Wavelet Sliding Mode Control of Uncertain Nonaffine Nonlinear Discrete Time Systems

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***Abstract*-In this paper, a wavelet based adaptive sliding mode control is designed for a class of discrete time uncertain nonlinear nonaffine systems. An equivalent affine like structure is first developed form the original nonaffine system and subsequently a wavelet based adaptive sliding mode tracking control scheme is developed for the affine like system. Wavelet neural network (WNN) is used to mimic the uncertainties present in the system. Proposed scheme is derived to guaranty the necessary and sufficient reaching condition for sliding mode control in presence of modeling uncertainties and mathematical inaccuracies. A numerical example is provided to verify the effectiveness of theoretical development.**

Keywords-Adaptive sliding mode control; wavelet networks; discrete time nonlinear systems

I. INTRODUCTION

Over last few years, several efforts on the development of adaptive control strategies for uncertain discrete time nonlinear systems have been cited in the literature. In these cases the common assumption was that the system is affine in input [1, 2]. However for discrete time nonaffine systems these strategies are not directly applicable. Commonly used strategy for this class of systems is to obtain an affine like structure for the original non affine discrete time system and subsequently a control strategy is developed [3,4].

Sliding mode control (SMC) is an efficient controlling strategy for uncertain systems. Along with order reduction other features offered by this control strategy are fast response, insensatization towards uncertainties and disturbances. The SMC is variable structure control algorithm which drives state trajectories toward a specific hyperplane and maintains the trajectories sliding on hyperplane until the origin of the state space is reached [20]. Few effective strategies for the development of SMC algorithms for discrete-time systems have been sited in the literature. These schemes preserve the distinguished features of sliding mode control and at the same time limits the undesired effects of chattering [5-11].

Modeling uncertainties and mathematical inaccuracies usually degrades the performance of the controller, especially for nonlinear and complex control problems [12, 13].In such cases, some system identification tool is augmented with baseline controller. Neural Networks (NNs) have been proved a very efficient system identification tool due to its universal approximation property and learning capability [13]. Recently some researchers are inclined towards the development of adaptive control strategies using wavelet neural network as a system identification tool [17-19]. A wavelet network consists of single layer of translated and dilated versions of mother wavelet function. Wavelet Neural Networks are having superior approximation capabilities than conventional neural networks due to features like orthogonality, multiresolution, space and frequency localization properties [14, 15].

In this work a wavelet based adaptive sliding mode control scheme is proposed to solve the tracking control problem of uncertain discrete time nonaffine nonlinear systems. Inspired by the approximation capabilities of the wavelet neural networks, this work utilizes the WNN as system identification tool.

The paper is organized as follows: section II highlights the approximation features of WNN, system formulation is described in section III and controller designing and stability aspects are discussed in section IV. Effectiveness of the proposed strategy is illustrated through an example in section V while section VI concludes the paper.

II. FUNDAMENTALS OF WAVELET NETWORKS

A) *Wavelet Neural Network*

Wavelet network is a single layer network consisting of translated and dilated versions of orthonormal father and mother wavelet function. Basis functions are used in wavelet network span $L^2(\Re)$ subspace. Due to its universal approximation property any function $f(x) \in L^2(\Re)$ can be approximated by linear combination of basis functions [10-11].

Orthonormality is a promising feature of wavelet bases, it assures that coefficient needed for reconstruction of any function are fixed and unique and can be tuned independent of other wavelet bases.

Wavelets are derived from the basic requirement of multiresolution analysis, which provides a mathematical framework to describe the increment in information from coarse approximation to finer approximation. Multiresolution analysis

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is basically a decomposition of space $S \in L^2(\Re)$, with following properties [16]

(a) Whole space S is constructed as a sequence of nested and closed finite dimensional subspace S_i

$$\cdots \subset S_{-1} \subset S_0 \subset S_1 \cdots \subset S_2 \subset \cdots \qquad \forall n \in Z$$

(b) $\bigcap_{n\in Z} S_n = \{0\}$

(c) $\bigcup_{n\in Z} S_n = L^2(\Re)$

So any function $f \in S$ can be approximated with desired accuracy by its projection $f_i = P_i f$ on S_i, i.e., $\lim_{i\to\infty} f_i = f$.

(d) $f(x) \in S_i \Leftrightarrow f(2x) \in S_{i+1}$
$f(x) \in S_i \Leftrightarrow f(x - 2^{-i}k) \in S_i$

(e) Multiscale structure provides an orthogonal split of S_{i+1} into low and high frequency parts S_i and W_i respectively.

$$S_{i+1} = S_i \oplus W_i$$

$$W_i \perp W_j \quad \text{if } i \neq j$$

$$W_i \subset S_j \quad \text{if } j > i$$

Decomposition of the whole space S can be expressed as

$$S = S_i \oplus W_i \oplus W_{i-1} \oplus W_{i-2} \cdots \oplus W_0 \oplus W_{-1} \cdots$$

Normally, the wavelet bases are derived using dyadic translation and binary dilation of scaling function $\phi \in S$ and wavelet function $\varphi \in S$. At any resolution j

$$\phi_{jq}(x) = 2^{j/2}\phi(2^j x - q) \qquad j, q \in Z$$

$$S_j = span\{\phi_{jq}(x), q \in Z\}$$

and $\varphi_{jq}(x) = 2^{j/2}\varphi(2^j x - q) \qquad j, q \in Z$

$$W_j = span\{\varphi_{jq}(x), q \in Z\}$$

It follows that any function $f(x(k))$ in S can be expressed as a wavelet series expansion

$$f(x(k)) = \sum_{j=N1}^{N2}\sum_{q=M1}^{M2}\left\langle \phi_{j,q}(x(k)), f(x(k))\right\rangle \phi_{j,q}(x(k)) \tag{1}$$

Convergence of the wavelet series can be expressed as

$$\lim_{\substack{N1,M1\to-\infty\\ N2,M2\to+\infty}} \left\| f(x(k)) - \sum_{j=N1}^{N2}\sum_{k=M1}^{M2}\left\langle \varphi_{j,q}(x(k)), f(x(k))\right\rangle \varphi_{j,q}(x(k)) \right\| = 0 \tag{2}$$

For nonlinear system modeling the structure of the wavelet network can not be taken infinitely large so truncating the wavelet series to finite numbers of resolutions and translates at each resolution the above expression can be approximated as

$$f(x(k)) = \sum_{j\geq J}^{N}\sum_{q=M1_j}^{M2_j}\left\langle \phi_{j,q}(x(k)), f(x(k))\right\rangle \phi_{j,q}(x(k)) + \varepsilon(x(k)) \tag{3}$$

where J is lowest resolution, $N \in \mathbb{Z}$ represents the highest resolution while $q = \left[M1_j, \cdots, M2_j\right] \in \mathbb{Z}$ represents the number of translates at jth resolution and $\varepsilon(x(k))$ is the approximation error defined as

$$\varepsilon(x(k)) = f(x(k)) - \sum_{j\geq J}^{N}\sum_{q=M1_j}^{M2_j}\left\langle \varphi_{j,q}(x(k)), f(x(k))\right\rangle \varphi_{j,q}(x(k)) \tag{4}$$

Owing to the property of multi resolution analysis (3) can be expressed as

$$f((x(k))) = \left\{\begin{array}{l} \sum_{q=M1_j}^{M2_j}\left\langle \phi_{J,q}(x(k)), f(x(k))\right\rangle \phi_{J,q}(x(k)) + \\ \sum_{j\geq J}^{N}\sum_{q=M1_j}^{M2_j}\left\langle \varphi_{j,q}(x(k)), f(x(k))\right\rangle \varphi_{j,q}(x(k)) + \varepsilon(x(k)) \end{array}\right\} \tag{5}$$

For a function of the form $f(x(k)) : \Re^n \to \Re$, wavelet network model can be extended to multidimensional wavelet network by tensor product of single dimensional wavelet bases [13].

$$\phi_{J,q}(x(k)) = \prod_{i=1}^{n}\phi_{J,q}(x_i(k)); \quad \varphi_{j,q}(x(k)) = \prod_{i=1}^{n}\varphi_{j,q}(x_i(k)) \tag{6}$$

$$\begin{aligned} f(x(k)) &= \left\{\begin{array}{l} \sum_{q=M1_j}^{M2_j}\left\langle \phi_{J,q}(x(k)), f(x(k))\right\rangle \phi_{J,q}(x(k)) + \\ \sum_{j\geq J}^{N}\sum_{q=M1_j}^{M2_j}\left\langle \varphi_{j,q}(x(k)), f(x(k))\right\rangle \varphi_{j,q}(x(k)) + \varepsilon(x(k)) \end{array}\right\} \\ &= \sum_{q=M1_j}^{M2_j}\alpha_{J,q}(k)\phi_{J,q}(x(k)) + \sum_{j\geq J}^{N}\sum_{q=M1_j}^{M2_j}\beta_{j,q}(k)\varphi_{j,q}(x(k)) + \varepsilon(x(k)) \end{aligned} \tag{7}$$

where $\alpha_{J,q}(k)$ $\beta_{j,q}(k)$ are weights of wavelet basis functions

Now (7) can be rewritten as

$$f(x(k)) = \alpha^T(k)\phi(x(k)) + \beta^T(k)\varphi(x(k)) + \varepsilon(x(k)) \tag{8}$$

where

$$\alpha(k) = \left[\alpha_{JM1_J}(k), .., \alpha_{JM2_J}(k)\right]^T$$

and

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$\beta(k)=\left[\beta_{JM1_J}(k),\cdots,\beta_{JM2_J}(k),\cdots,\beta_{NM1_N}(k),\cdots\beta_{NM2_N}(k)\right]^T$ are the scaling and wavelet weight vectors respectively. $\varphi(x(k))=\begin{bmatrix}\varphi_{JM1_J}(x(k)),\cdots,\varphi_{JM2_J}(x(k)),\cdots \\ \cdots,\varphi_{NM1_N}(x(k)),\cdots,\varphi_{NM2_N}(x(k))\end{bmatrix}^T$ and $\phi(x(k))=\left[\phi_{JM1_J}(x(k)),..,\phi_{JM2_J}(x(k))\right]^T$ are wavelet and scaling vectors respectively.

It can be shown that, for an arbitrary constant $\lambda>0$, there exist a finite integer J_N and real constant optimal weight vectors α^*,β^* such that the unknown nonlinear function $f(x(k))$ can be approximated as follows

$$f(x(k))=\left\{\begin{array}{l}\sum_{q=M1_J}^{M2_J}\alpha^*{}_{J,q}(k)\phi_{J,q}(x(k))+\sum_{j\geq J}^{J_N}\sum_{q=M1_j}^{M2_j}\beta^*{}_{j,q}(k)\varphi_{j,q}(x(k)) \\ +\varepsilon(x(k))\end{array}\right\}$$
$$=\alpha^{*T}\phi(x(k))+\beta^{*T}\varphi(x(k))+\varepsilon(x(k))\ \forall x(k)\in\Omega\subset\Re^n \qquad (9)$$

where $\varepsilon(x(k))$ denotes the approximation error and is assumed to be bounded by $|\varepsilon(x(k))|\leq\lambda$ and Ω is a compact set.

Optimal parameter vectors needed for best approximation of the function are difficult to determine so defining an estimate function as

$$\hat f(x(k))=\hat\alpha^T\phi(x(k))+\hat\beta^T\varphi(x(k)) \qquad (10)$$

where $\hat\alpha,\hat\beta$ are the estimates of α^*,β^* respectively. Defining the estimation error as

$$\tilde f(x(k))=f(x(k))-\hat f(x(k))= \left\{\tilde\alpha^T(k)\phi(x(k))+\tilde\beta^T(k)\varphi(x(k))+\varepsilon(x(k))\right\} \qquad (11)$$

where $\tilde\alpha(k)=\alpha^*-\hat\alpha(k),\tilde\beta(k)=\beta^*-\hat\beta(k)$

By properly selecting the number of resolutions, the estimation error $\tilde f(x(k))$ can be made arbitrarily small on the compact set so that the bound $\left\|\tilde f(x(k))\right\|\leq\tilde f_m$ holds for all $x\in\Omega\subset\Re^n$.

The residual part $\varepsilon(x)$ can be assumed to be bounded by a linear in parameter function

$$|\varepsilon(x(k))|\leq\gamma^T z(k) \qquad (12)$$

where $\gamma\in\Re^4$ represents unknown optimal weight vector while $z(k)=\left[1,\|x(k)\|,\|x(k)\|\|\hat\alpha(k)\|,\|x(k)\|\|\hat\beta(k)\|\right]^T$. Assuming that $\hat\gamma(k)$ be the estimate of γ, estimation error will be $\tilde\gamma(k)=\gamma-\hat\gamma(k)$ .Adaptation laws for the online tuning of $\hat\alpha(k),\hat\beta(k)$ and $\gamma(k)$ will be derived in following section.

III. SYSTEM FORMULATION

Consider a discrete time nonlinear nonaffine system of the form

$$\begin{aligned}x_1(k+1)&=x_2(k)\\x_2(k+1)&=x_3(k)\\&\vdots\\x_n(k+1)&=g(x(k)u(k))\\y(k)&=x_1(k)\end{aligned} \qquad (13)$$

where $x(k)=\left[x_1(k),x_2(k),...,x_n(k)\right]^T\in\Re^n,u(k)\in\Re,y(k)\in\Re$ are state vector, control input and output respectively. $g(x(k),u(k)):\Re^{n+1}\to\Re$ is the unknown nonlinear function of state variable and input. In this work unknown system dynamics is approximated by a wavelet network.

By applying the transformation, system (13) can be expressed in an affine like form

$$\begin{aligned}x_1(k+1)&=x_2(k)\\x_2(k+1)&=x_3(k)\\&\vdots\\x_n(k+1)&=g(x(k)u(k))-u(k)+u(k)=f(x(k),u(k))+u(k)\\y(k)&=x_1(k)\end{aligned} \qquad (14)$$

In transformed system (14), uncertainty $f(x(k),u(k))$ is approximated by a wavelet network.

The objective is to design adaptive sliding mode controller to achieve the desired tracking performance simultaneously nullifying the effect of modeling inaccuracies.

IV. WAVELET SLIDING MODE CONTROLLER DESIGN

Let $\bar y_d(k)\in\Re^n$ be the desired trajectory vector and assuming that its past values for previous $(n-1)$ instances are known.

Defining the state tracking error vector

$$e(k)=x(k)-\bar y_d(k) \qquad (15)$$

with $e_i(k)=x_i(k)-y_d(k+i-n) \qquad i=1,\cdots,n$

So the error dynamics of the system (13) becomes

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$$e_i(k+1) = e_{i+1}(k) \quad 1 \le i \le n-1$$
$$e_n(k+1) = f(x(k),u(k)) + u(k) - y_d(k+1) \tag{16}$$

A linear functional sliding surface is defined as

$$s(k) = ce(k) \tag{17}$$

where $c = [c_1, c_2, \cdots, c_n] \in \Re^n$ is a vector of positive constant values, selected such that the poles of the systems are located inside the unit circle. Then $s(k+1)$ is defined as

$$s(k+1) = \left\{ \begin{array}{l} c_1 e_2(k) + c_2 e_3(k) + \cdots + c_{n-1} e_n(k) + c_n f_n(x(k),u(k)) \\ + c_n(u(k) - y_d(k+1)) \end{array} \right\} \tag{18}$$

In this expression the component $c_n f_n(x(k),u(k))$ is modeled by using a wavelet network.

For discrete time systems an inequality of the form

$$s(k)\left[s(k+1) - s(k)\right] < 0 \tag{19}$$

is necessary but not sufficient condition to be used as reaching law, as it does not assures the convergence towards the sliding surface.

In order to assure reaching condition constraint imposed by following inequality is also required to be satisfied

$$|s(k+1)| < |s(k)| \tag{20}$$

By combining (18) and (19) an efficient sliding mode control law can be constructed. [6]

Defining the control effort as

$$u(k) = u_{eq}(k) + u_r(k) \tag{21}$$

where the equivalent control tern is defined as

$$u_{eq}(k) = \left(\begin{array}{l} y_d(k+1) - \dfrac{1}{c_n}(c_1 e_2(k) + c_2 e_3(k) + \cdots + c_{n-1} e_n(k) - \\ \hat{f}(x(k), \bar{y}_d(k)) - \mu s(k)) \end{array} \right) \tag{22}$$

here $\hat{f}(x(k), \bar{y}_d(k))$ is the wavelet approximation of uncertain term $c_n f_n(x(k),u(k))$ and $0 < \mu < 1$.

The robust control term is defined as

$$u_r(k) = \frac{1}{c_n}(-\hat{\gamma}^T(k) z(k) \operatorname{sgn}(s(k))) \tag{23}$$

With the help of the proposed tuning laws presented in the next part of this section, the error term $\tilde{f}(k)$ is reduced to a small arbitrary value which is further attenuated by robust control term $u_r(k)$.

Weight update rules for wavelet network parameters and weight parameters for adaptive approximation of residual term are based on Lyapunov based adaptation methodology and are given as

$$\begin{array}{ll} \hat{\alpha}(k+1) = \alpha(k) + \Delta\alpha(k) & \Delta\alpha(k) = -\tau_1 s(k)\phi(x(k)) \\ \hat{\beta}(k+1) = \beta(k) + \Delta\beta(k) & \Delta\beta(k) = -\tau_2 s(k)\varphi(x(k)) \\ \hat{\gamma}(k+1) = \gamma(k) + \Delta\gamma(k) & \Delta\gamma(k) = -\tau_3 |s(k)| z(k) \end{array} \tag{24}$$

where τ_1, τ_2, τ_3 are the learning rates with positive constants.

Theorem: For the system of the form (13), with sliding surface (17) , if weight parameters are adaptively tuned as per laws proposed in (24) then the wavelet based sliding mode control law (21), (22) and (23) guarantees the convergence of every trajectory of closed loop system to the sliding surface satisfying the inequalities (19) and (20).

V. Proof: Consider a Function of the Form

$$\Delta V(k) = s(k)(s(k+1) - s(k)) + \frac{1}{\tau_1}\tilde{\alpha}^T(k)(\tilde{\alpha}(k+1) - \tilde{\alpha}(k)) + \frac{1}{\tau_2}\tilde{\beta}^T(k)(\tilde{\beta}(k+1) - \tilde{\beta}(k)) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)(\tilde{\lambda}(k+1) - \tilde{\gamma}(k)) \tag{25}$$

Substituting control law $u(k)$ (21), (22) in above equation

$$\Delta V(k) = s(k)(\tilde{f} - \mu s(k) + c_n u_r) + \frac{1}{\tau_1}\tilde{\alpha}^T(k)(\hat{\alpha}(k+1) - \alpha(k)) + \frac{1}{\tau_2}\tilde{\beta}^T(k)(\hat{\beta}(k+1) - \beta(k)) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)(\hat{\gamma}(k+1) - \gamma(k))$$

$$\Delta V(k) = s(k)(\tilde{f}(x(k)) - \mu s(k) + c_n u_r) + \frac{1}{\tau_1}\tilde{\alpha}^T(k)\Delta\hat{\alpha}(k) + \frac{1}{\tau_2}\tilde{\beta}^T(k)\Delta\hat{\beta}(k) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

Substituting $\tilde{f}(x(k))$ (11) and adaptation laws for $\Delta\hat{\alpha}(k)$ and $\Delta\hat{\beta}(k)$ (24) in above equation,

$$\Delta V(k) = s(k)(\varepsilon(x(k)) - \mu s(k) + c_n u_r) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

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$$\le |s(k)||\varepsilon(x(k))| \text{-} \mu s^2(k) + c_n u_r s(k) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

Substituting $|\varepsilon(x(k))|$ (12) in above equation

$$\le |s(k)|\gamma^T z(k) \text{-} \mu s^2(k) + c_n u_r s(k) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

$$\le |s(k)|(\tilde{\gamma}^T(k) + \gamma^{*T}(k))z(k) \text{-} \mu s^2(k) + c_n u_r s(k) + \frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\gamma(k)$$

Substituting u_r and adaptation laws for $\Delta\hat{\gamma}(k)$ (23) in above equation

$$\le \text{-}\mu s^2(k) \tag{26}$$

Therefore $\Delta V(k)$ is negative which implies the convergence of system trajectories to sliding surface and boundedness of all the closed loop signals.

VI. SIMULATION RESULTS

Simulation is performed to verify the effectiveness of proposed wavelet based sliding mode control strategy. Considering a system of the form

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= x_3(k) \\ x_3(k+1) &= \frac{0.62 x_1(k)\sin(2x_1(k))}{10 + x_2^2(k)} + \sin(u(k)) + \frac{0.1 x_2(k)u(k)}{(10 + x_1(k))} \\ y(k) &= x_1(k) \end{aligned} \tag{27}$$

System belongs to the class of discrete time uncertain nonlinear systems defined by (9) with $n=3$.The sampling time T is taken as 0.05 sec . The proposed controller strategy is applied to this system with an objective to solve the tracking problem of system.

The desired trajectory is taken as

$$y_d = \begin{cases} 0.8\,\mathrm{sgn}(\sin 1.3\pi kT) & 0 \le k \le 400 \\ \sin(.5\pi kT) & 400 < k \le 500 \\ \cos(.5\pi kT) & 501 < k \le 600 \\ 0.7\,\mathrm{sgn}(\sin 2\pi kT) & 600 < k \le 800 \\ 0.8\cos(1.5\pi kT) & 800 < k \le 1000 \end{cases} \tag{28}$$

Initial conditions are taken as $x(0) = [1.8, 1.2, 1.5]^T$. Controller parameters are taken as $c = [0.1, 0.075, 0.234]; \mu = 0.1$. Wavelet network used for modeling the uncertainties is constructed by using three dimensional Daubechies wavelet (db3), J is kept 2 with $M_{2_J} = 7$ while N is selected as 5 and translates are made double when resolution is increased by 1.Wavelet parameters for wavelet network are tuned online using the proposed adaptation laws, initial conditions for all the wavelet parameters are set to zero. To avoid chattering $\mathrm{sgn}(s(k))$ is replaced by following saturation function

$$r(k) = \begin{cases} s(k) & |s(k)| \le 0.05 \\ \mathrm{sgn}(s(k)) & |s(k)| > 0.05 \end{cases} \tag{29}$$

Simulation results are shown in Fig.1 and Fig.2. Fig.1 reflects the efficient tracking performance of the proposed controller scheme. Due to fast and efficient learning ability of wavelet network, system response rapidly tracks the desired trajectory with rapidly decaying transient observed during initial phase of the simulation. Tracking efficiency of the proposed scheme is also illustrated by inserting bounded spikes in the desired trajectory. Fig.2 shows tracking error and sliding function for the system under consideration. As observed from the figure tracking error and sliding surface are always close to zero.

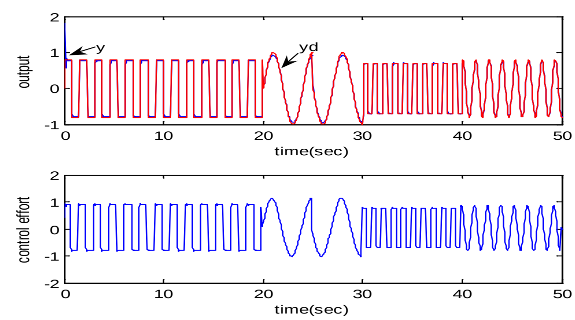

Figure 1 System Output and Control Effort

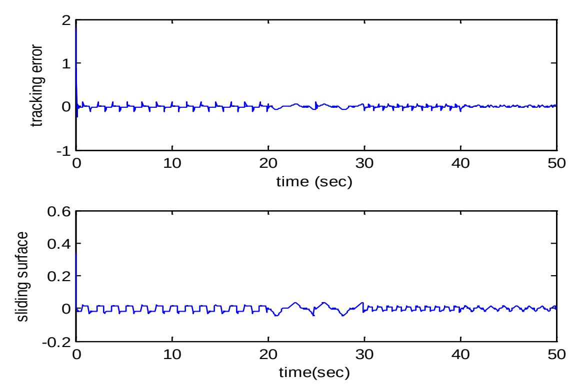

Figure 2 Tracking Error and Sliding Surface

VII. CONCLUSION

A wavelet based sliding mode control scheme is proposed for a class of discrete time uncertain nonlinear systems. Wavelet networks are used for approximating the uncertain system dynamics. Adaptation laws are developed for online tuning of the wavelet parameters. The theoretical analysis is validated by the simulation results.

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