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Propagation of Torsional Surface Waves in a Non-Homogeneous Crustal Layer over a Viscoelastic Mantle

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ABSTRACT

The present paper studies the possibility of propagation of torsional surface waves in a non-homogeneous isotropic crustal layer lying over a viscoelastic mantle. Both rigidity and density of the crustal layer are assumed to vary exponentially with depth. Separation of variable method has been used to get the analytical solutions for the dispersion equation for the torsional surface waves. Further, in the absence of non-homogeneity and internal friction, this equation is in complete agreement with the classical result of Love. Also, the effects of non-homogeneity, internal friction (viscoelastic parameter), rigidity, wave number and time period on the phase velocity of torsional surface waves have shown graphically.

Mathematics Subject Classification: 74J15

Keywords: Non-homogeneous media, Torsional wave, Rigidity, Viscoelasticity.

1 INTRODUCTION

The formulations and solutions of many problems of linear wave-propagation for homogeneous media are available in the literature of continuum mechanics of solids. In recent years, however, sufficient interest has arisen in the problem connected with bodies whose mechanical properties are functions of space, i.e. non-homogeneous bodies. This interest is mainly due to the advent of solid rocket propellants, polymeric materials and growing demand of engineering and industrial applications.

The theory of visco-elasticity is of interest in the broad field of solid mechanics and of special interest in seismology, engineering, exploration geophysics, and acoustics for consideration of wave propagation in layered media with arbitrary amounts of intrinsic absorption, ranging from low-loss models of the deep Earth to moderate-loss models for soils and weathered rock.

Studies of wave propagation in the earth stratum under loads have usually assumed that the earth behaves to a first approximation as an ideal elastic or viscoelastic material. The stratura may be of finite depth or it may be so deep compared to the size of the loaded area that it can be regarded as a half-space. In either case the complete solutions to elastic or viscoelastic problems are known when material parameters are treated independent of position.

The formation of earth strate in nature tends, however, to result in depth variations of these parameters and this may be due principally, either to stratification of different materials or to the effect of superincumbent pressure. The case of homogeneous viscoelastic half-space has been treated extensively in the literature. By comparison little has been done on the non-homogenous viscoelastic half-space when the material characteristics vary continuously with depth. This fact is likely to be true when the effects of over-burden pressure predominate. Furthermore these studies have been restricted to particular variations which have allowed the resulting simplification to be exploited.
Since our earth is a spherical body having finite dimension and the elastic waves generated must receive the effect of the boundaries. Naturally, this phenomenon leads us to the investigation of boundary waves or surface waves, i.e. the waves, which are confined to some surface during their propagation. Although much literature is available on the propagation of surface waves i.e. Rayleigh waves, Love waves and Stoneley waves etc., in a non-homogeneous elastic media in the monographs of [15]Stoneley, [2]Bullen, [8]Ewing et al., [10]Hunters and [11]Jeffreys. But very little literature is available on the propagation of One another type of surface wave called torsional surface wave whose amplitudes decaying exponentially with distance from the free surface and horizontally polarized but give a twist to the medium when it propagates. These waves often propagate during the earthquakes and become responsible to some extent for the destruction on the earth crustal layer. Therefore seismologists have started taking a keen interest on the propagation of torsional surface waves in different geo-media.
Disturbances in a generalized viscoelastic and elastic medium are being undertaken by many authors ([7]De SK and Sengupta PR, [5]Datta BK) due to its utilitarian aspects in various branches of science and technology particularly in geophysics and seismology.[18]Wang YZ and Tsai TJ studied the static and dynamic analysis of visco-elastic plate by FEM. [12]Manolis and Shaw studied harmonic wave propagation in viscoelastic media with stochasticity. A number of researchers ([14]Sharma JN, Singh D and Kumar R) attempted several problems of waves and vibrations in viscoelastic media. [4]Chattopadhyay A, Gupta S, Sharma V K., Kumari P studied the propagation of shear waves in viscoelastic medium at irregular boundaries.[3]Vlastislav Cerveny investigated plane waves in viscoelastic anisotropic media. Behaviour of waves interfaces in anisotropic media by three approaches by [16] Vaclav Vavrycuk. Further References have also been made to [6]Davini et al., [13]Gupta et al. and [1,17]Akbarov et al and

Vishwakarma et al. for their excellent work on the propagation of torsional surface waves.

In present study, we consider the propagation of torsional surface wave in a Non-homogeneous isotropic substratum overlying a viscoelastic mantle. Both rigidity and density of the crustal layer are assumed to vary exponentially with depth. Due to the presence of iron-alloy and silicate materials in earth composition which are having remarkable property that, under wide range of pressure and temperature conditions, these materials behaves elastically under the influence of small-magnitude transient forces but viscously under the influence of long duration forces i.e. why we choose viscoelastic mantle for the present study. An attempt has been made to investigate the effect of viscoelastic material on the phase velocity of torsional wave. Using variable separable method, dispersion equation has been obtained. Further this equation is in complete agreement with the corresponding classical results in the absence of non-homogeneity and internal friction.

2. Statement of the problem

Consider a non-homogeneous layer of finite thickness H, over a viscoelastic mantle. The origin of the cylindrical co-ordinate system (r, θ, z) is located at the interface separating the layer from the mantle, and the z-axis is directed downwards (as shown in fig. 1). In the layer, rigidity and density has been taken as $\mu = \mu_0 e^{2pz}$ and $\rho = \rho_0 \, e^{2pz}$ respectively, which are constant at interface and p be the non-homogeneity parameter, whereas in the viscoelastic mantle μ, ρ and μ' represents the rigidity, density and internal friction (viscoelastic parameter) respectively.

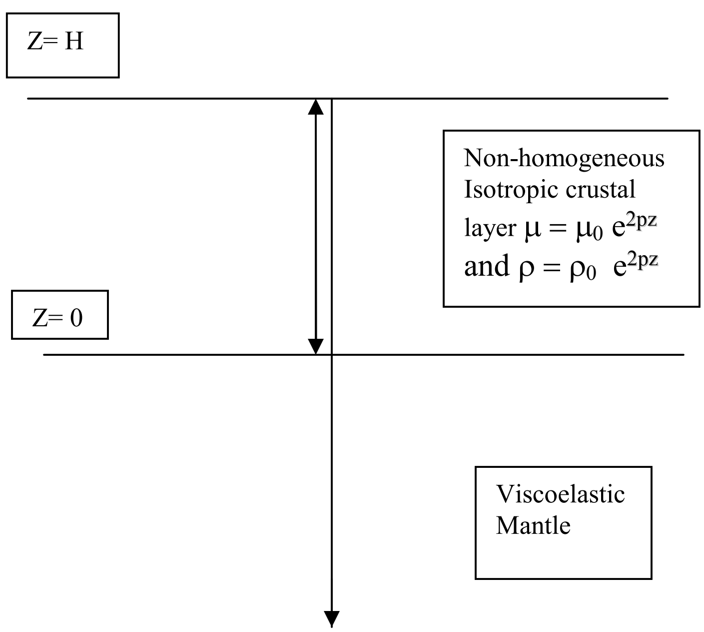

Figure 1

3. Solution of layer

To study the torsional surface waves with cylindrical co-ordinate system having z-axis towards the interior of the viscoelastic mantle. If r and θ be radial and circumferential co-ordinates respectively and if the wave travels along the radial direction only, the equation of motion may be written as (Biot 1965)

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta}+\frac{\partial \sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=\rho\frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial \sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta}+\frac{\partial \sigma_{\theta z}}{\partial z}+\frac{2\sigma_{r\theta}}{r}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{1}$$

$$\frac{\partial \sigma_{rz}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta}+\frac{\partial \sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=\rho\frac{\partial^2 w}{\partial t^2},$$

where σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, σ_{rz}, $\sigma_{r\theta}$ and $\sigma_{\theta z}$ are the respective stress components and "u, v, w" are the respective displacement components. Now, the stress-strain relations are given by

$$\sigma_{rr}=\lambda\Omega+2\mu e_{rr},\ \sigma_{\theta\theta}=\lambda\Omega+2\mu e_{\theta\theta},$$
$$\sigma_{zz}=\lambda\Omega+2\mu e_{zz},\ \sigma_{r\theta}=2\mu e_{r\theta}, \tag{2}$$
$$\sigma_{rz}=2\mu e_{rz},\ \sigma_{\theta z}=2\mu e_{\theta z},$$

where λ and μ are Lame's constant and $\Omega=\left(\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{u}{r}+\frac{\partial w}{\partial z}\right)$ denotes the dilatation.

The strain-displacement relations are

$$e_{rr}=\frac{1}{2}\frac{\partial u}{\partial r},\ e_{\theta\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{u}{r}\right),\ e_{zz}=\frac{1}{2}\frac{\partial w}{\partial z},$$

$$e_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial \theta}+\frac{\partial v}{\partial r}-\frac{v}{r}\right),\ e_{\theta z}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial \theta}\right), \tag{3}$$

$$e_{rz}=\frac{1}{2}\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right)$$

The torsional wave is characterized by the displacements

$$u=0,\ w=0,\ v=v(r,z,t) \tag{4}$$

Now, considering eqn. (2), (4), we get

$$\frac{\partial \sigma_{r\theta}}{\partial r}+\frac{\partial \sigma_{z\theta}}{\partial z}+\frac{2\sigma_{r\theta}}{r}=\rho(z)\frac{\partial^2 v}{\partial t^2} \tag{5}$$

where v (r, z, t) is the displacement along θ direction.

For an elastic medium, the stresses are related to displacement by

$$\sigma_{r\theta}=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)\text{ and }\sigma_{\theta z}=\mu\frac{\partial v}{\partial z}$$

If the material parameters which are involved in torsional wave propagation are

$$\mu=\mu_0\,e^{2pz}\text{ and }\rho=\rho_0\,e^{2pz} \tag{6}$$

Eq. (5) takes the form

$$\mu_0\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)v+\frac{\partial}{\partial z}\left(\mu_0\frac{\partial v}{\partial z}\right)=\rho_0\frac{\partial^2 v}{\partial t^2} \tag{7}$$

We may assume the solution of (7) as

$$v = V(z)\, J_1(kr)\, e^{i\omega t} \tag{8}$$

where V is the solution of

$$\frac{d^2V}{dz^2}+2p\frac{dV}{dz}-K^2\left(1-\frac{c^2}{c_s^{\,2}}\right)V=0, \tag{9}$$

in which $c = \frac{\omega}{k}$ and $c_S = \left(\frac{\mu_0}{\rho_0}\right)^{1/2}$ and J_1 (kr) is the Bessel function of first kind and first order.

Now, since the layer is homogeneous and isotropic, eq. (9) reduces to

$$\frac{d^2V}{dz^2}-K^2\left(1-\frac{c^2}{c_0^{\,2}}\right)V = 0. \tag{10}$$

The solution of eq. (9) may be given as

$$V= A_1 e^{(-p+m)z} + A_2\, e^{-(p+m)z},$$

where $m^2 = p^2 + k^2\left(1-\frac{c^2}{c_s^{\,2}}\right)$ and A_1 and A_2 are arbitrary constants.

Hence the displacement in the upper non-homogeneous layer is

$$v = v_0 \text{ (say)} =[A_1 e^{(-p+m)z} + A_2\, e^{-(p+m)z}]J_1(kr)\, e^{i\omega t}. \tag{11}$$

4. Solution of viscoelastic mantle

In the mantle, assuming that the torsional surface wave travels in radial direction and that of all the mechanical properties associated with it are independent of θ. For torsional surface wave, u = w = 0 and v = v(r, z, t) and the equation of motion for viscoelastic voigt type may be written as (Biot 1965)

$$\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)=\rho\frac{\partial^2 v}{\partial t^2} \tag{12}$$

where μ is the modulus of rigidity of the medium and μ' is the internal friction or viscoelastic parameter.

For the wave propagating along r direction one may assume the solution of eq. (12) as

$$v = V(z)\, J_1(kr)\, e^{i\omega t} \tag{13}$$

where V is the solution of

$$\frac{d^2V}{dz^2}-K^2\left\{1-\frac{c_1^{\,2}}{c_2^{\,2}(1+iA)}\right\}V(z)=0 \tag{14}$$

in which $c_1 = \frac{\omega}{k}$, $c_2 = \left(\frac{\mu}{\rho}\right)^{1/2}$, $A = \frac{\omega\mu'}{\mu}$ and $\omega = \frac{2\pi}{T}$

The solution of eq. (14) satisfying the condition $\lim_{z\to\infty} V(z) = 0$ is

$$V = \left[B_1 \cos\left(\alpha_1\alpha_3 z\right) - B_2 \sin\left(\alpha_1\alpha_3 z\right)\right] e^{-\alpha_1\alpha_2 z} \tag{15}$$

where

$$\alpha_1 = \left[\frac{k^2\gamma_1}{c_2^2\left(1 + A^2\right)}\right]^{1/2}, \ \alpha_2 = \cos\left(\frac{\theta}{2}\right), \ \alpha_3 = \sin\left(\frac{\theta}{2}\right)$$

$$\gamma_1 = \left[\left\{c_2^2\left(1 + A^2\right) - c_1^2\right\}^2 + \left(Ac_1^2\right)^2\right]^{1/2}, \ \theta = \tan^{-1}\left[\frac{A}{\left(\frac{c_2^2}{c_1^2} + A\frac{c_2}{c_1} - 1\right)}\right]$$

Therefore the final solution of eq. (12) may be written as

$$v = v_1 \text{ (say)} = \left[B_1 \cos\left(\alpha_1\alpha_3 z\right) - B_2 \sin\left(\alpha_1\alpha_3 z\right)\right] J_1(kr)\, e^{i\omega t - \alpha_1\alpha_2 z} \tag{16}$$

5. Boundary Conditions

The following boundary conditions must be satisfied

(1) At the free surface $z = -H$, stress is vanishing so that

$$\mu_0 \frac{\partial v_0}{\partial z} = 0 \text{ at } z = -H \tag{17a}$$

(2) At the interface $z = 0$, continuity of stress component gives,

$$\mu_0 \frac{\partial v_0}{\partial z} = \left(\mu + \mu' \frac{\partial}{\partial t}\right)\left(\frac{\partial v_1}{\partial z}\right) \text{ at } z = 0 \tag{17b}$$

(3) Continuity of the displacement component gives,

$v_0 = v_1$ at $z = 0$ (17c)

$$\text{(4)} \left(\mu + \mu' \frac{\partial}{\partial t}\right)\left(\frac{\partial v_1}{\partial z}\right) = 0 \text{ at } z = 0 \tag{17d}$$

Now, using eq. (11), eq. (17a) becomes,

$$A_1(-p+m)e^{-mH} - A_2(p+m)e^{mH} = 0. \tag{18a}$$

From eq. (11), eq. (16) and eq. (17b), we have

$$[A_1(-p+m) - A_2(p+m)]\mu_0 = B_1(-\mu\alpha_1\alpha_2 - \mu'\alpha_1\alpha_2 i\omega) + B_2(-\mu\alpha_1\alpha_3 - \mu'\alpha_1\alpha_3 i\omega). \tag{18b}$$

Similarly, we have from eq. (17c) and eq. (17d)

$$A_1 + A_2 = B_1 \tag{18c}$$

$$\cot\left(\frac{\theta}{2}\right) B_1 + B_2 = 0 \tag{18d}$$

Eliminating the arbitrary constants A_1, A_2, B_1 and B_2, we have

$\det(a_{ij}) = 0$, $i, j = 1, 2, 3, 4,$

where

$$a_{11} = (-p+m)e^{-mH},\ a_{12} = (p+m)e^{mH},\ a_{13} = 0,\ a_{14} = 0,$$

$$a_{21} = \mu_0\,(-p+m),\ a_{22} = \mu_0\,(p+m),\ a_{23} = \left(\mu+\mu' i\omega\right)\alpha_1\alpha_2,\ a_{24} = \left(\mu+\mu' i\omega\right)\alpha_1\alpha_3,$$

$$a_{31} = 1,\ a_{32} = 1,\ a_{33} = -1,\ a_{34} = 0,$$

$$a_{41} = 0,\ a_{42} = 0,\ a_{43} = -\cot\left(\frac{\theta}{2}\right),\ a_{44} = -1, \tag{19}$$

Expanding the above determinant, and Equating the real part of above equation, we get

$$\tan\left(kH\sqrt{\left(\frac{c^2}{c_0^2}-1\right)-e^2}\right) =$$

$$2\frac{\mu}{\mu_0}\frac{\left[\left\{\left(1+A^2\right)-\frac{c^2}{c_1^2}\right\}^2+\left(A\frac{c^2}{c_1^2}\right)^2\right]^{1/4}\cos\left(\frac{\theta}{2}\right)}{\sqrt{1+A^2}\ \left(\frac{c^2}{c_0^2}-1\right)}\left[e\tan\left(kH\sqrt{\left(\frac{c^2}{c_0^2}-1\right)-r^2}\right)+\left(\sqrt{\left(\frac{c^2}{c_0^2}-1\right)-e^2}\right)\right]$$

where e = p/k. (20)

Equation (20) is the dispersion equation giving the velocity of torsional surface wave in non-homogeneous isotropic layer over a viscoelastic mantle. Clearly frequency equation depends on the non-homogeneity of the material medium, phase velocity, wave number, rigidity and internal friction.

When non-homogeneity and internal friction μ' is neglected, the mantle becomes perfectly elastic, homogeneous and isotropic. Then the eq. (20) reduces to

$$\tan\left(kH\sqrt{\frac{c^2}{c_0^2}-1}\right) = \left(\frac{\mu}{\mu_0}\right)\left(\sqrt{1-\frac{c^2}{c_1^2}}\Big/\sqrt{\frac{c^2}{c_0^2}-1}\right)$$

which is in complete agreement with the corresponding classical result of Love wave in a homogeneous layer over an elastic homogeneous mantle.

6. Numerical Computation and Discussion

Since equation (20) gives the dispersion equation for torsional surface waves in non-homogeneous isotropic layer over a viscoelastic mantle. This equation is very helpful in order to study the effects of non-homogeneity, internal friction (viscoelastic parameter), rigidity, wave number and time period on the phase velocity of torsional surface waves. The graphs have been plotted from the dispersion equation (20) by using the numerical data given by [9]Gubbin. The value of $c_0/c_1 = 1.2$ has been kept fixed in all the graphs.

Figures 2-7, has been plotted for dimensionless phase velocity along y-axis against dimensionless wave number along x-axis. Figures 2-7, shows the effect of non-homogeneity, rigidity of the layer and the internal friction (viscoelastic parameter) on the torsional surface wave propagation.

Here, T = 0.15 seconds and $\mu/\mu_0 = 0.4$. The value of dimensionless internal friction (μ/μ') for curve no. 1, curve no.2 and curve no. 3 has been taken as 10, 50 and 100 respectively.

For figure1-3, non-homogeneity ratio e = 0.002, 0.02, 0.1, it has been found that as the internal friction μ'decreases the phase velocity increases for a particular wave number. It also shows that the phase velocity decreases as the wave number increases. Further as e > 1, the effects of non-homogeneity on the torsional surface wave propagation are quite different from previous three cases. These effects are shown by figures 5-7.

Figures 8-13, have been plotted horizontal axis as dimensionless wave number against vertical axis as dimensionless phase velocity. Figures 8-13 shows the effect of non-homogeneity, rigidity of the layer and the mantle (Internal friction ratio) on the torsional surface wave propagation. Here, $\mu/\mu' = 10$ and T= 0.15 seconds. The value of rigidity ratio μ/μ_0 has been taken as 0.2, 0.4, 0.6 and 0.8 for curve no. 1, curve no. 2, curve no. 3 and curve no.4 respectively. From figures 8-10, as the non-homogeneity ratio e varies from 0.1 to 100, it is observed that the phase velocity has same value for all the four curves after that as the rigidity of the layer increases the phase velocity slightly decreases uniformly at a particular wave number.

Figures 14-18, have been plotted for dimensionless phase velocity along y-axis against time period along x-axis. Figures 14-18, shows the effect of non-homogeneity, internal friction ratio and time period on the torsional wave propagation. Here, the values of kH= 0.5 and μ/μ_0= 0.5 are fixed. For figures 14-18, the value of dimensionless internal friction (μ/μ') has been taken as 70 respectively. For figure14, e = 0.01, 0.02, 0.03, it has been found that the dimensionless initial phase velocity of torsional surface wave in the non-homogeneous substratum would be -1.3835 under the above considered value. It firstly slightly increases as the time period increases upto3 and then constant as the time period increases further.

For figures15-17, e = 0.1,0.2,0.3,10,11,12,100,110,120, it has been found that the dimensionless phase velocity of torsional surface wave vary from -5 to -227.5 and remains constant for a particular non-homogeneity ratio, as the time period T increases.

For figure18, e =1000, 1100, 1200, it has been found that the dimensionless initial phase velocity of torsional surface wave in the non-homogeneous substratum would vary from -1.0049 to -1.0059, under the above considered value. It firstly slightly decreases as the time period increases upto3 and then constant as the time period increases further.

Figures19-24, have been plotted for dimensionless phase velocity along y-axis against time period along x-axis. Figure 19-24 shows the effect of non-homogeneity, rigidity ratio and time period on the torsional wave propagation. Here, the values of kH= 0.5 and μ/μ'= 0.5 are fixed. For figures 19-23, the value of dimensionless rigidity ratio (μ/μ_0) has been taken as 0.6 and 1.4 respectively. For figures19-20, e =0.1, s = 0.6, 1.4, it has been found that the dimensionless phase velocity of torsional surface wave increases from -3.3578 to -3.3570 and -3.3578 to -3.3575, as the time period increases to 1.1, then slightly decreases as the time period increases to 1.6 and then remains constant as the time period T increases further.

For figures 21-22, e =100, s = 0.6, 1.4, it has been found that the dimensionless phase velocity of torsional surface wave is -227 and remains constant as the time period increases further.

For figures 23-24, e =1000, s = 0.6, 1.4, it has been found that the dimensionless phase velocity of torsional surface wave decreases from -1 to -1.003 and -1 to -1.0012 as time period T increases.

It has also been noticed that the phase velocity decreases as the rigidity ratio increases.

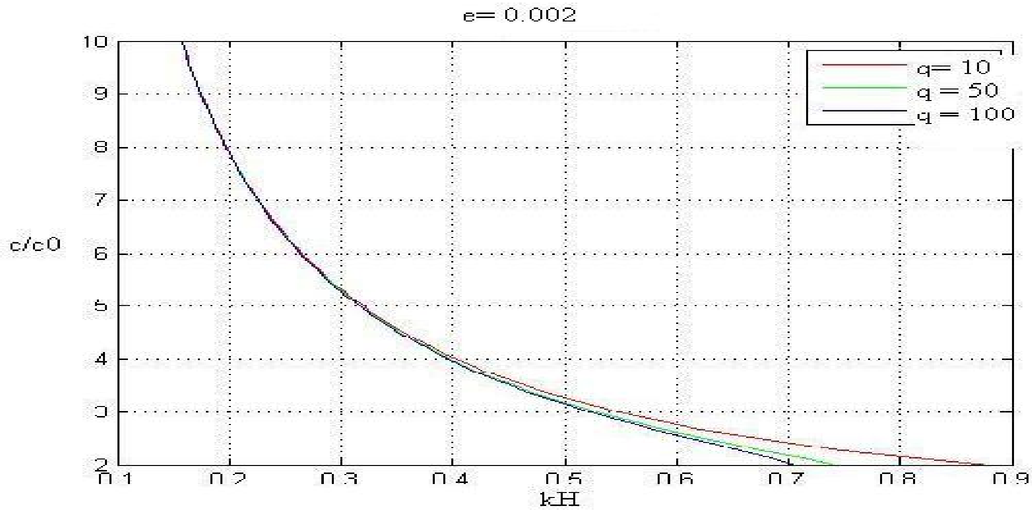

Figure2. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, μ/μo=0.4, e = 0.002 and q = μ/μ' = 10, 50, 100.

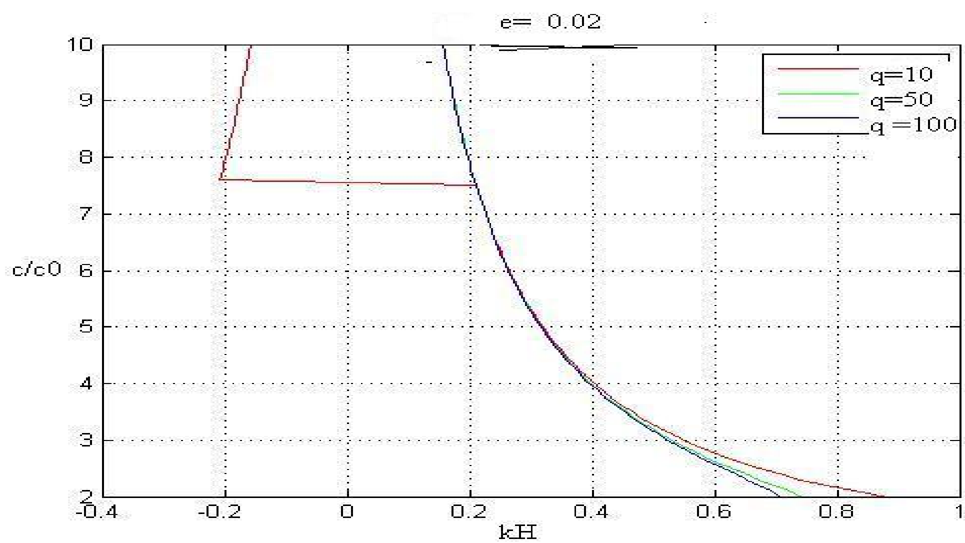

Figure3. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, μ/μo=0.4, e = 0.02 and q = μ/μ' = 10, 50, 100.

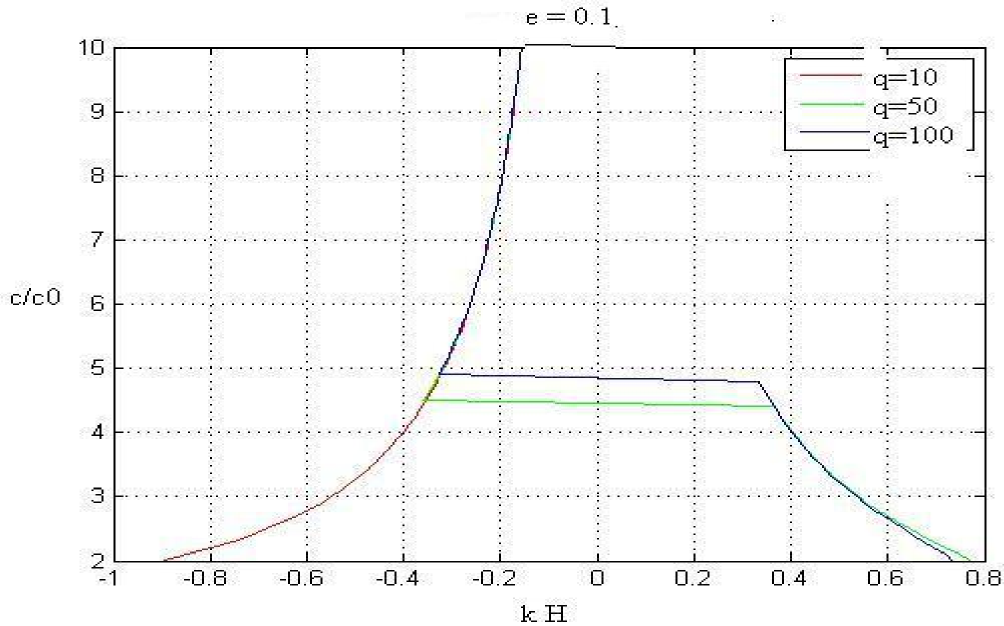

Figure4. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, μ/μo=0.4, e = 0.1 and q = μ/μ' = 10, 50, 100.

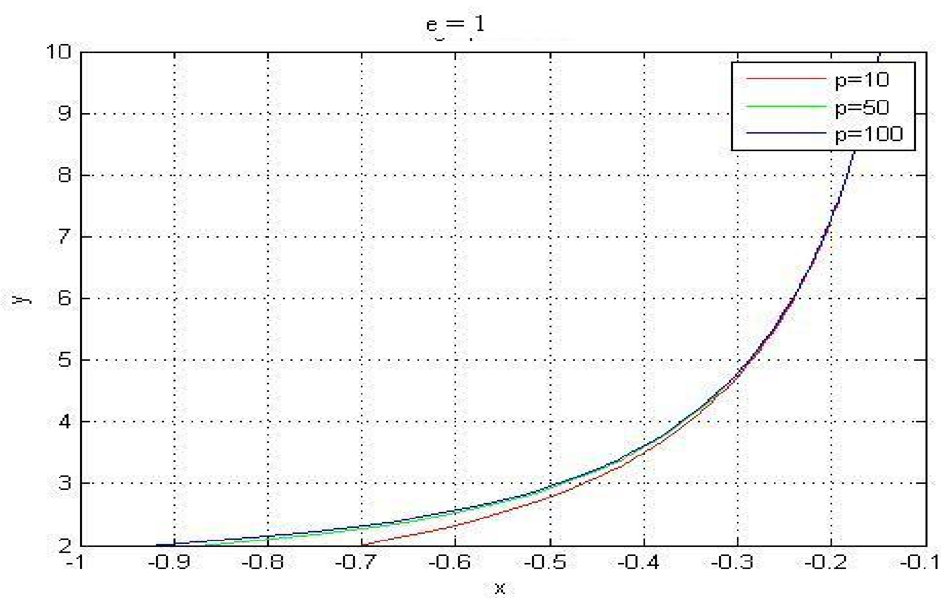

Figure5. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, μ/μo=0.4, e = 1.0 and q = μ/μ' = 10, 50, 100.

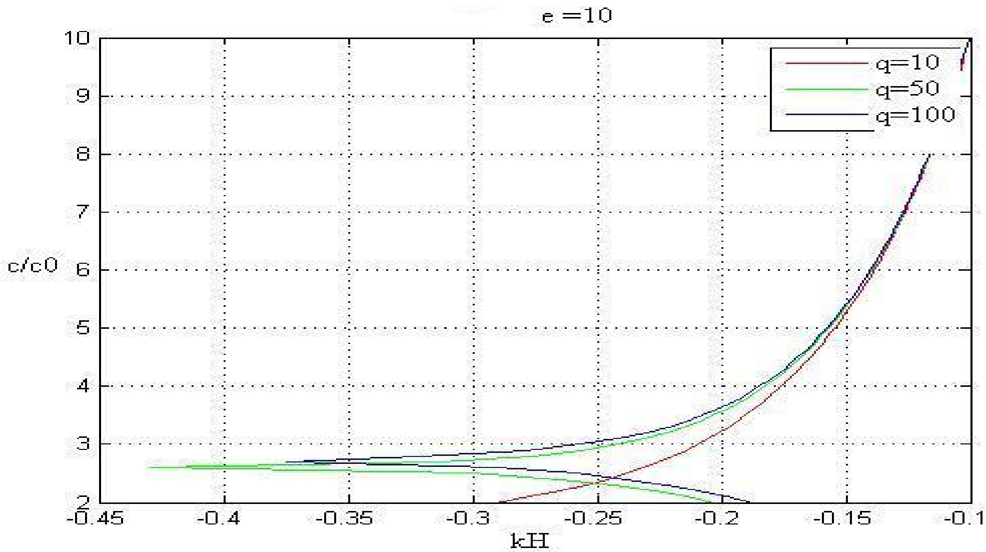

Figure6. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, $\mu/\mu o$=0.4, e = 10 and q = μ/μ' = 10, 50, 100.

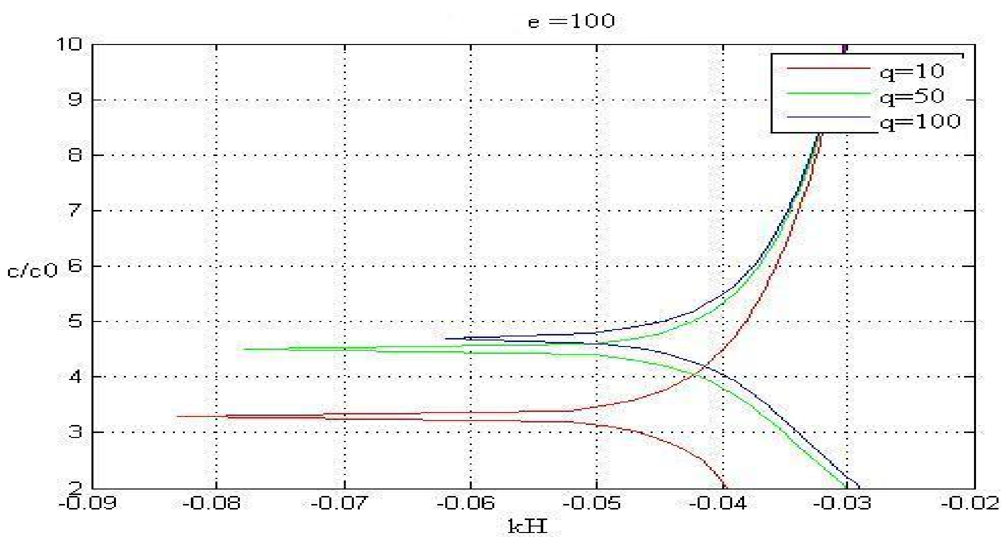

Figure7. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, $\mu/\mu o$=0.4, e = 100 and q = μ/μ' = 10, 50, 100.

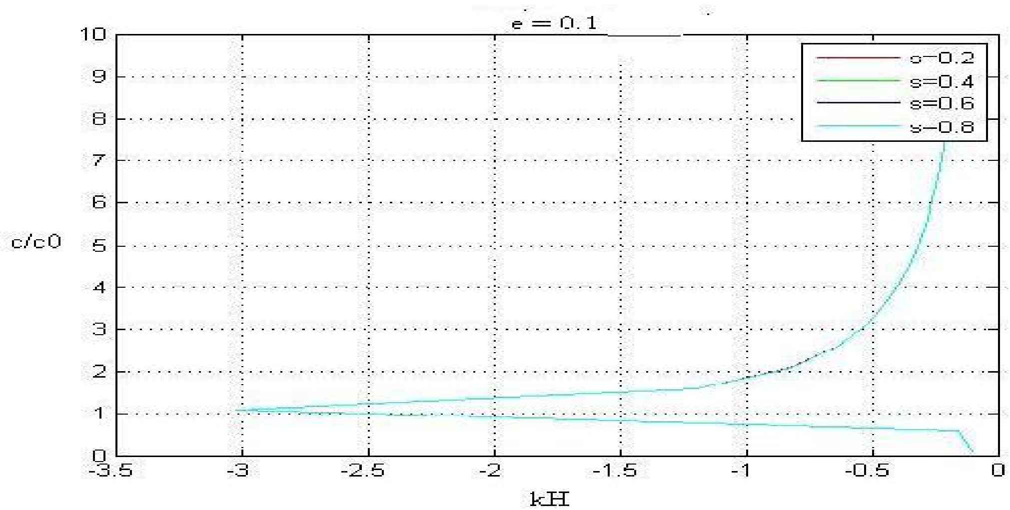

Figure8. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, $\mu/\mu' = 10$, $e = 0.1$ and $s = \mu/\mu o$=0.2, 0.4, 0.6, 0.8.

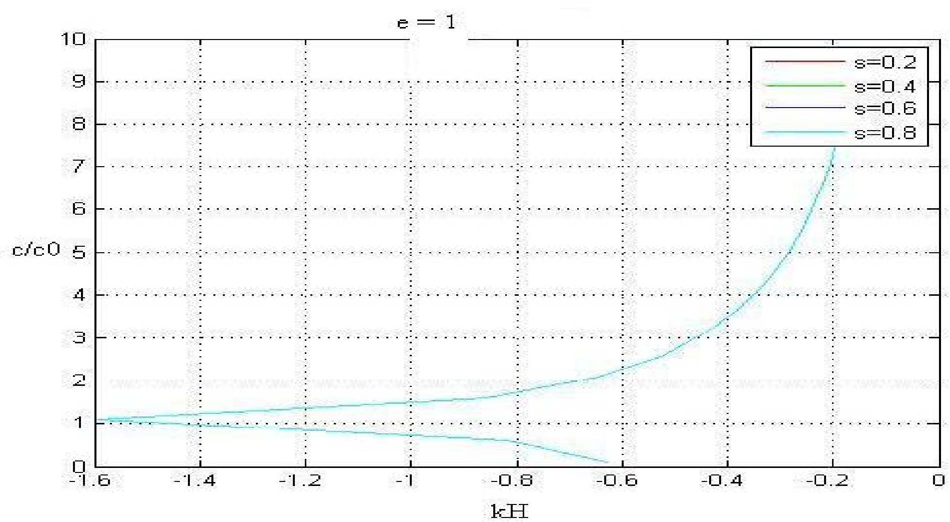

Figure9. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, $\mu/\mu' = 10$, $e = 1$ and $s = \mu/\mu o$=0.2, 0.4, 0.6, 0.8.

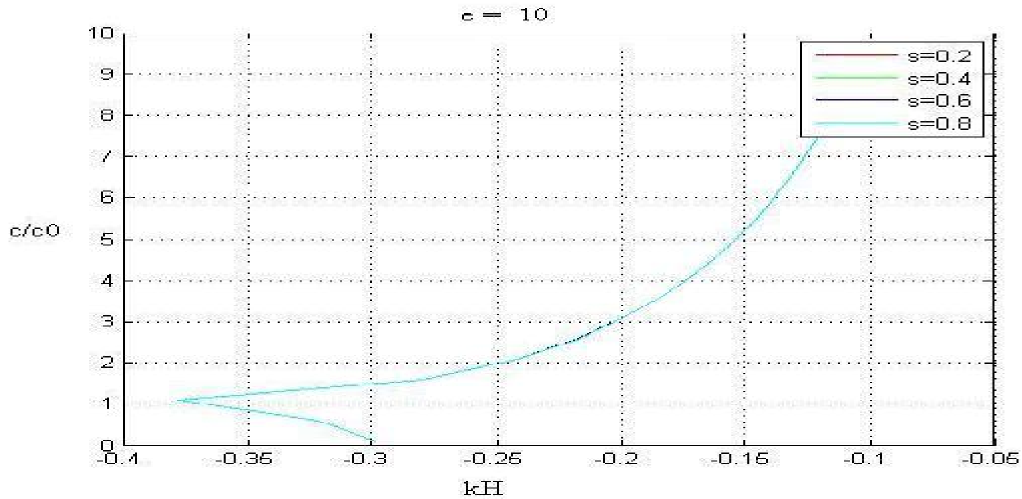

Figure10. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, μ/μ' = 10, e = 10 and s = μ/μo=0.2, 0.4, 0.6, 0.8.

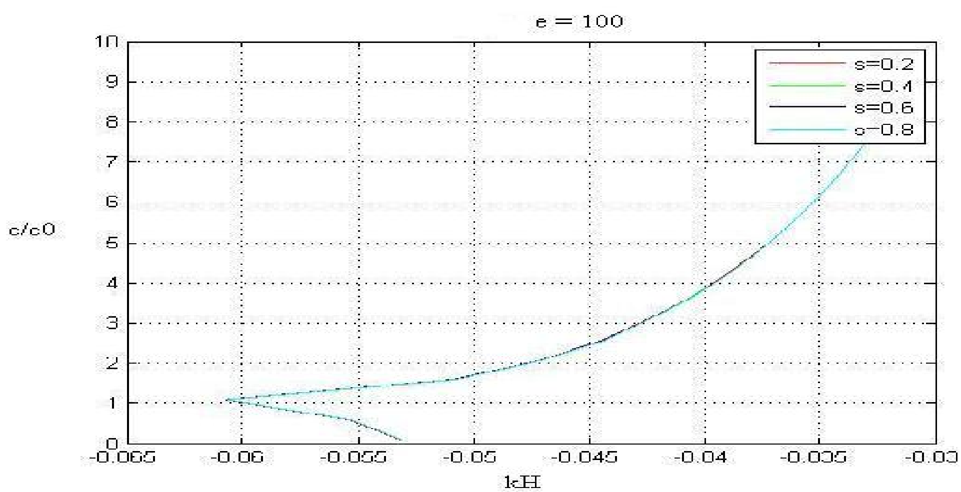

Figure11. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, μ/μ' = 10, e = 100 and s = μ/μo=0.2, 0.4, 0.6, 0.8.

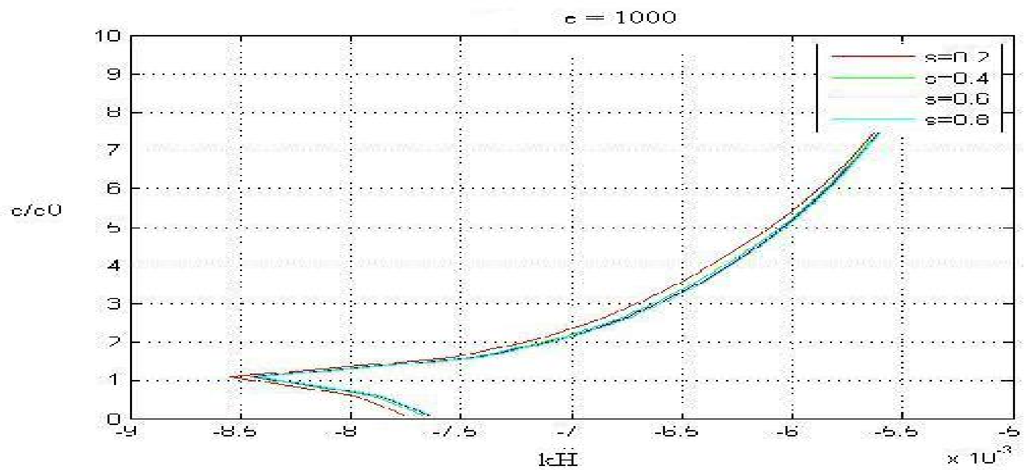

Figure12. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, $\mu/\mu' = 10$, e = 1000 and s = $\mu/\mu o$=0.2, 0.4, 0.6, 0.8.

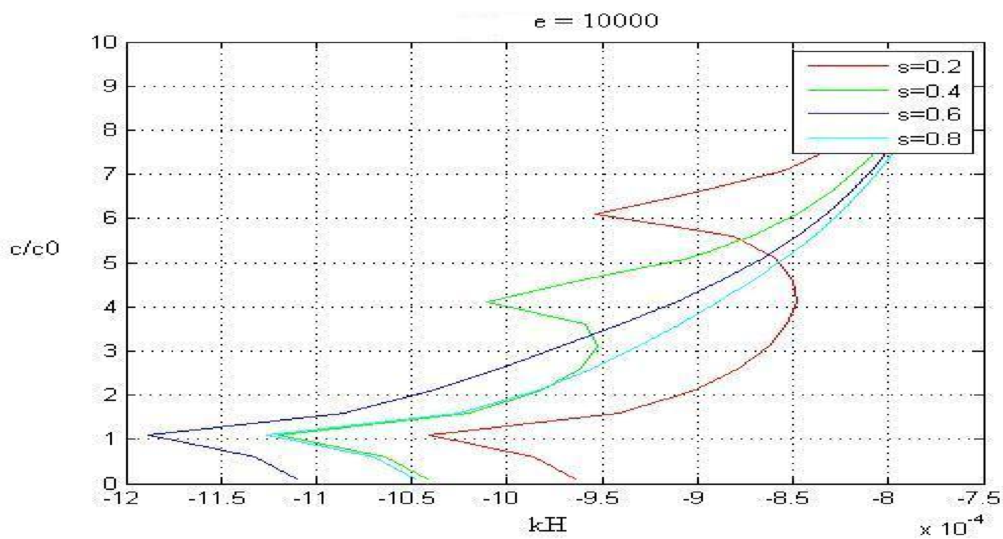

Figure13. Dimensionless phase velocity against dimensionless wave no. for T=0.15 sec, $\mu/\mu' = 10$, e = 10000 and s = $\mu/\mu o$=0.2, 0.4, 0.6, 0.8.

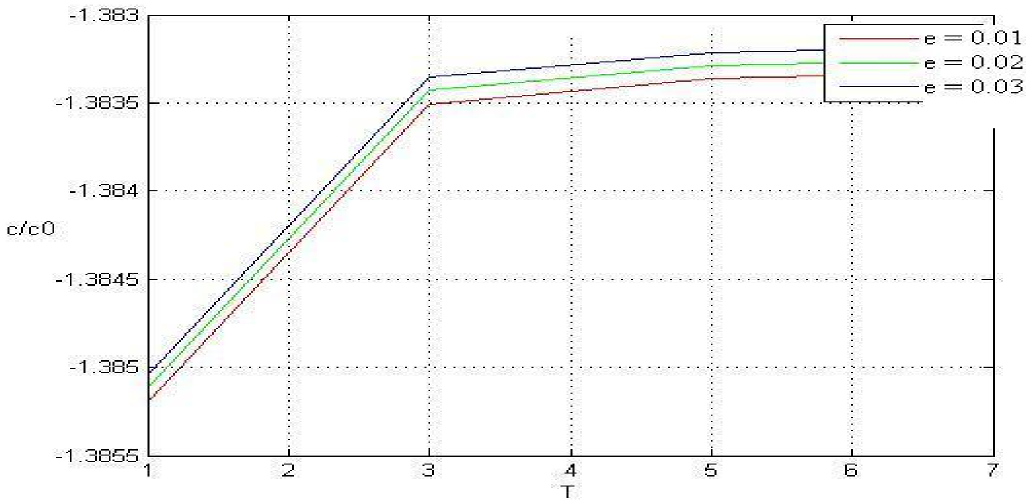

Figure 14 Dimensionless phase velocity against time period with kH=0.5, μ/μo= 0.5, μ/μ' = 70 and e = 0.01, 0.02, 0.03.

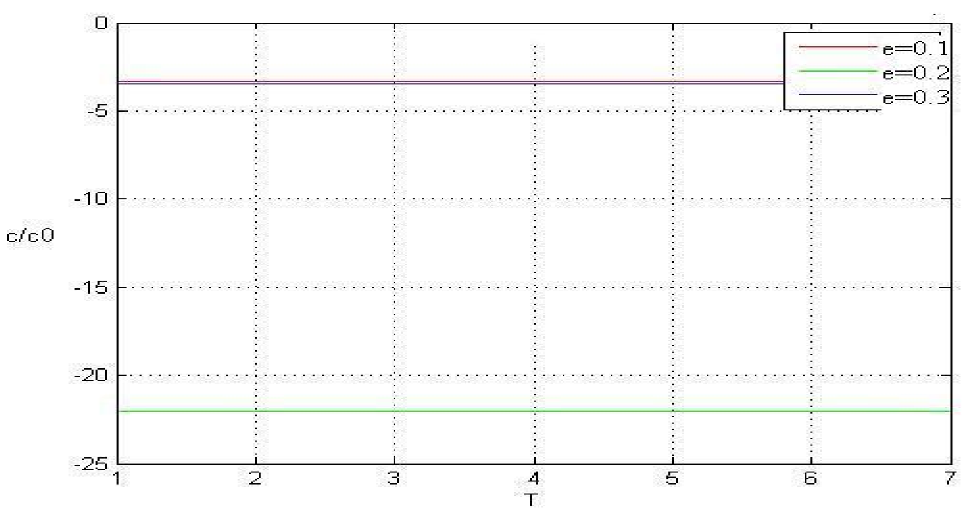

Figure 15 Dimensionless phase velocity against time period with kH=0.5, μ/μo= 0.5, μ/μ' = 70 and e = 0.1, 0.2, 0.3.

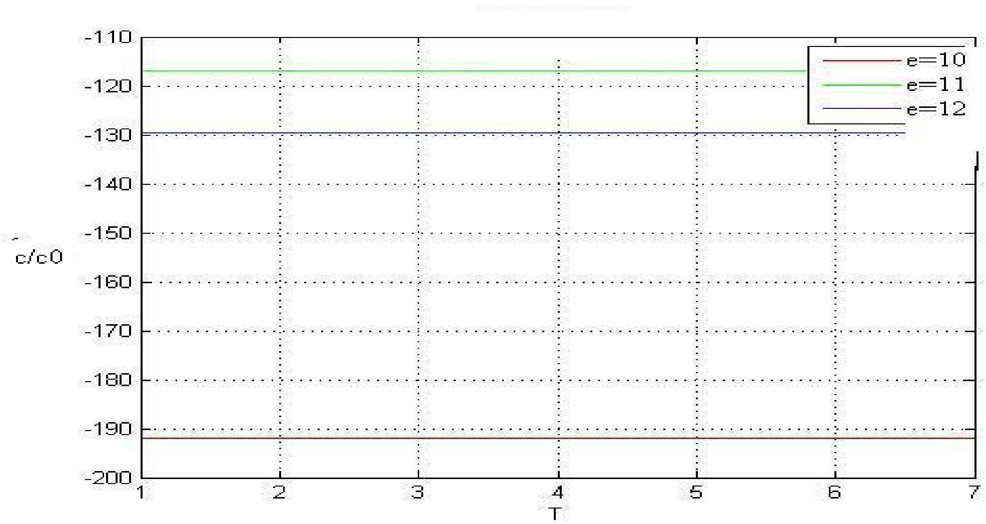

Figure 16 Dimensionless phase velocity against time period with kH=0.5, μ/μo= 0.5, μ/μ' = 70 and e = 10, 11, 12.

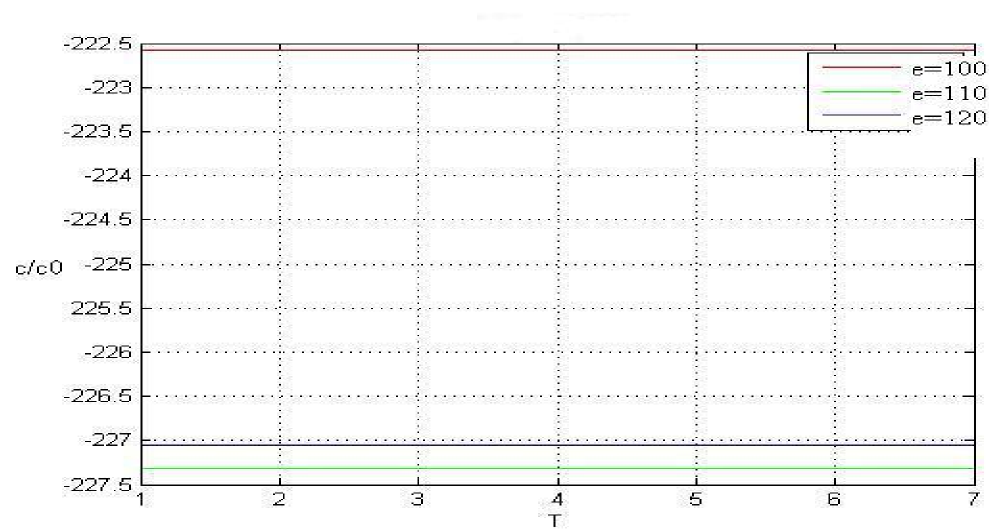

Figure 17 Dimensionless phase velocity against time period with kH=0.5, μ/μo= 0.5, μ/μ' = 70 and e = 100, 110, 120.

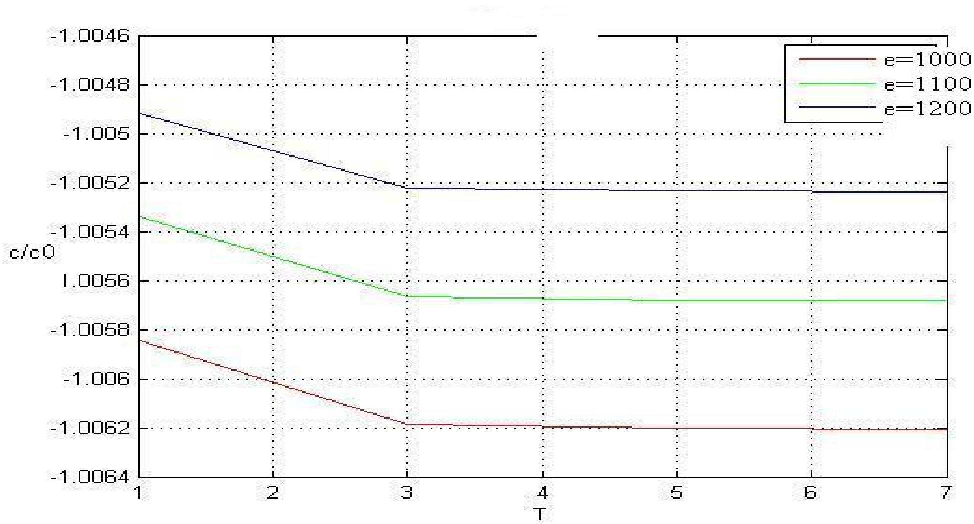

Figure 18 Dimensionless phase velocity against time period with kH=0.5, μ/μo= 0.5, μ/μ' = 70 and e = 1000, 1100, 1200.

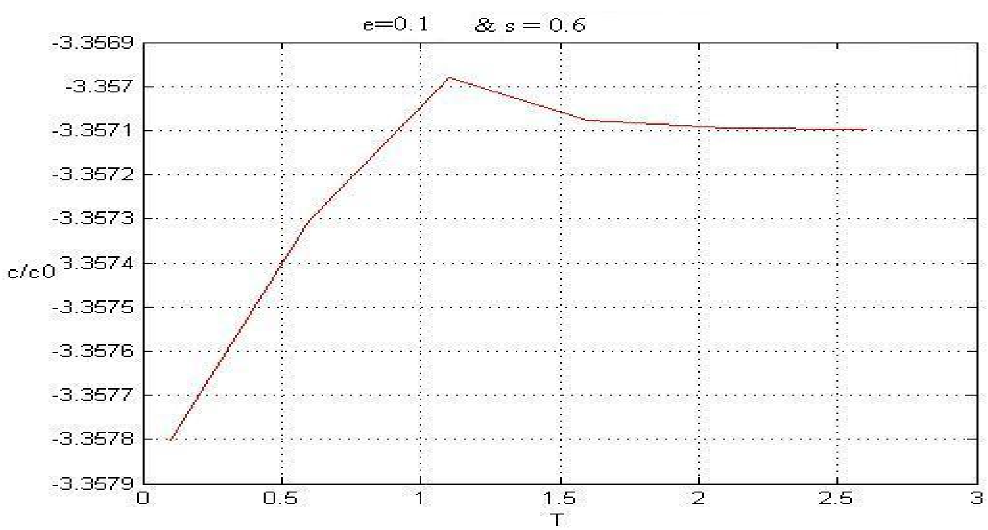

Figure 19 Dimensionless phase velocity against time period with kH=0.5, μ/μ' = 0.5, e = 0.1 and μ/μo= 0.6.

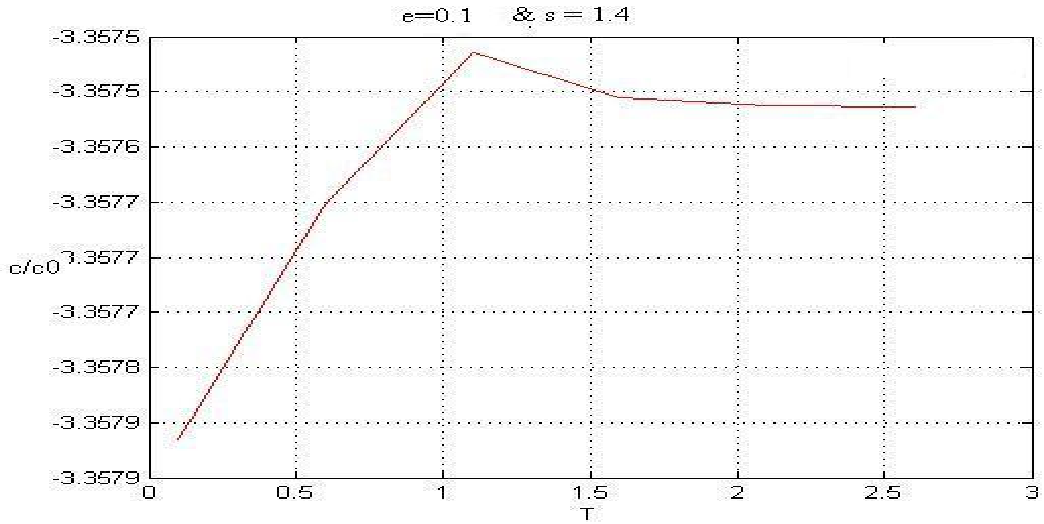

Figure 20 Dimensionless phase velocity against time period with kH=0.5, μ/μ' = 0.5, e = 0.1 and μ/μo= 1.4.

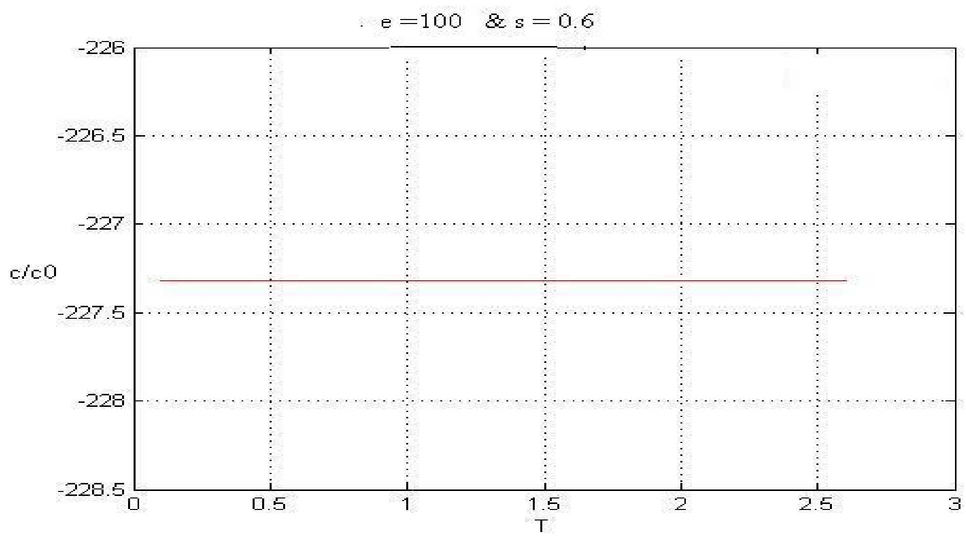

Figure 21 Dimensionless phase velocity against time period with kH=0.5, μ/μ' = 0.5, e = 100 and μ/μo= 0.6.

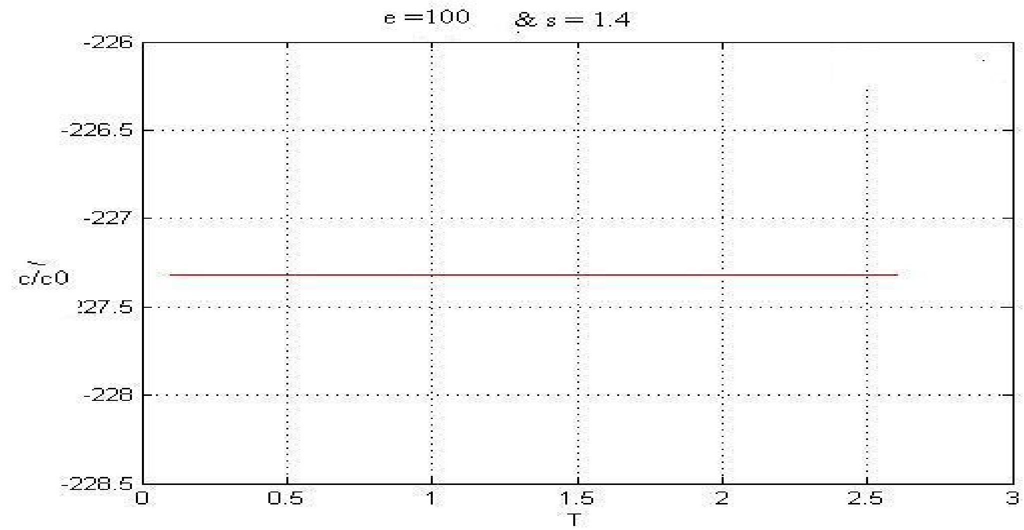

Figure 22 Dimensionless phase velocity against time period with kH=0.5, μ/μ' = 0.5, e = 100 and μ/μo= 1.4.

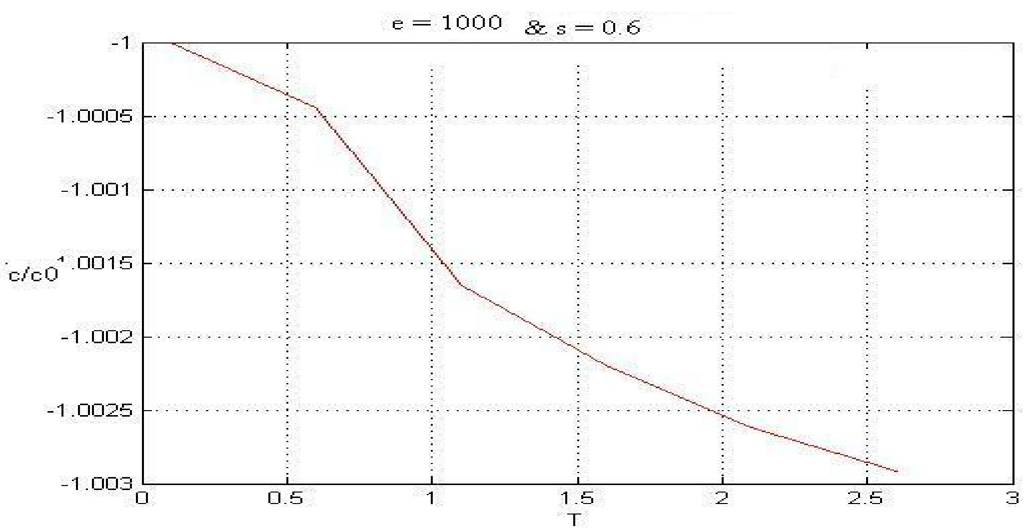

Figure 23 Dimensionless phase velocity against time period with kH=0.5, μ/μ' = 0.5, e = 1000 and μ/μo= 0.6.

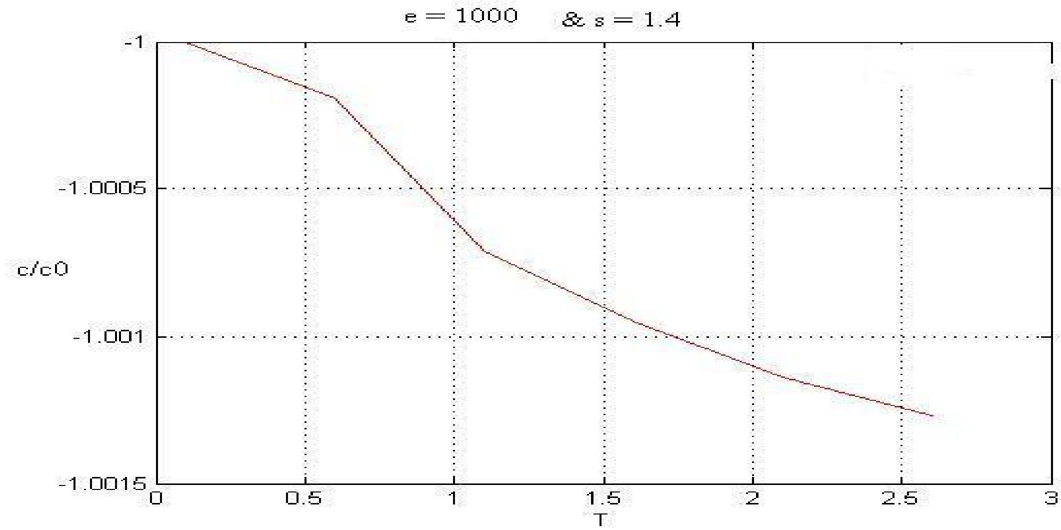

Figure 24 Dimensionless phase velocity against time period with kH=0.5, μ/μ' = 0.5, e = 1000 and μ/μo= 1.4.

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