ &= \max\{(\mu_{\tilde{F}(\alpha)} \cup \mu_{\tilde{G}(\beta)})(x), (\mu_{\tilde{F}(\alpha)} \cup \mu_{\tilde{G}(\beta)})(y)\}\\ &= \max\{(\mu_{\tilde{O}(\alpha,\beta)}(x), \mu_{\tilde{O}(\alpha,\beta)}(y)\}\end{aligned}$$

and

$$\begin{aligned}\lambda_{\tilde{O}(\alpha,\beta)}(xy) =&(\lambda_{\tilde{F}(\alpha)} \cap \lambda_{\tilde{G}(\beta)})(xy) = \min\{\lambda_{\tilde{F}(\alpha)}(xy), \lambda_{\tilde{G}(\beta)}(xy)\}\\ \leq& \min\{\min\{\lambda_{\tilde{F}(\alpha)}(x), \lambda_{\tilde{F}(\alpha)}(y)\}, \min\{\lambda_{\tilde{G}(\beta)}(x), \lambda_{\tilde{G}(\beta)}(y)\}\}\\ =& \min\{(\lambda_{\tilde{F}(\alpha)} \cap \lambda_{\tilde{G}(\beta)})(x), (\lambda_{\tilde{F}(\alpha)} \cap \lambda_{\tilde{G}(\beta)})(y)\}\\ =& \min\{(\lambda_{\tilde{O}(\alpha,\beta)}(x), \lambda_{\tilde{O}(\alpha,\beta)}(y)\}.\end{aligned}$$

It follows that $(\tilde{F}, X)\tilde{\vee}(\tilde{G}, Y)$ is an intuitionistic fuzzy soft ideal over S.

The case for $(\tilde{F}, X)\tilde{\sqcup}(\tilde{G}, Y)$ can be similarly proved.

Theorem 3.9 *If $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ are two intuitionistic fuzzy soft ideals over a semigroup S, then so is $(\tilde{F}, X)\tilde{\cap}(\tilde{G}, Y)$.*

Proof. By Definition 2.8, we can write $(\tilde{F}, X)\tilde{\cap}(\tilde{G}, Y) = (\tilde{H}, Z)$, where $Z = X \cup Y$ and

$$\tilde{H}(\alpha) = \begin{cases} \tilde{F}(\alpha) & \text{if } \alpha \in X - Y, \\ \tilde{G}(\alpha) & \text{if } \alpha \in Y - X, \\ \tilde{F}(\alpha) \cap \tilde{G}(\alpha) & \text{if } \alpha \in X \cap Y. \end{cases}$$

for all $\alpha \in Z$.

Now for any $\alpha \in Z$ and $x, y \in S$, we consider the following cases.

Case 1: $\alpha \in X - Y$. Then

$$\mu_{\tilde{H}(\alpha)}(xy) = \mu_{\tilde{F}(\alpha)}(xy) \geq \max\{\mu_{\tilde{F}(\alpha)}(x), \mu_{\tilde{F}(\alpha)}(y)\} = \max\{\mu_{\tilde{H}(\alpha)}(x), \mu_{\tilde{H}(\alpha)}(y)\}$$

and

$$\lambda_{\tilde{H}(\alpha)}(xy) = \lambda_{\tilde{F}(\alpha)}(xy) \leq \min\{\lambda_{\tilde{F}(\alpha)}(x), \lambda_{\tilde{F}(\alpha)}(y)\} = \min\{\lambda_{\tilde{H}(\alpha)}(x), \lambda_{\tilde{H}(\alpha)}(y)\}.$$

Case 2: $\alpha \in Y - X$. Analogous to the proof of Case 1, we have $\mu_{\tilde{H}(\alpha)}(xy) \geq \max\{\mu_{\tilde{H}(\alpha)}(x), \mu_{\tilde{H}(\alpha)}(y)\}$ and $\lambda_{\tilde{H}(\alpha)}(xy) \leq \min\{\lambda_{\tilde{H}(\alpha)}(x), \lambda_{\tilde{H}(\alpha)}(y)\}$.

Case 3: $\alpha \in X \cap Y$. Then $\tilde{H}(\alpha) = \tilde{F}(\alpha) \cap \tilde{G}(\alpha)$. Analogous to the proof of Theorem 3.7 we have $\mu_{\tilde{H}(\alpha)}(xy) \geq \max\{\mu_{\tilde{H}(\alpha)}(x), \mu_{\tilde{H}(\alpha)}(y)\}$ and $\lambda_{\tilde{H}(\alpha)}(xy) \leq \min\{\lambda_{\tilde{H}(\alpha)}(x), \lambda_{\tilde{H}(\alpha)}(y)\}$.

Thus, in any case, we have $\mu_{\tilde{H}(\alpha)}(xy) \geq \max\{\mu_{\tilde{H}(\alpha)}(x), \mu_{\tilde{H}(\alpha)}(y)\}$ and $\lambda_{\tilde{H}(\alpha)}(xy) \leq \min\{\lambda_{\tilde{H}(\alpha)}(x), \lambda_{\tilde{H}(\alpha)}(y)\}$, and so $(\tilde{F}, X)\tilde{\cap}(\tilde{G}, Y)$ is an intuitionistic fuzzy soft ideal over S.

Theorem 3.10 *If $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ be two intuitionistic fuzzy soft ideals over a semigroup G, then so is $(\tilde{F}, X)\tilde{\cup}(\tilde{G}, Y)$.*

Proof. The proof is similar to that of Theorem 3.9.

Denote by $\mathscr{IFSI}(S, E)$ the set of all intuitionistic fuzzy soft ideals over S. From Theorems 3.7-3.10, we have the following results.

Theorem 3.11 $(\mathscr{IFSI}(S,E),\widetilde{\cup},\widetilde{\sqcap})$ *is a complete distributive lattice under the ordering relation* "$\tilde{\subseteq}$".

Proof. For any $(\tilde{F},X),(\tilde{G},Y)\in\mathscr{IFSI}(S,E)$, by Theorems 3.7 and 3.10, $(\tilde{F},X)\widetilde{\cup}(\tilde{G},Y)\in\mathscr{IFSI}(S,E)$ and $(\tilde{F},X)\widetilde{\sqcap}(\tilde{G},Y)\in\mathscr{IFSI}(S,E)$. It is obvious that $(\tilde{F},X)\widetilde{\cup}(\tilde{G},Y)$ and $(\tilde{F},X)\widetilde{\sqcap}(\tilde{G},Y)$ are the least upper bound and the greatest lower bound of $(\tilde{F},X)$ and $(\tilde{G},Y)$, respectively. There is no difficulty in replacing $\{(\tilde{F},X),(\tilde{G},Y)\}$ with an arbitrary family of $\mathscr{IFSI}(S,E)$ and so $(\mathscr{IFSI}(S,E),\widetilde{\cup},\widetilde{\sqcap})$ is a complete lattice. Now we prove that the following distributive law

$$(\tilde{F},X)\widetilde{\sqcap}((\tilde{G},Y)\widetilde{\cup}(\tilde{H},Z))=((\tilde{F},X)\widetilde{\sqcap}(\tilde{G},Y))\widetilde{\cup}((\tilde{F},X)\widetilde{\sqcap}(\tilde{H},Z))$$

holds for all $(\tilde{F},X),(\tilde{G},Y),(\tilde{H},Z)\in\mathscr{IFSI}(S,E)$. Suppose that

$(\tilde{F},X)\widetilde{\sqcap}((\tilde{G},Y)\widetilde{\cup}(\tilde{H},Z))=(I,X\cap(Y\cup Z))$,

$((\tilde{F},X)\widetilde{\sqcap}(\tilde{G},Y))\widetilde{\cup}((\tilde{F},X)\widetilde{\sqcap}(\tilde{H},Z))=(J,(X\cap Y)\cup(X\cap Z))=(J,X\cap(Y\cup Z))$.

Now for any $\alpha\in X\cap(Y\cup Z)$, it follows that $\alpha\in X$ and $\alpha\in Y\cup Z$. We consider the following cases.

Case 1: $\alpha\in X,\alpha\notin Y$ and $\alpha\in Z$. Then $I(\alpha)=F(\alpha)\cap H(\alpha)=J(\alpha)$.

Case 2: $\alpha\in X,\alpha\in Y$ and $\alpha\notin Z$. Then $I(\alpha)=F(\alpha)\cap G(\alpha)=J(\alpha)$.

Case 3: $\alpha\in X,\alpha\in Y$ and $\alpha\in Z$. Then $I(\alpha)=F(\alpha)\cap(G(\alpha)\cup H(\alpha))=(F(\alpha)\cap G(\alpha))\cup(F(\alpha)\cap H(\alpha))=J(\alpha)$.

Therefore, I and J are the same operators, and so

$$(\tilde{F},X)\widetilde{\sqcap}((\tilde{G},Y)\widetilde{\cup}(\tilde{H},Z))=((\tilde{F},X)\widetilde{\sqcap}(\tilde{G},Y))\widetilde{\cup}((\tilde{F},X)\widetilde{\sqcap}(\tilde{H},Z)).$$

This completes the proof.

Theorem 3.12 $(\mathscr{IFSI}(S,E),\widetilde{\sqcup},\widetilde{\cap})$ *is a complete distributive lattice under the ordering relation* "$\tilde{\subseteq}'$", *where for any* $(\tilde{F},X),(\tilde{G},Y)\in\mathscr{IFSI}(S,E)$, $(\tilde{F},X)\tilde{\subseteq}'(\tilde{G},Y)$ *if and only if* $Y\subseteq X$ *and* $F(\alpha)\subseteq G(\alpha)$ *for any* $\alpha\in Y$.

Proof. The proof is similar to that of Theorem 3.11.

Now we consider the intuitionistic fuzzy soft sets over a definite parameter set. Let $A\subseteq E$, S be a semigroup and

$$\mathscr{IFS}_A(S)=\{(\tilde{F},X)\in\mathscr{IFSI}(S,E)|\tilde{F}:A\to\mathscr{IF}(S)\}$$

the set of intuitionistic fuzzy soft ideals over S and the parameter set A. It is trivial to verify that $(\tilde{F},X)\widetilde{\cup}(\tilde{G},X)$, $(\tilde{F},X)\widetilde{\cap}(\tilde{G},X)$, $(\tilde{F},X)\widetilde{\sqcup}(\tilde{G},X)$, $(\tilde{F},X)\widetilde{\sqcap}(\tilde{G},X)\in\mathscr{IFS}_A(S)$ for all $(\tilde{F},X),(\tilde{G},X)\in\mathscr{IFS}_A(S)$.

Corollary 3.13 $(\mathscr{IFS}_A(S),\widetilde{\cup},\widetilde{\sqcap})$ *and* $(\mathscr{IFS}_A(S),\widetilde{\sqcup},\widetilde{\cap})$ *are sublattices of* $(\mathscr{IFSI}(S,E),\widetilde{\cup},\widetilde{\sqcap})$ *and* $(\mathscr{IFSI}(S,E),\widetilde{\sqcup},\widetilde{\cap})$, *respectively.*

Definition 3.14 Let $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ be two intuitionistic soft sets over a semigroup S. The *product* of $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ is defined to be the intuitionistic soft set $(\tilde{F} \circ \tilde{G}, Z)$, where $Z = X \cup Y$ and

$$\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x) = \begin{cases} \mu_{\tilde{F}(\alpha)}(x) & \text{if } \alpha \in X - Y, \\ \mu_{\tilde{G}(\alpha)}(x) & \text{if } \alpha \in Y - X, \\ \sup\limits_{x=ab} \min\{\mu_{\tilde{F}(\alpha)}(a), \mu_{\tilde{G}(\alpha)}(b)\} & \text{if } \alpha \in X \cap Y, \end{cases}$$

and

$$\lambda_{(\tilde{F}\circ\tilde{G})(\alpha)}(x) = \begin{cases} \lambda_{\tilde{F}(\alpha)}(x) & \text{if } \alpha \in X - Y, \\ \lambda_{\tilde{G}(\alpha)}(x) & \text{if } \alpha \in Y - X, \\ \inf\limits_{x=ab} \max\{\lambda_{\tilde{F}(\alpha)}(a), \lambda_{\tilde{G}(\alpha)}(b)\} & \text{if } \alpha \in X \cap Y, \end{cases}$$

for all $\alpha \in Z$ and $x \in S$. This is denoted by $(\tilde{F} \circ \tilde{G}, Z) = (\tilde{F}, X) \circ (\tilde{G}, Y)$.

Theorem 3.15 *If $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ are intuitionistic fuzzy soft ideals over a semigroup S, then so is $(\tilde{F}, X) \circ (\tilde{G}, Y)$.*

Proof. Let $(\tilde{F}, X)$ and $(\tilde{G}, Y)$ be intuitionistic fuzzy soft ideals over S. Then for any $\alpha \in A \cup B$ and $x, y \in S$, we consider the following cases.

Case 1: $\alpha \in X - Y$. Then

$$\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy) = \mu_{\tilde{F}(\alpha)}(xy) \geq \max\{\mu_{\tilde{F}(\alpha)}(x), \mu_{\tilde{F}(\alpha)}(y)\} = \max\{\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x), \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(y)\}.$$

Case 2: $\alpha \in Y - X$. Analogous to the proof of Case 1, we have

$$\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy) \geq \max\{\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x), \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(y)\}.$$

Case 3: $\alpha \in X \cap Y$. Then

$$\begin{aligned} \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(y) &= \sup_{y=ab} \min\{\mu_{\tilde{F}(\alpha)}(a), \mu_{\tilde{G}(\alpha)}(b)\} \leq \sup_{xy=xab} \min\{\mu_{\tilde{F}(\alpha)}(xa), \mu_{\tilde{G}(\alpha)}(b)\} \\ &\leq \sup_{xy=cd} \min\{\mu_{\tilde{F}(\alpha)}(c), \mu_{\tilde{G}(\alpha)}(d)\} = \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy). \end{aligned}$$

Similarly, we have $\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x) \leq \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy)$, and so

$$\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy) \geq \max\{\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x), \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(y)\}.$$

Thus, in any case, we have $\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy) \geq \max\{\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x), \mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(y)\}$. In a similar way, we may prove that $\lambda_{(\tilde{F}\circ\tilde{G})(\alpha)}(xy) \leq \min\{\lambda_{(\tilde{F}\circ\tilde{G})(\alpha)}(x), \lambda_{(\tilde{F}\circ\tilde{G})(\alpha)}(y)\}$.

Therefore, $(\tilde{F}, X) \circ (\tilde{G}, Y)$ is an intuitionistic fuzzy soft ideal over S.

Let S be an ordered semigroup with an identity e. Denote by $\mathscr{IFSIT}(S, E)$ the set of all intuitionistic fuzzy soft ideals over S such that $\mu_{\tilde{F}}(e) = 1$ and $\lambda_{\tilde{F}}(e) = 0$ for all $(\tilde{F}, X) \in \mathscr{IFSIT}(S, E)$. Then we have the following result.

Theorem 3.16 *Let S be a semigroup with an identity e. Then $(\mathscr{IFSIT}(S,E),\circ,\tilde{\sqcap})$ is a complete lattice under the relation "$\tilde{\subseteq}$".*

Proof. Let $(\tilde{F},X),(\tilde{G},Y)\in\mathscr{IFSIT}(S,E)$. It follows from Theorems 3.9 and 3.15 that $(\tilde{F},X)\tilde{\sqcap}(\tilde{G},Y)\in\mathscr{IFSIT}(S,E)$ and $(\tilde{F},X)\circ(\tilde{G},Y)\in\mathscr{IFSIT}(S,E)$. It is clear that $(\tilde{F},X)\tilde{\sqcap}(\tilde{G},Y)$ is the greatest lower bound of $(\tilde{F},X)$ and $(\tilde{G},Y)$. We now show that $(\tilde{F},X)\circ(\tilde{G},Y)$ is the least upper bound of $(\tilde{F},X)\circ(\tilde{G},Y)$. For any $\alpha\in A$ and $x\in S$, we consider the following cases.

Case 1: $\alpha\in X-Y$. Then $\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x)=\mu_{\tilde{F}}(x)$ and $\lambda_{(\tilde{F}\circ\tilde{G})(\alpha)}(x)=\lambda_{\tilde{F}}(x)$.

Case 2: $\alpha\in X\cap Y$. Then $(F\circ G)(\alpha)=F(\alpha)\circ G(\alpha)$. Since $\mu_{\tilde{G}(\alpha)}(e)=1$ and $\lambda_{\tilde{G}(\alpha)}(e)=0$, we have

$$\mu_{(\tilde{F}\circ\tilde{G})(\alpha)}(x)=\sup_{x=ab}\min\{\mu_{\tilde{F}(\alpha)}(a),\mu_{\tilde{G}(\alpha)}(b)\}\geq\min\{\mu_{\tilde{F}(\alpha)}(x),\mu_{\tilde{G}(\alpha)}(e)\}=\mu_{\tilde{F}(\alpha)}(x)$$

and

$$\lambda_{(\tilde{F}\circ\tilde{G})(\alpha)}(x)=\inf_{x=ab}\max\{\lambda_{\tilde{F}(\alpha)}(a),\lambda_{\tilde{G}(\alpha)}(b)\}\leq\max\{\lambda_{\tilde{F}(\alpha)}(x),\lambda_{\tilde{G}(\alpha)}(e)\}=\lambda_{\tilde{F}(\alpha)}(x),$$

and so $(\tilde{F},X)\tilde{\subseteq}(\tilde{F},X)\circ(\tilde{G},Y)$. Similarly, we have $(\tilde{G},Y)\tilde{\subseteq}(\tilde{F},X)\circ(\tilde{G},Y)$. Now, let $(\tilde{H},Z)\in\mathscr{IFSIT}(S,E)$ be such that $(\tilde{F},X)\tilde{\subseteq}(\tilde{H},Z)$ and $(\tilde{G},Y)\tilde{\subseteq}(\tilde{H},Z)$. Then, it is easy to see that $(\tilde{F},X)\circ(\tilde{G},Y)\tilde{\subseteq}(\tilde{H},Z)\circ(\tilde{H},Z)\tilde{\subseteq}(\tilde{H},Z)$. Hence $(\tilde{F},X)\vee(\tilde{G},Y)=(\tilde{F},X)\circ(\tilde{G},Y)$. There is no difficulty in replacing $\{(\tilde{F},X),(\tilde{G},Y)\}$ with an arbitrary family of $\mathscr{IFSIT}(S,E)$ and so $(\mathscr{IFSIT}(S,E),\circ,\tilde{\sqcap})$ is a complete lattice under the relation "$\tilde{\subseteq}$".

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