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A Deterministic Algorithm for Generating Optimal Three-Stage layouts of Homogenous Strip Pieces

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Abstract:

Purpose: The time required by the algorithms for general layouts to solve the large-scale two-dimensional cutting problems may become unaffordable. So this paper presents an exact algorithm to solve above problems.

Design/methodology/approach: The algorithm uses the dynamic programming algorithm to generate the optimal homogenous strips, solves the knapsack problem to determine the optimal layout of the homogenous strip in the composite strip and the composite strip in the segment, and optimally selects the enumerated segments to compose the three-stage layout.

Findings: The algorithm not only meets the shearing and punching process need, but also achieves good results within reasonable time.

Originality/value: The algorithm is tested through 43 large-scale benchmark problems. The number of optimal solutions is 39 for this paper's algorithm; the rate of the rest 4 problem's solution value and the optimal solution is 99. 9%, and the average consumed time is only 2. 18seconds. This paper's pattern is used to simplify the cutting process. Compared with the classic three-stage, the two-segment and the T-shape algorithms, the solutions of the algorithm are better than that of the above three algorithms. Experimental results show that the algorithm to solve a large-scale piece packing quickly and efficiency.

Keywords: two-dimensional layout, homogenous strip, dynamic programming recursion

1. Introduction

The unconstrained two-dimensional cutting (UTDC) problem refers to a series of small shape (or part) non-overlapping on a rectangular panel and the optimization objective of the problems is to find an arrangement for maximizing the material usage. UTDC problem is widely used in the leather, wood, metal and other manufacturing industries. Although many researchers have studied the UTDC problem, from the theory of computational complexity theory, layout problem have been proved to be a quiet difficult combinatorial optimization problem (Cui, 2013; Han, Bennell & Zhao, 2013; Thomas & Chaudhari, 2013; He & Wu, 2013; Liu & Liu, 2011; Ji, Lu & Cha, 2012; Huang & Liu, 2006; Jiang, Lv & Liu, 2008).

According to the UTDC problem, the layouts can be divided into the general layouts and the specific layouts. On the one hand, when the layouts have no any constraint, the layouts are called the general layouts (Gilmore & Gomory, 1965; Beasley, 1985; Cui, Wang & Li, 2005; Seong & Kang, 2003; Hifi & Zissimopoulos, 1996; Alvarez-Valdes, Parajon & Tamarit, 2002); on the other hand, when the layouts must meet some specific production request, the layouts are called the specific layout.

Now, there are some exact algorithms for the general layouts (Gilmore & Gomory, 1965; Cui et al., 2005). But the computation results in the references indicate that the computation time of these algorithms cannot be intolerable for solving the large scale UTDC problems. So many researchers have committed to study the specific layouts. The specific layouts have three advantages: meeting the practical production technology; high computation efficiency; the results are close to the optimal results.

There are many advanced specific layouts, for example, Hifi (2001) proposed the classic two-stage and the three-stage layout; Fayard and Zissimopoulos (1995) presented the two-segment layout; Cui (2004a) proposed the T-shape layout. Through analysis, the T-shape layout is the superset of the two-stage layout, and is the subset of the classic three-stage layout; the two-segment is the superset of the T-shape layout, and is the subset of the classic three-stage layout.

This paper propose a new layout — the three-stage layout based on the homogenous stripe (3HS). The 3HS layout can meet the need of the cutting technology in the practical production.

3HS layout is the superset of the classic three-stage, two-segment, T-shape and the classic two-stage layout, and we will introduce it in the section 2.4.

The layout decides the layout value. The sequence of the above layouts value from largest to smallest is follows: the general layout, the classic three-stage layout, the two-segment layout, the T-shape layout, and the classic two-stage layout. This paper’s 3HS layout is between the general layout and the classic three-stage layout.

This paper will introduce 3HS layout in part 2; the exact algorithm for generating the 3HS layout in part 3; the experiments and results analysis in part 4; conclusion in part 5.

2. 3HS layout

2.1. Homogenous stripe

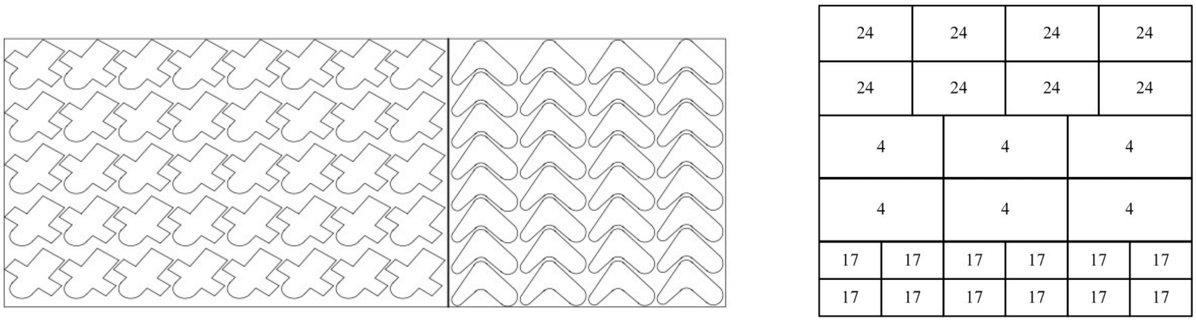
The homogenous stripe consists of the same size with same dimension. Figure 1(a) shows horizontal homogenous rectangular stripes, and its width is the blank width. Figure 1(b) shows vertical homogenous irregular stripes, and its width is the blank length.

(a) The horizontal homogenous stripe (b) The vertical homogenous stripe

Figure 1. The homogenous stripe

2.2. Composite strip

The composite strip consists of the homogenous stripe. The composite strip can be divided into the homogenous stripe by series of cuts. When cutting, each knife cuts single horizontal or vertical homogenous stripe. Figure 2 show the composite strip of the rectangular blanks and irregular blanks; the Figure 2(a) is the X composite strip of the irregular blanks, and the Figure 2(b) is Y composite strip of the rectangular blanks.



(a) The X composite strip (b) The Y composite strip

Figure 2. The composite strip

Figure 3 shows the process of its being cut. The arrow is the cut station, and the number is the cuts sequence. After the composite strip cut into homogenous stripe, the blank is been separated from homogenous stripe by the punch.

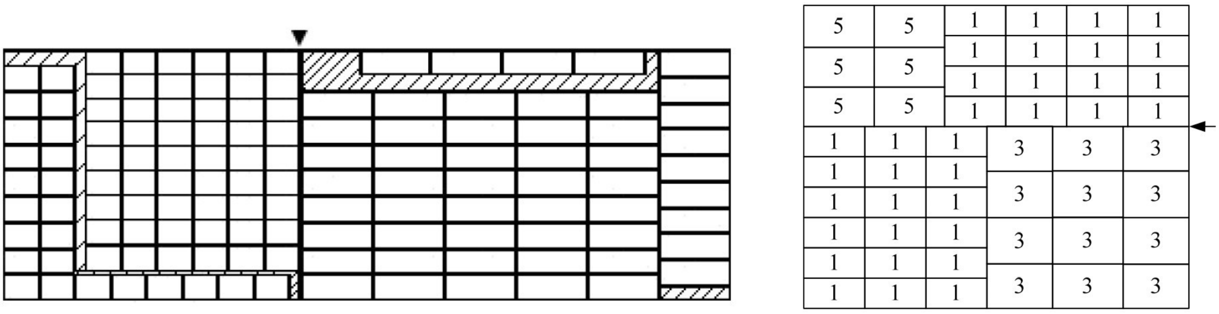
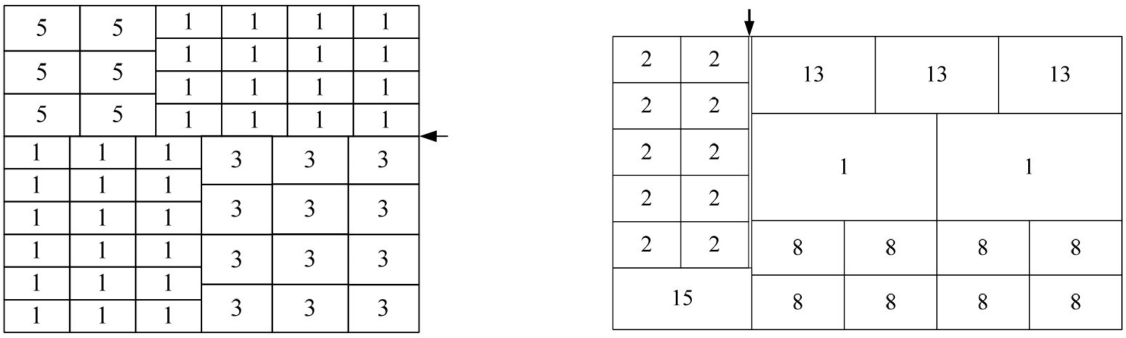


Figure 3. The cutting process of composite strip

2.3. Segment

The segment consists of composite strips. The X-segment includes series of X composite strips from up to bottom (Figure 4(a)), and the Y-segment includes series of Y composite strips from left to right (Figure 4(b)). In Figure 4, the arrow indicates the composite strip boundary line. In fact, from the concept, when the Y composite strip in Y-segment is viewed as X-segment, the Y-segment becomes the X-segment; in other words, Y-segment is a specific X-segment.



(a) The X-segment (b) The Y-segment

Figure 4. The segment

2.4. 3HS layout

Figure 5 shows the 3HS layouts. Each 3HS layout composes of many segments. In 3HS layout, if it consists of some horizontal X-segments from left to right, it is called 3HSX layout (Figure 5(a)); if it consists of some vertical Y-segments from up to bottom, it is called 3HSY layout (Figure 5(b)).

(a) The 3HSX layout (b) The 3HSY layout

Figure 5. The types of the 3HS layout

Figure 6 shows 3HSX layout, and the arrow indicates the boundary line. The 3HS layout can be divided into composite strips by three stages, and composite strips can be divides into blanks by other two stages or more stages. In Figure 6, first, vertical 1 divides the sheet into three segments; second, horizontal 2 divides the segments into composite strips; third, vertical 3 divides the composite strips into homogenous strips; last, each is divided into blanks from the process is same to Figure 1.

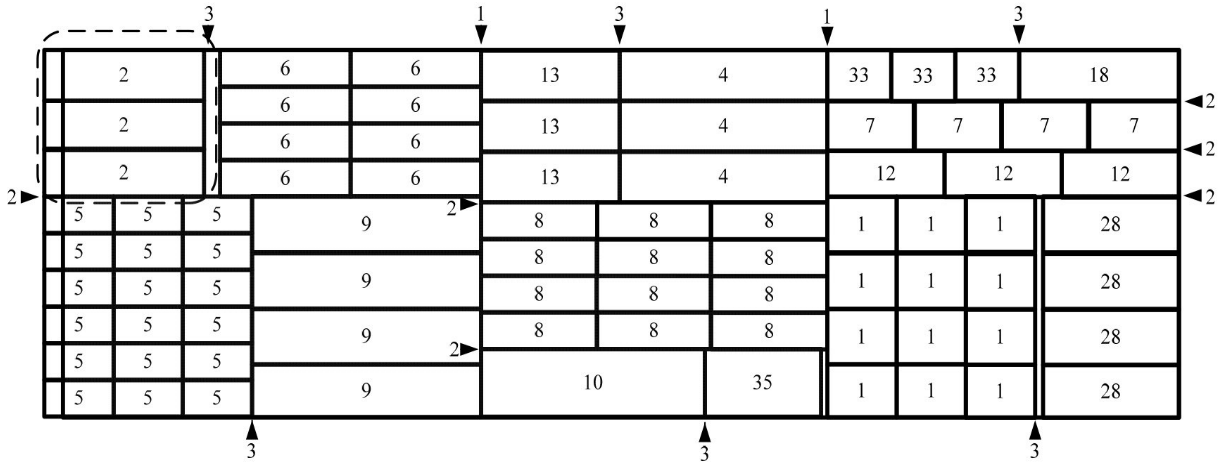


Figure 6. The 3HSX layout and its cutting process

In 3HS layout, if each blank takes place of the homogenous strip, the 3HS layout turns into the classic three-stage layout; if the number of the segment is 2, the 3HS layout becomes the two-segment layout; if segments are X-segment and Y-segment, the 3HS layout turns into the T-shape layout. In addition, the T-shape layout is the superset of the classic two-stage layout (Cui, 2004a). Thus, the 3HS layout is the superset of the classic three-stage, two-segment, T-shape, and the classic two-stage layout. In other words, the solution of 3HS layout is better than that of the above four layouts.

3. The algorithm for generating 3HS layout

3.1. Notes and functions

Table 1 lists the various notes and functions used by the algorithms. Most of the notes and functions will be introduce again when used for the first time, this table help readers quickly finding.

L, W	Length and width of sheet
l_i, v_i	Length and value of ith blank, $i = 1,\ldots,m$
w_i, w_{0i}, w_{1i}	w_i is the length of ith regular blank, w_{0i}, w_{1i} are the initial step and progressive step of ith irregular blank, $i = 1,\ldots,m$
$P_s^{(i)}$, $Q_s^{(i)}$	Normal length and width of homogenous strip
P, Q	Normal length and width of composite strip
$P_{ssegment}$	Normal size of segment
$n_s^{(i)}(x,y)$	Maximum number of ith blank in $x \times y$ homogenous strip
$s(x,y)$	Maximum value of $x \times y$ homogenous strip
$f_s^1(x,y)$	Value of $x \times y$ X composite strip
$f_s^2(x,y)$	Value of $x \times y$ Y composite strip
$g_s^1(x,y)$	Value of $x \times y$ X-segment
$g_s^2(x,y)$	Value of $x \times y$ Y-segment
$v_{SX\text{-}3STAGE}$	Maximum value of the optimal 3HSX layout
$v_{SY\text{-}3STAGE}$	Maximum value of the optimal 3HSY layout
$v_{S\text{-}3STAGE}$	Maximum value of the optimal 3HS layout

Table 1. Notes and function

3.2. The steps of algorithm

Supposed the size of sheet and blank are integer, and the blank direction is fixed. The algorithm of 3HS layout (3HSA) includes the following steps:

Step 1. Determining the optimal homogenous strip by dynamic programming algorithm;

Step 2. Solving the optimal homogenous strip layout in composite strip by knapsack problem;

Step 3. Solving the optimal composite strip in segment by knapsack problem;

Step 4. Determining the optimal 3HSX layout by knapsack problem;

Step 5. Determining the optimal 3HSY layout by knapsack problem;

Step 6. Solving the optimal 3HS layout.

3.3. The normal size

The normal sizes have been used by many scholars (Ji et al., 2012; Beasley, 1985; Hifi, 2001; Fayard & Zissimopoulos, 1995; Cui, 2004a). The normal size is the length and width linear combination of blank. The layout references (Cui, 2004a) have proved that the blank maximum number of rectangle $x \times y$ is equal to the blank maximum value of rectangle $x_0 \times y_0$, and x_0 is the optimal normal size that is lessen than x, and y_0 is the optimal normal size that is lessen than y. To different layout, according to normal size features, we should define it appropriately to improve the solving speed.

Definition 1. The homogenous strip normal size

According to above description, the homogenous strip consists of blanks with same shape, and the blank direction is fixed. Therefore, the homogenous strip length normal size $P_s^{(i)}$ is the length linear combination of each blank. The equation is follows:

$$P_{\mathrm{s}}^{(i)} = \{x = z_i l_i ; z_i \in \boldsymbol{N}; 0 \le x \le L;\} \tag{1}$$

(1) The homogenous width normal size of regular blank $Q_s^{(i)}$ is follows:

$$Q_{\mathrm{s}}^{(i)} = \{y = z_i w_i ; z_i \in \boldsymbol{N};\ 0 \le y \le W\} \tag{2}$$

(2) The homogenous width normal size of irregular blank $Q_s^{(i)}$ is follows:

$$Q_{\mathrm{s}}^{(i)} = \{y = z1_i w0_i + z2_i w1_i ;\ \ z1_i, z2_i \in \boldsymbol{N};\ 0 \le y \le W\} \tag{3}$$

The 0 and L are added to the normal size sequence. The $P_s^{(i)} = p_1^{\mathrm{s}}, p_2^{\mathrm{s}}, \ldots, p_M^{\mathrm{s}}$ represents the homogenous strip length normal size of ith blank, and M is the number of normal size; and the $Q_s^{(i)} = q_1^{\mathrm{s}}, q_2^{\mathrm{s}}, \ldots, q_N^{\mathrm{s}}$ represents the homogenous strip width normal size of ith blank, and N is the number of normal size.

Definition 2. The composite strip normal size

According to above description, the composite strip composes of homogenous strips. So, the composite strip length normal size P is the length linear combination of each blank:

$$P = \left\{ x = \sum_{i=1}^{m} z_i l_i ;\ \ z_i \in \boldsymbol{N};\ \ i = 1, \ldots, m;\ \ 0 \le x \le L \right\} \tag{4}$$

(1) The composite strip width normal size of regular blank Q is follows:

$$Q = \left\{ y = \sum_{i=1}^{m} z_i w_i ;\ z_i \in \boldsymbol{N};\ i = 1, \ldots, m;\ 0 \le y \le W \right\} \tag{5}$$

(2) The composite strip width normal size of irregular blank Q is follows:

$$Q = \left\{ \sum_{i=1}^{m} z1_i w0_i + z2_i w1_i ;\ \ z1_i, z2_i \in \boldsymbol{N};\ \ i = 1, \ldots, m;\ \ 0 \le y \le W \right\} \tag{6}$$

The 0 and L are added to the normal size sequence. The p_1, p_2,...,p_M represents the composite strip length normal size, and M is the number of normal size; the q_1, q_2,...,q_N represents the composite strip width normal size, and N is the number of normal size.

Definition 3. The segment normal size

According to above description, the segment consists of composite strip. Therefore, the segment normal width $P_{ssegment}$ is the collection of composite strip length normal size:

$$P_{\text{ssegment}} = P = \{p_1^{\text{sseg}}, p_2^{\text{sseg}}, \ldots, p_M^{\text{sseg}}\} \tag{7}$$

If both the segment width and length belong to $P_{ssegment}$, then the segment is a normal segment.

3.4. The value of homogenous strip $x \times y$

(1) Solving the maximum number that the homogenous strip $x \times y$ includes blanks

Assume that $n_s^{(i)}(x, y)$ is the maximum number of ith blank in the homogenous $x \times y$, and there is following recursive formula, and $x \in P_s^{(i)}$, $y \in Q_s^{(i)}$:

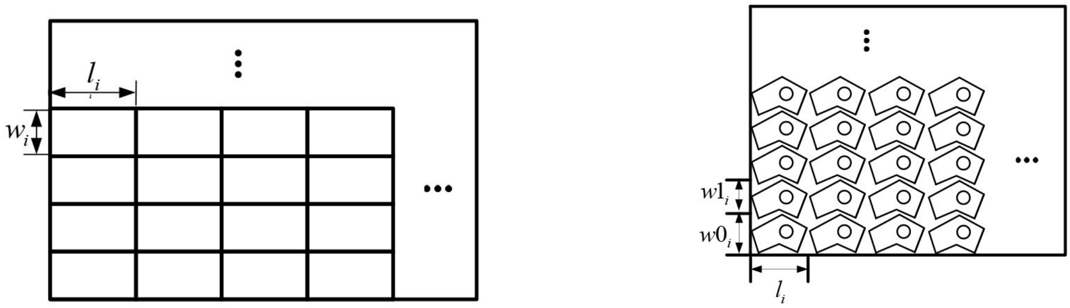
- The maximum number of ith regular blank in the homogenous $x \times y$:

$$n_{\text{s}}^{(i)}(x, y) = \left\{ \text{int}(x/l_i) \times \text{int}(y/w_i);\ \ x \in P_{\text{s}}^{(i)};\ \ y \in Q_{\text{s}}^{(i)} \right\} \tag{8}$$

- The maximum number of ith irregular blank in the homogenous $x \times y$:

$$n_{\text{s}}^{(i)}(x, y) = \left\{ \text{int}(x/l_i) \times [1 + \text{int}(x - w0_i)/w1_i];\ \ x \in P_{\text{s}}^{(i)};\ \ y \in Q_{\text{s}}^{(i)} \right\} \tag{9}$$

Figure 7 shows the blanks number of the homogenous strip $x \times y$.



(a) The rectangular blank (b) The irregular blank

Figure 7. The blanks number of the homogenous strip $x \times y$

(2) Determining the blank maximum value in homogenous $x \times y$

Suppose $s(x, y)$ is the maximum value in homogenous $x \times y$, and v_i is the ith blank value, then:

$$s(x,y)=\left\{\max[n_s^{(i)}(x,y)\times v_i];\ x\in P_s^{(i)};\ y\in Q_s^{(i)}\right\} \tag{10}$$

3.5. Determining the homogenous strip optimal layout in composite strip

(1) Determining the homogenous strip optimal layout in X composite strip

Suppose $f_s^1(x, y)$ is the value of X composite strip $x \times y$, and $x \in P;\ y \in Q$:

$$f_s^1(x,y)=\left\{\max\left[\sum_{i=1}^{M}k_i s(p_i,y)\right];\quad \sum_{i=1}^{M}k_i p_i \le x;\quad k_i\in \boldsymbol{N}\right\} \tag{11}$$

The solution of above knapsack problem can refer to literature (Kellerer, Pferschy & Pisinger, 2004).

(2) Determining the composite strip optimal layout in Y composite strip

Suppose $f_s^2(x, y)$ is the value of Y composite strip $x \times y$, and $x \in P;\ y \in Q$:

$$f_s^2(x,y)=\left\{\max\left[\sum_{i=1}^{N}k_i s(x,q_i)\right];\quad \sum_{i=1}^{N}k_i q_i \le y;\quad k_i\in \boldsymbol{N}\right\} \tag{12}$$

3.6. Determining the section optimal layout in segment

Assume that $g_s^1(x, y)$ is the value of X-segment $x \times y$, and $g_s^2(x, y)$ is the value of Y-segment $x \times y$. So, there is following formula, and $x, y \in P_{ssegment}$:

$$g_s^1(x,y) = \left\{ \max\left[\sum_{i=1}^{N} k_i f_s^1(x, q_i) \right]; \quad \sum_{i=1}^{N} k_i q_i \le y; \quad k_i \in \boldsymbol{N} \right\} \tag{13}$$

The following equation determines $g_s^2(x, y)$:

$$g_s^2(x,y) = \left\{ \max\left[\sum_{i=1}^{M} k_i f_s^2(p_i, y) \right]; \quad \sum_{i=1}^{M} k_i p_i \le x; \quad k_i \in \boldsymbol{N} \right\} \tag{14}$$

3.7. The optimal 3HS layout

Suppose $v_{SX\text{-}3STAGE}$ is the value of optimal 3HSX layout:

$$v_{SX\text{-}3STAGE} = \left\{ \max\left[\sum_{i=1}^{M} k_i g_s^1\left(p_i^{sseg}, W\right) \right]; \quad \sum_{i=1}^{M} k_i p_i^{sseg} \le L; \quad k_i \in N \right\} \tag{15}$$

Suppose $v_{SY\text{-}3STAGE}$ is the value of optimal 3HSY layout:

$$v_{SY\text{-}3STAGE} = \left\{ \max\left[\sum_{i=1}^{M} k_i g_s^2\left(L, p_i^{sseg}\right) \right]; \quad \sum_{i=1}^{M} k_i p_i^{sseg} \le W; \quad k_i \in N \right\} \tag{16}$$

Suppose $v_{S\text{-}3STAGE}$ is the value of optimal 3HS layout:

$$v_{S\text{-}3STAGE} = \max\left(v_{SX\text{-}3STAGE}, v_{SY\text{-}3STAGE}\right) \tag{17}$$

3.8. The steps of generating the optimal 3HS layout

The algorithm for contains the following steps:

Step 1. Determining the normal of homogenous strip, composite strip and segment from Sect. 3.3.

Step 2. Determining the optimal homogenous strip from Sect. 3.4.

Step 3. Determining the optimal composite strip by equations (11) and (12).

Step 4*.* Determining the optimal segment by equations (13) and (14).

Step 5*.* Determining the optimal 3HS layout from Sect. 3.7.

3.9. The time complexity of the 3HSA

The time it takes for determining the normal size of composite strip and section from Sect. 3.3 is $O(mL)$.

The time it takes for determining the optimal homogenous strip from Sect. 3.4 is $O(mLW)$.

The time it takes for determining the optimal composite strip with equation (11) and (12) is $O(LW^2 + WL^2)$.

The time it takes for determining the optimal segment with equation (13) and (14) is $O(L^2 + W^2)$.

Therefore, the total time it takes for determining the optimal 3HS layout is $O(LW^2 + WL^2 + L^2 + W^2)$. Because $mL << mLW$, $W^2 << LW^2$ and $L^2 << L^2W$, therefore, the time complexity is $= O[LW\,(m + L + W)]$.

4. The computation results

As we known, there is no report about the algorithm for generating 3HS layout. The section illustrates the efficiency of this paper algorithm by 43 conventional benchmarks. The benchmark problems use computer with Pentium 4 CPU, clock speed with 2.8 GHz, main memory with 512MB. The problems can be downloaded from website http://www.laria.u-picardie.fr/hifi/OR-Benchmark. The section compares the 3HS layout with the classic three-stage, two-segment, and T-shape and general layouts.

3HS The algorithm of generating optimal 3HS layout

3STAGE Hifi's (Hifi, 2001) algorithm of generating optimal three-stage layout

2SEGMENT The algorithm of Reference (Fayard & Zissimopoulos, 1995) to generate optimal two-segment layout

T-shape The algorithm of Reference (Cui, 2004a) to generate optimal T-shape layout

GENERAL The algorithm of Reference (Cui, Wang & Li, 2005) to generate optimal general layout

According to the above description, the sequence for layout value of above layouts is follows: GENERAL, 3HS, 3STAGE, 2SEGMENT, T-shape. Suppose V_N, V_{3HS}, V_{3STAGE}, $V_{2SEGMENT}$ and $V_{T\text{-}shape}$ is layout value of the above five algorithms respectively. Therefore, V_N V_{3HS} V_{3STAGE} $V_{2SEGMENT}$ $V_{T\text{-}shape}$. Table 2 shows the experiment results, and the note "▲" indicates that the layout value reaches the optimal result. The Table 3 and Table 4 show statistical results.

ID	V_N	V_{3HS}	V_{3STAGE}	$V_{2SEGMENT}$	$V_{T\text{-}shape}$
H	12,348	▲	12,192	12,192	12,132
HZ1	5,226	▲	▲	▲	▲
M1	15,024	▲	▲	▲	▲
M2	73,176	▲	72,564	72,564	72,564
M3	142,817	▲	▲	▲	▲
M4	265,768	▲	▲	▲	▲
M5	577,882	▲	▲	▲	▲
B	8,997,780	▲	▲	▲	▲
U1	22,370,130	22,368,528	22,351,950	22,351,950	22,351,950
U2	20,232,224	▲	20,194,715	20,118,655	20,118,655
U3	48,142,840	48,095,058	48,095,058	48,042,264	48,029,748
UU1	242,919	▲	241,260	241,260	241,260
UU2	595,288	▲	▲	▲	▲
UU3	1,072,764	▲	▲	▲	▲
UU4	1,179,050	1,178,295	1,178,295	1,178,295	1,178,295
UU5	1,868,999	▲	1,868,985	1,868,985	1,868,985
UU6	2,950,760	▲	▲	▲	▲
UU7	2,930,654	▲	▲	▲	▲
UU8	3,959,352	▲	▲	▲	▲
UU9	6,100,692	▲	▲	▲	▲
UU10	11,955,852	▲	▲	▲	▲
UU11	13,157,811	13,147,305	13,146,050	13,141,175	13,127,726
HZ2	8,226	▲	▲	▲	▲
MW1	3,882	▲	▲	▲	▲
MW2	24,950	▲	▲	▲	▲
MW3	37,068	▲	▲	▲	▲
MW4	59576	▲	▲	▲	▲
MW5	189,924	▲	▲	▲	▲
BW	2,307,817	▲	▲	▲	▲
W1	162,867	▲	▲	▲	161,424
W2	35,159	▲	34,656	34,656	34,656
W3	234,108	▲	▲	▲	▲
UW1	6,036	▲	▲	▲	▲
UW2	8,468	▲	▲	▲	▲
UW3	6,302	▲	6,226	6,226	6,226
UW4	8,326	▲	▲	▲	▲
UW5	7,780	▲	▲	▲	▲
UW6	6,615	▲	▲	▲	▲
UW7	10,464	▲	▲	▲	▲
UW8	7,692	▲	▲	▲	▲
UW9	7,038	▲	▲	▲	▲
UW10	7,507	▲	▲	▲	▲

Table 2. The computation results of different layouts

From tables, we can draw conclusions: 1) The optimal results of this paper's algorithm are equal or very close to the general algorithm; 2) The optimal results of this paper's algorithm are better than the classic three-stage, two-segment, T-shape.

Layouts	3HS	3STAGE	2SEGMENT	T-shape
The optimal number of problems	39	32	32	31

Table 3. The optimal number of different layouts

Table 3 lists the optimal number of different layouts, and these statistical data come from Table 2. In 43 classical benchmark problems, the number of 3HS layout's optimal results is 39, and the results ratio of the rest 4 problems and optimal is 99.9%; the number of 3STAGE, 2SEGMENT and T-shape layout's optimal results is 32, 32 and 31 respectively. Therefore, the results of this paper algorithm are better than other layouts.

	3STAGE	2SEGMENT	T-Shape
3HS	9	10	10
3STAGE		3	5
2SEGMENT			4

Table 4. The better number problem of different layouts

Table 4 lists the optimal number of different layouts, and these statistical data come from Table 2. In 43 classical benchmark problems, 1) there are 9 problems that the 3HS layout is better than 3STAGE and 2SEGMENT, and 10 problems for T-shape; 2) there are 3 problems that the 3STAGE layout is better than 2SEGMENT and 5 problems for T-shape; 3) there are 4 problems that the 2SEGMENT layout is better than T-shape. The 3HSA total time it takes for solving 43 problems from table 2 is 93.74s, and each problem's average time is 2.18s. Therefore, the time is reasonable in practical application.

5. Conclusions

It is very difficult to solve UTDC problem. Although there are exact algorithms, the practical computation results indicate these algorithms only solve small scale problems efficiently. These algorithm's time it takes for solving large scale problems is unaffordable. Therefore, people usually solve the problem by two types algorithms, first, the algorithms for generating specific layouts, which not only meet the practical production technology, but also solve large scale problems efficiently within reasonable time, for example, the classic three-stage layout, two-segment layout and T-shape layout; second, the results of genetic algorithm is close to general layout algorithm.

The paper presents an exact algorithm for generating 3HS layout. On the one hand, 3HSA is a specific layout algorithm and its optimization result is better than the classic three-stage, two-segment and T-shape layout, and 3HSA not only improves sheet utilization within reasonable time, but also meets the shearing and punching process need. On the other hand, 3HSA is the heuristic algorithm, and the computations results show that the optimization result of 3HSA is very close that of general algorithm. Therefore, 3HSA can solve a large-scale rectangular piece packing efficiency.

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