Study of a Novel Rapid Transfer Alignment Algorithm

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***Abstract*—Transfer alignment of strapdown inertial navigation system (SINS) for the ship-borne equipment, always utilize the message from master inertial navigation system on the ship. Rapid transfer alignment methods such as "velocity plus attitude" and "velocity plus rate" has been developed by researchers in this area, for the improvement of the rapidness of the transfer alignment, but the accuracy is poor when the flexure of the ship is grievous under abominable environment, the previous research shows that the flexure along different axis of the ship is always not the same, aiming at this problem, theoretical analysis of the effect about the flexure along every axis on the transfer alignment accuracy has been done, find out that the rate of the ship, which is disturbed by flexure rate seriously, depressed the transfer alignment accuracy much more than the other two axis, based on the theoretical analysis, a new rapid transfer alignment prototype named "velocity plus partial rate" matching is presented, by subtracting that rate of the ship from the conventional rate measurement message. Sufficiently utilizing the sway maneuver and avoiding the influence of the flexure, can obtain higher precision and less computational cost than the "velocity plus rate" matching rapid transfer alignment, while the flexure of a certain axis is severity. Simulation system is developed for the evaluation of the presented algorithm, results show that the presented method can obtain higher accuracy than the "velocity plus rate" matching rapid transfer alignment under worse environment, and evidently reduce the computational cost while applying Kalman filter for the estimation task.**

Keywords—Strapdown inertial navigation system; initial alignment; transfer alignment; optimal estimate; Kalman filter

I. INTRODUCTION

Strapdown inertial navigation systems are equipped in many types of ship-borne equipment, followed by the development of technology [1]-[3]. Initial alignment of an inertial navigation system is the process whereby the orientations of the axes are determined with respect to the reference axes system. There are many complications that make alignment both times consuming and complex. However, accurate alignment is crucial, if precision navigation is to be achieved over long periods of time without any form of aiding [3]. Transfer alignment is the process of matching the slave inertial navigation system to the master using natural or deliberately-induced maneuvers of the vehicle [1], [2], and is the first choice of initial alignment of air-borne or ship-borne equipments' strapdown inertial navigation system. The velocity and the precision of the initial alignment directly affect the ability of rapid response and the accuracy of the weapon, so improving the velocity and precision of the initial alignment is always the goal of the researchers in this area[4],[5]. Kalman filtering, which is recognized as one of the most powerful traditional techniques of estimation, is applied in modern initial alignment [6].

The horizon error about the "platform" of the inertial navigation system will cause the measurement of the special force wrongly, and propagated as east and north velocity error [2], the velocity matching transfer alignment relies on the maneuver of the vehicle, but it is impossible for large scale ships, so velocity matching only transfer alignment couldn't accomplish the alignment task here. The ship will suffer certain of movement about pitch or roll, because of the wind and the wave on sea. By matching the angular rate of the ship can finish the transfer alignment process quickly [2] , [7], and [8] .

The redistribution of freight and fuel on the ship, the non-uniform heating of different parts of the ship under sun will cause the static angular deformation of the ship's deck, also dynamic angular deformations of the ship's deck are caused by motion disturbances, sea waves, helm's operation, etc. [9].Although exact deformation model can be built theoretically for the Kalman filter, and then estimated the errors, but it is difficult in practice. Kain studied the suboptimal filter where the effects of the wide-band flexure process were masked by a white noise process [10].

Motivated by the works presented above, avoiding the exact modelling of the flexure, this paper studied a new rapid transfer alignment methods named "velocity plus partial rate" matching, clear the angular rate message which is disturbed by the flexure seriously from the angular rate measurement message.

The rest of this paper is organized as follows: section Ⅱ introduces the notations and related definitions used in this paper, sectionⅢ present s the "velocity plus angular rate" rapid transfer alignment error model, section Ⅳ analyses the relationship between the estimation accuracy of the misalignment and the flexure of the ship, based the theoretical analysis above, the novel rapid transfer alignment named "velocity plus partial angular rate" matching is studied in section Ⅴ, then the simulation system and the simulation have been done in sectionⅥ, finally in sectionⅧ gives the conclusion of this paper.

II. NOTATIONS AND RELATED DEFINITIONS

INS: inertial navigation system

DCM: direction cosine matrix

KF: Kalman filter

Navigation frame (n-frame): the navigation frame defined as local level frame at the true position

Master INS frame (m-frame): frame fixed to the master INS

Real slave INS frame (sr-frame): frame fixed to the slave INS

Computed slave INS frame (sc-frame): frame fixed to the computed slave INS

$\boldsymbol{C}_m^{sc}$: DCM from m-frame to sc-frame

$\boldsymbol{C}_m^{n}$: DCM from m-frame to n-frame, attitude matrix of master INS

$\boldsymbol{C}_{sc}^{n}$: DCM from sc-frame to n-frame, computed attitude matrix of slave INS

$\boldsymbol{C}_m^{sr}$: DCM from m-frame to sr-frame

The relationships among these frames are show in figure1:

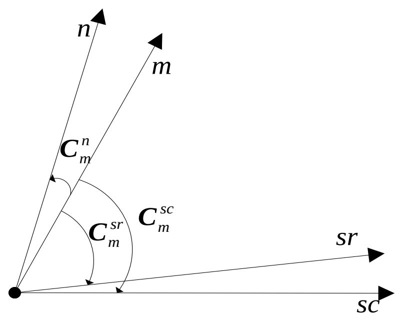

Fig. 1 Relationships among the defined frames

III. VELOCITY PLUS ANGULAR RATE MATCHING RAPID TRANSFER ALIGNMENT ERROR MODEL

A. *Error Model of Rapid Transfer Alignment*

In transfer alignment the master inertial navigation system is often of higher precision and its navigation error can be neglected, the slave inertial navigation system is often of lower precision. Here we simply give the rapid transfer alignment error model Kain introduced in [10]:

$$\delta\dot{\boldsymbol{V}} = \boldsymbol{C}_{sc}^{n}(\boldsymbol{\psi}_m - \boldsymbol{\psi}_a)\times \hat{\boldsymbol{f}}_{sr}^{sr} + \boldsymbol{C}_{sc}^{n}(\boldsymbol{f}_f^{sr} + \nabla^{sr}) \tag{1}$$

$$\dot{\boldsymbol{\psi}}_m = (\boldsymbol{\psi}_m - \boldsymbol{\psi}_a)\times \hat{\boldsymbol{\omega}}_{nsr}^{sr} + \boldsymbol{\omega}_{fs}^{sr} + \boldsymbol{\varepsilon}^{sr} \tag{2}$$

$$\dot{\boldsymbol{\psi}}_a = \boldsymbol{\eta}_a \tag{3}$$

Here $\delta\boldsymbol{V}$ is the computed velocity difference between the master and slave velocities compensated by the lever arm effect from the master to slave. $\boldsymbol{\psi}_m$ represents the time varying angular change between the two available (master and slave) direction cosine matrices, it can be obtained from:

$$\boldsymbol{C}_n^{sc}(t)\boldsymbol{C}_m^{n}(t) = \begin{bmatrix} 1 & \psi_{mz}(t) & -\psi_{my}(t) \\ -\psi_{mz}(t) & 1 & \psi_{mx}(t) \\ \psi_{my}(t) & -\psi_{mx}(t) & 1 \end{bmatrix} = I - [\boldsymbol{\psi}_m(t)\times] \tag{4}$$

Note that at the initiation of transfer alignment, we make the assumption that $\boldsymbol{C}_{sc}^{n}(0) = \boldsymbol{C}_m^{n}(0)$, that is, we initialize the slave direction cosine matrix with the master direction cosine matrix. Thus $\boldsymbol{\psi}_m(0) = \boldsymbol{0}$.

$\boldsymbol{\psi}_a$ is the real misalignment between the master and the slave inertial navigation system, and $\boldsymbol{\eta}_a$ is white noise process added to $\boldsymbol{\psi}_a$.

B. *State Space Presentation of The Dynamics*

Kalman filter is often utilized as the estimator of the transfer alignment, before this the state dynamics should be introduced first [11], so we define the state vector of rapid transfer alignment as:

$$\boldsymbol{X} = [\delta V_x;\delta V_y;\delta V_z;\psi_{mx};\psi_{my};\psi_{mz};\psi_{ax};\psi_{ay};\psi_{az}] \tag{5}$$

$\delta\boldsymbol{V},\boldsymbol{\psi}_m,\boldsymbol{\psi}_a$ are defined in the last section, the subscript x, y and z represent the axes of the vehicle, then the system state equation can be presented:

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{W} \tag{6}$$

$\boldsymbol{W}$ is system noise, $\boldsymbol{A}$ is the system matrix according to the error model about(1)-(3).

C. *"Velocity Plus Rate" Matching Rapid Transfer Alignment*

"Velocity plus attitude" is the rapid transfer alignment developed by kain himself in [10], in this method the measurement are $\delta\boldsymbol{V}$ and $\boldsymbol{\psi}_m$, both of them are "measurable" quantities computed from the master and slave inertial navigation systems, $\delta\boldsymbol{V}$ is the computed velocity difference between the master and slave velocities accounting for the master to slave lever arm, $\boldsymbol{\psi}_m$ can be achieved from equation (4), and both of them are contained in the state vector, so in this method the observation vector is:

$$\boldsymbol{Z} = [\delta\boldsymbol{V}_x;\delta\boldsymbol{V}_y;\delta\boldsymbol{V}_z;\boldsymbol{\psi}_{mx};\boldsymbol{\psi}_{my};\boldsymbol{\psi}_{mz}] \tag{7}$$

The observation matrix is:

$$\boldsymbol{H} = \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \tag{8}$$

The author introduced the "velocity plus rate" matching rapid transfer alignment in [8], "velocity" is also used as the measurement, the other measurement is the angular rate between master and slave inertial navigation systems,

The measured angular rate of master and slave inertial navigation is $\boldsymbol{Z}^m$ and $\boldsymbol{Z}^s$, respectively:

$$\boldsymbol{Z}^m = \boldsymbol{\omega}_{im}^m \tag{9}$$

$$\boldsymbol{Z}^s = \hat{\boldsymbol{\omega}}_{is}^s \tag{10}$$

The angular rate of slave SINS can be presented as the angular rate of the master's added with the flexure rate and the measurement error about the gyros:

$$\begin{aligned}\boldsymbol{Z}^s &= \hat{\boldsymbol{\omega}}_{is}^s \\ &= \boldsymbol{\omega}_{im}^s + \boldsymbol{\omega}_f^s + \boldsymbol{\varepsilon}^s \\ &= \boldsymbol{C}_m^{sr}\boldsymbol{\omega}_{im}^m + \boldsymbol{\omega}_f^s + \boldsymbol{\varepsilon}^s\end{aligned} \tag{11}$$

$\boldsymbol{C}_m^{sr}$ is the DCM from m-frame to sr-frame, if the misalignment angular is small, it can be:

$$\boldsymbol{C}_m^{sr} = \mathbf{I}_{3\times3} - (\boldsymbol{\psi}_a \times) \tag{12}$$

So the difference between the angular rate of the slave and master INS is:

$$\begin{aligned}\delta\boldsymbol{Z}_\omega &= \boldsymbol{Z}^s - \boldsymbol{Z}^m \\ &= [I - (\boldsymbol{\psi}_a \times)]\boldsymbol{\omega}_{im}^m + \boldsymbol{\omega}_f^s + \boldsymbol{\varepsilon}^s - \boldsymbol{\omega}_{im}^m \\ &= -\psi_a \times \boldsymbol{\omega}_{im}^m + \boldsymbol{\omega}_f^s + \boldsymbol{\varepsilon}^s \\ &= \boldsymbol{\omega}_{im}^m \times \boldsymbol{\psi}_a + \boldsymbol{\omega}_f^s + \boldsymbol{\varepsilon}^s\end{aligned} \tag{13}$$

In this method the observation vector is:

$$\boldsymbol{Z} = [\delta V_x; \delta V_y; \delta V_z; \delta Z_{\omega x}; \delta Z_{\omega y}; \delta Z_{\omega z}] \tag{14}$$

The observation matrix is:

$$\boldsymbol{H} = \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & & \\ & 0 & -\omega_{imz}^m & \omega_{imy}^m \\ \mathbf{0}_{3\times6} & \omega_{imz}^m & 0 & -\omega_{imx}^m \\ & -\omega_{imy}^m & \omega_{imx}^m & 0 \end{bmatrix} \tag{15}$$

This "velocity plus angular rate" matching rapid transfer alignment has performed well as is shown in the previous research.

IV. ANALYSIS OF THE FLEXURE AND THE ESTIAMTION ACCURACY OF THE MISALIGNMENT

The research of Titterton D H and Mochalov A V shows that the flexure along the roll axis is much more serious than the other two[2], [8], that is to say, the error induced to the transfer alignment which is caused by flexure of this axis is larger. Based on the "velocity plus angular rate" matching transfer alignment introduced above, we studied the novel rapid transfer alignment algorithm named "velocity plus partial rate" matching, clearing the angular rate along roll axis, which is disturbed by the flexure seriously. The analysis of the relationship between the flexure and the accuracy of the transfer alignment is done as following.

Innovated by reference [2] consider the two axis sets shown in figure 2, correspond to the orientations of the master frame and the slave frame at two locations remote from each other on a ship. In the left is the master frame $O_m XY$,which is taken to be aligned perfectly with the pitch, roll and yaw axes of the ship, and $\dot{\phi}_p, \dot{\phi}_r, \dot{\phi}_y$ are the angular rate of pitch, roll and yaw respectively, while the right in the figure is the slave frame $O_s XYZ$, the corresponding angular rate are $\dot{\phi}_p + \delta\dot{\phi}_p, \dot{\phi}_r + \delta\dot{\phi}_r, \dot{\phi}_y + \delta\dot{\phi}_y$, with the flexure angular rate $\delta\dot{\phi}_p, \delta\dot{\phi}_r, \delta\dot{\phi}_y$.

$$\phi_r - roll; \quad \phi_p - pitch \quad \phi_y - yaw$$

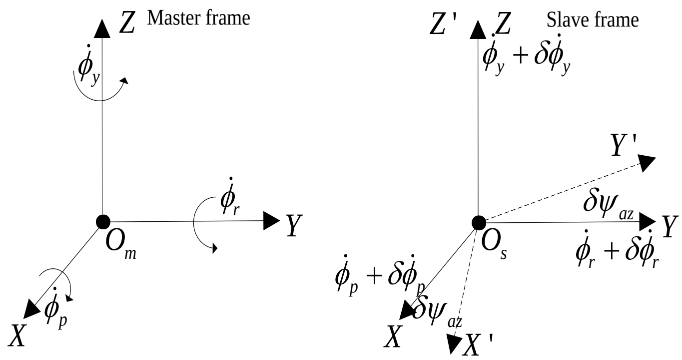

Fig. 2 Illustration of the effects of ship flexure on axis alignment

Here we study the effect of different matching message on the estimation accuracy of transfer alignment. Effect of pitch rata angular matching on the misalignment of Z axis $\delta\psi_{az}$ is analysed first. The pitch angular rate measured by the master system is $\dot{\phi}_p$, while the slave system measured can be written as:

$$(\dot{\phi}_p + \delta\dot{\phi}_p)\cos\delta\psi_{az} - (\dot{\phi}_r + \delta\dot{\phi}_r)\sin\delta\psi_{az} \tag{16}$$

Now the pitch angular rate matching is:

$$\begin{aligned}\delta Z &= (\dot{\phi}_p + \delta\dot{\phi}_p)\cos\delta\psi_{az} - (\dot{\phi}_r + \delta\dot{\phi}_r)\sin\delta\psi_{az} - \dot{\phi}_p \\ &= \dot{\phi}_p(\cos\delta\psi_{az} - 1) - \dot{\phi}_r\sin\delta\psi_{az} + \delta\dot{\phi}_p\cos\delta\psi_{az} \\ &\quad - \delta\dot{\phi}_r\sin\delta\psi_{az}\end{aligned} \tag{17}$$

Get the first order of $\delta\psi_{az}$:

$$\delta Z = -(\dot{\phi}_r + \delta\dot{\phi}_r)\delta\psi_{az} + \delta\dot{\phi}_p \tag{18}$$

In this case, the measurement difference settles to zero when:

$$\delta\psi_{az} = \delta\dot{\phi}_p / (\dot{\phi}_r + \delta\dot{\phi}_r) \tag{19}$$

From the results above, we can see that the residual yaw misalignment will reduce as the roll rate of the ship becomes larger, or as the flexure about the pitch axis becomes smaller.

By the same way, while applying the roll rate as the measurement, the effect of flexure on the yaw misalignment is:

$$\delta\psi_{az} = \delta\dot{\phi}_r / (\dot{\phi}_p + \delta\dot{\phi}_p) \tag{20}$$

That is to say, the residual yaw misalignment will reduce as the pitch rate of the ship becomes larger, or as the flexure about roll axis, measurement is this case, becomes smaller. The flexure along the roll axis of the ship is always much larger than the pitch one, also the roll rate is larger than the pitch one, so, according to (19) and(20), pitch angular rate matching transfer alignment is benefit for the estimation of the misalignment along Z axis.

Consider the estimation accuracy of misalignment along X axis, utilize the yaw angular rate as the matching message, we have:

$$\delta\psi_{ax} = \delta\dot{\phi}_y / (\dot{\phi}_r + \delta\dot{\phi}_r) \tag{21}$$

While apply the roll angular rate as the matching message, we have:

$$\delta\psi_{ax} = \delta\dot{\phi}_r / (\dot{\phi}_y + \delta\dot{\phi}_y) \tag{22}$$

The flexure along the roll axis is much larger than the one along the yaw axis, and the roll rate of the ship is larger then the yaw rate, from the comparison of (21) and(22) , we can find that the yaw angular rate matching is benefit for the estimation accuracy about the misalignment on X axis.

Finally consider the relationship between the measurement message and the estimation accuracy along Y axis; first consider the pith angular rate matching:

$$\delta\psi_{ay} = \delta\dot{\phi}_p / (\dot{\phi}_y + \delta\dot{\phi}_y) \tag{23}$$

Then consider the yaw angular rate matching:

$$\delta\psi_{ay} = \delta\dot{\phi}_y / (\dot{\phi}_p + \delta\dot{\phi}_p) \tag{24}$$

Form (23) and(24), we can see that the misalignment estimation accuracy along Y axis, limited by the roll and yaw rate, the values of these two date are in the same level, so the estimation accuracy along Y axis will lower than the other two axes.

V. VELOCITY PLUS PARTIAL RATE MATCHING TRANSFER ALIGNMENT

From the analysis above, while consider the improvement of misalignment estimation accuracy along Z and X axis, we chose pitch rate matching and yaw rate matching respectively, and abandon the roll rate matching simultaneously. So we present a novel rapid transfer alignment method, which clear the roll rate message from the transfer alignment measurement.

Now the state vector and the system matrix is the same as shown in section Ⅲ (5)and (6). But the measurement vector is changed to:

$$Z_{pw} = [\delta V_x; \delta V_y; \delta V_z; \delta W_x; \delta W_z] \tag{25}$$

The rate measurement matrix is:

$$H_{p\omega} = \begin{bmatrix} 0_{2\times 6} & 0 & -\omega_{imz}^m & \omega_{imy}^m \\ & -\omega_{imy}^m & \omega_{imx}^m & 0 \end{bmatrix} \tag{26}$$

Thus the whole measurement matrix is：

$$H_p = \left[H_v; H_{p\omega} \right] \tag{27}$$

All these composed the “velocity plus partial rate” matching transfer alignment method.

VI. SIMULATION ANALYSIS

In order to compare the estimation accuracy and convergence rate of the misalignment of the proposed algorithm, simulation has been done in this section. The simulation system is developed according to ref [12], as shown in figure 3, it consists of motion model, where the motion manner is set; the ship motion simulation is used to give the ship’s maneuver under the motion model; then the motion is sensed by master and slave IMU respectively, which contains gyro and accelerometer, here the flexure of the body and lever arm effect are simulated and sensed by the slave IMU only; then strapdown inertial navigation algorithm is utilized to compute the navigation message, such as velocity, attitude and position of the body, let the mast SINS much more accurate than the slave one, and is well initialized, then by matching the velocity and partial rate message from master SINS and slave SINS to obtain the measurement message of transfer alignment, then the classical Kalman filter is utilized to estimate the misalignment[11].

In simulation, we choose one classically utilized condition of the inertial navigation, the initial position is latitude $\varphi_0 = 30°$ and longitude $\lambda_0 = 108°$, the initial velocity is $V_x = V_y = 25$ m/s, the initial attitude of the ship is $[5, -5, 30]°$, the time period for INS data update is 10ms, and the alignment filter period is 50ms. The motion of the ship represents as:

$$\begin{cases} \theta = \theta_m \sin(\omega_\theta t + \theta_0) \\ \gamma = \gamma_m \sin(\omega_\gamma t + \gamma_0) \\ \psi = \psi_m \sin(\omega_\psi t) + \psi_0 \end{cases} \tag{28}$$

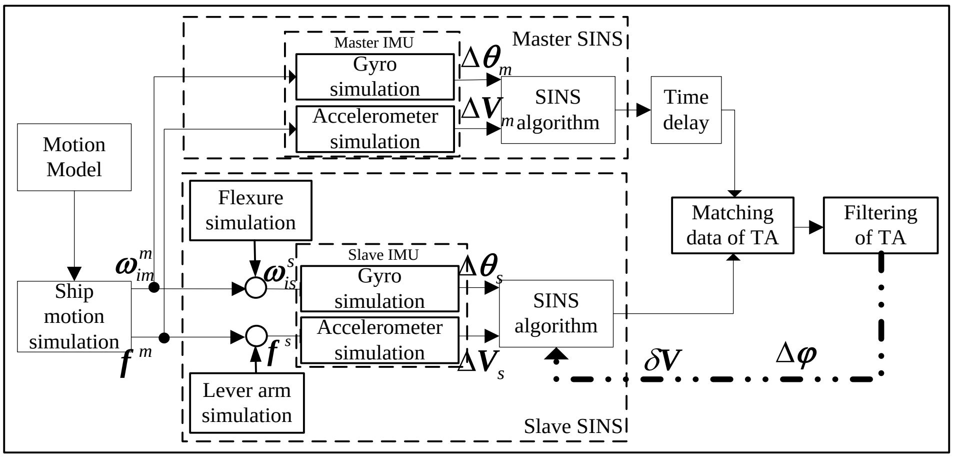

Fig. 3 Block diagram of transfer alignment simulation system

TABLE I. ESTIMATION ERROR OF THE MISALIGNMENT

Methods	Estimation error (deg)		
	X	Y	Z
Velocity plus rate	0.187718	-0.256952	-0.183439
Velocity plus partial rate	0.063602	-0.101714	0.143451

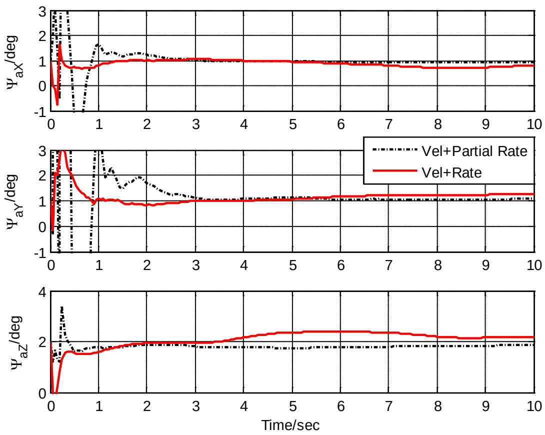

Fig. 4 Estimation of misalignment angle

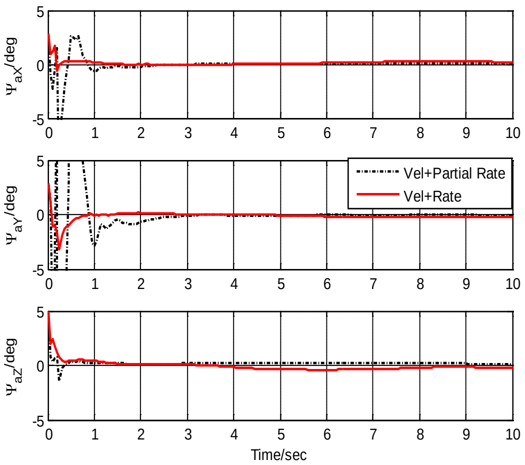

Fig. 5 Estimation error of misalignment angle

Simulations have been done and the simulation results are show n from figure 4 and figure 5. From simulation results in fig.4 and fig.5, we can see that while clear the roll rate message, which is disturbed by flexure seriously, from the rate measurement, the transfer alignment accuracy is improved obviously. The estimation error of the two methods is shown in table Ⅰ for detail.

VII. CONCLUSION

Aiming at the problem that the estimation accuracy of "velocity plus rate" matching transfer alignment method effected by the flexure of the ship seriously, by theoretically analysing the effect of every angular rate message along three axes on the transfer alignment accuracy, presented a novel rapid transfer alignment name "velocity plus partial rate" matching, clearing the angular rate message which is disturbed flexure seriously, then design a simulation system for the evaluation of this method, results show that the presented algorithm can reduce the effect of the flexure, and thus improve the accuracy of transfer alignment.

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