Study of Faulty Perturbed Systems: Application on Flexible Structures

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***Abstract*- An experimental study on controlled faulty perturbed flexible structures is developed using an active mass damper actuator, where the flexible structure is subject to external perturbation and sensor faults. To attenuate the disturbance effects on the flexible structure, we present three robust controllers: one is based on dynamics LMI control technique design, other is an improvement of the first one but adding a chattering term, and the last one is the second one but with the chattering gain adjusted dynamically. According to experiments, where a flexible two level building with active mass damper (by Quanser), external perturbation, and sensor faults, evidence that the proposed LMI controller with chattering term where its gain is dynamically adjusted presents the best closed-loop system behavior.**

***Keywords*- Robust Control; Structural Systems; LMI Techniques; Adaptive Control**

I. INTRODUCTION

In this paper, a study of control strategies is developed for active mass damper systems subject to external perturbation. This is conceptually similar to active mass damper being studied in earthquake mitigation research facilities to reduce damage from earthquakes on high rise buildings [1, 2, 3]. For the purpose of maintaining the seismic response of structures within safety levels, service and comfort limits, the combination of passive base isolators and feedback controllers has been proposed in recent years [1,4]. In these systems, the presence of structural or dynamic faults is relevant.

The Fault Tolerant Control (FTC) is related to the design of a reconfiguring unit which adjusts the structure and the tuning of the controller in order to preserve pre-specified performances also for the faulty system [5]. Many strategies can be followed at this stage. Mechanical reconfiguration, such as switching between redundant hardware or mechanical parameters, adaptive robust control, supervisory switching control, are just a few examples of different strategies which can pursue to deal with faulty plants. Generally speaking, FTC systems can be categorized into two main groups: active and passive. In a common way, FTC systems based on robust control are called passive [5]. This technique is designed considering a set of presumed. Faults modes and the resulting control system performance tend to be conservative. In the literature, passive fault-tolerant control system is also known as reliable control systems or control systems with integrity [4]. However, it is relatively easy to design the controller for the presumed faults because they do not rely on online controller adjustment [6]. On the other hand, if the FTC system uses adaptive control methods that change the parameters or the control objective, the FTC approach is called active (see [7] for details). In this case, some automatic adjustment are done trying to reach the control objectives and the price to pay for the ability to deal with unknown faults is that overall system becomes more complicated [8].

Fig. 1 Quanser shaking Table II

The purpose is to design a control system to counteract the effects of the external disturbance on the structure. Here, a solution is presented using the robust H∞ control theory [9] by the method of linear matrix inequality (LMI) [10, 11, 12] that can be applied to structural control [2]. Then, to improve controller performance, a study is made. First, a swing-up controller is designed, in terms of the sign of the local velocity, where Lyapunov theory is used to prove stability [13]. Also, adaptive controller is designed to improve the swing-up control performance. An experimental shaking table is used to simulate earthquakes exciting the flexible modes of a tall structure [3]. It is composed by a flexible two levels building with active mass damper at the top of the building (see Fig. 1). Only acceleration measurements at level one and two are available, and the position measurement of the cart (the active mass damper) situated at the building top. However, a velocity observer is introduced to define these controllers in terms of the cart position. Experiments are done to study the effectiveness of the LMI-chattering controller when sinusoidal chirp external perturbation and system faults are present.

The paper is structured as follows. Section II presents the mathematical model of the structural system, modeled using Lagrangian formulation. Section III presents the control objective, where the three controllers are presented. In

Sections IV and V, experiments are carried out. Finally, conclusions are stated.

II. STRUCTURAL MODEL

The model is derived using Lagragian formulation, where the dynamic equations are obtained and then a linear model is derived by linearizing about the quiescent (latent) point [3]. The states are defined as:

$$x = \left[x_c x_{f1} x_{f2} \dot{x}_c \dot{x}_{f1} \dot{x}_{f2}\right]^T \quad (1)$$

where x_{fi} is the position of the floor i=1, 2 (floor deflection), and x_c is the cart position (see Figure 1). The linear model about the quiescent point is defined by [3]:

$$\dot{x}(t) = A\,x(t) + B\,u(t) \quad (2)$$

where $u(t)$ is the control input and the matrices are defined as [3]:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 278.43 & -18.69 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & -766.49 & 5.98 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 3.01 & 0 & -0.96 \end{bmatrix}^T \quad (3)$$

The available measurements are x_c, $\ddot{x}_{f1}$ and $\ddot{x}_{f2}$. That means we can introduce as the measured output the variable $y(t) = \left[x_c(t) \quad \ddot{x}_{f1}(t) \quad \ddot{x}_{f2}(t)\right]^T$ and from Systems (2)-(3) the next definition is derived:

$$y(t) = C\,x(t) + D\,u(t) \quad (4)$$

with

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & 766.49 & 5.98 & 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 & -0.96 \end{bmatrix}^T.$$

III. CONTROL DESIGN

In the structure presented in Figure 1, the active damper is located at the top of the building. For this reason, we want to study the effect of external disturbance $w(t) \in L_2$ on the cart position and floor two acceleration, where the cart is located. Therefore, using the H_∞ theory, a performance variable (virtual output $z(t)$ is defined as $z(t) = \left[x_c(t) \quad \ddot{x}_{f2}(t) \quad u(t) \right]$. The state-space representation of System (2)-(4) yields

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) + B_1\,w(t) \\ z(t) &= C_1\,x(t) + D_{12}\,u(t) \\ y(t) &= C\,x(t) + D\,u(t) + D_{21}\,w(t) \end{aligned} \quad (5)$$

Where

$$B_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C_1 = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} 0 & 0 & 0.1 \end{bmatrix}^T$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

Matrix B_1 is defined to take into account that the external perturbation $w(t)$ is produced on the ground. Matrices C_1 and D_{12} are defined to increase the weight of the cart position (where the controller is located) in front of the external perturbation. The above dynamic model satisfies the standard H∞assumptions [5]. An H∞ controller $u(t)$ is designed as a dynamic control strictly proper [6]:

$$K : \begin{cases} \eta(t) = A_k\,\varphi(t) + B_k\,y(t) \\ u(t) = C_k\,\varphi(t) \end{cases} \quad (6)$$

with $\varphi(t) \in R^6$. Considering thevariable $\eta(t)^T = (x(t)^T, \varphi(t)^T)$, the closed-loop Systems (5)-(6) is defined by:

$$\begin{cases} \dot{\eta}(t) = \underbrace{\begin{bmatrix} A & BC_k \\ B_k C & A_k + B_k D C_k \end{bmatrix}}_{A} \eta(t) + \underbrace{\begin{bmatrix} B_1 \\ B_k D_{21} \end{bmatrix}}_{B} w(t) \\ z(t) = \underbrace{\left(C_1 \quad D_{12} C_k\right)}_{E\infty} \eta(t) \\ y(t) = \underbrace{\left(C \quad DC_k\right)}_{E2\infty} \eta(t) + D_{12} w(t) \end{cases} \quad (7)$$

We will design the dynamic control (6) that stabilizes the closed-loop System (7) under L_2 disturbances. Considering a Lyapunov function $V_1(t)$ such that for any nonzero $\eta(t)$ and input $w(t) \in L_2$ the following condition holds (see [10] for details):

$$\frac{d}{dt} V_1(t) + \gamma^{-1} z(t)^T z(t) - \gamma\, w(t)^T w(t) < 0 \quad (8)$$

Then, an H_∞ performance bound for the closed-loop system (7) is ensured.

Definition (H_∞Controller [10]). If there exist matrices A_k,

B_k and C_k such that (8) holds, the control law $u(t)$ (6) is said to be an H_∞ controller for the System (5), that is, the system is internally stable[1] with H_∞ norm less than γ, i.e., $\|z\|_\infty \le \gamma^2 \|w\|_\infty$ for $w \in L_2$.

The H_∞ characterization of the robust control begins by considering the next Lyapunov function:

$$V_1(t) = \eta(t)^T P\, \eta(t) \tag{9}$$

with $P > 0$. Imposing the H_∞ Condition (8), we obtain:

$$(\mathrm{A}\eta + \mathrm{B}w)P\eta + (*) + \gamma^{-1}\eta^T \mathrm{E}_\infty^T \mathrm{E}_\infty \eta - \gamma w^T w < 0.$$

We can rewrite the previous inequality as a matrix by applying the Schur complement [12]:

$$\begin{bmatrix} \mathrm{A}^T P + P\mathrm{A} & 0 & 0 \\ \mathrm{B}^T P & -\gamma & 0 \\ \mathrm{E}_\infty & 0 & -\gamma \end{bmatrix} < 0 \tag{10}$$

This matrix inequality is solvable numerically using LMI-techniques included in the robust control toolbox of Matlab [14]. Given the continuous-time plant (5), the best H_∞ performance γ is computed solving (10), as well as an H_∞controller K in equation (10), that internally stabilizes the plant, yielding a closed-loop gain no larger than γ. These results can be summarized in the next proposition.

Proposition 1 (Dynamic-LMI control). If there exists a matrices A_k, B_k and C_k such that the matrix Inequality (10) is feasible, the control law $u(t)$ in Equation (6) is an H_∞ controller for the system in Equation (5).

A. Swing-Up Component

To improve the performance of the dynamic-LMI controller (6), a swing-up term is added to $u(t)$:

$$u_{ls}(t) = C_k \eta(t) + u_s(t) \tag{11}$$

with

$$u_s(t) = -\delta\, sign(\dot{x}_c(t)) \tag{12}$$

where$\delta$is a positive constant design parameter. Locally, the cart moves as a single degree-of-freedom system with mass *m*:

$$m\ddot{x}_c(t) = u_{chat}(t) + w_c(t) \tag{13}$$

where $w_c \in L_2$ is the local perturbation on the cart. To prove the local stability of System (5) with Controllers (11) – (12), we use Lyapunov theory. First, for System (13), consider Lyapunov function $V_2 = \frac{1}{2} m\dot{x}_c^2$. Then, for the whole system (5), define the Lyapunov function $V = V_1 + V_2$, where V_1 is the H_∞ Lyapunov Function (9), verifying the H_∞ Inequality (8). Asymptotic stability of the unperturbed closed-loop System (5) with (11) – (12) is concluded:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \dot{V}_1 + m\dot{x}_c\ddot{x}_c = \dot{V}_1 - \delta|\dot{x}_c| < 0.$$

B. Adaptive Strategy

Now, our objective is to improve the performance of the Controllers (11) – (12), an adaptive term is introduced to estimate the design parameter δ used in (12):

$$\begin{aligned} u_{la}(t) &= C_k \eta(t) + u_a(t) \\ u_a(t) &= -\delta(t)\ \mathrm{sgn}(\dot{x}_c(t) + \phi(t)) \\ \dot{\delta}(t) &= -\alpha \log(1+\delta(t)) + k_r \frac{1+\delta(t)}{1+\log(1+\delta(t))} |\dot{x}_c(t)| \\ \dot{\phi}(t) &= -\beta_1 \phi(t) - \beta_2 \dot{x}_c(t) \end{aligned} \tag{14}$$

where α, k_r, β_1 and β_2 are positive constant design parameters. This dynamic is based on reference [13] and $\delta(t) > 0$ is ensured for $t > t_0$, if $\delta(t_0) > 0$. With this adaptive law, it was seen experimentally that the control was de-phased. To solve this problem, a dynamic phase $\phi(t)$ is added to center $\dot{x}_c(t)$, with positive constant parameters β_1 and β_2. Locally, the cart moves as a single degree-of-freedom system with mass *m*:

$$m\ddot{x}_c(t) = u_a(t) + w_c(t) \tag{15}$$

where $w_c \in L_2$is the local perturbation on the cart. To prove that the System (5) with Controller (14) is internally stable, we use Lyapunov theory. Consider the $V_1(t)$ (9) and define the next Lyapunov function:

$$V = V_1 + \tfrac{1}{2} m(\dot{x}_c + \varphi)^2 + (1+\delta)\log(1+\delta) \tag{16}$$

The time derivative of V (16) for the unperturbed closed-loop(5) with (14) system is

$$\begin{aligned} \dot{V} &= \dot{V}_1 + m(\dot{x}_c + \phi)(\ddot{x}_c + \dot{\phi}) + \dot{\delta}(1 + \log(1+\delta)) \\ &= \eta^T A^T P \eta + (*) - \delta\, \mathrm{sgn}(\dot{x}_c + \phi)(\dot{x}_c + \phi) + \\ &\quad + m(\dot{x}_c + \phi)(-\beta_1\phi - \beta_2\dot{x}_c) - \alpha \log(1+\delta)(1 + \log(1+\delta)) \\ &\quad + k_r(1+\delta)|\dot{x}_c| - \alpha \log(1+\delta) \\ &\le (k_r - m(\beta_1 + \beta_2)|\phi| - m\beta_2|\dot{x}_c|)|\dot{x}_c| \\ &\le (k_r - m\beta_2|\dot{x}_c|)|\dot{x}_c| \end{aligned}$$

using that $\beta_1 > 0$, $\beta_2 > 0$ and $\delta > 0$. If the state $\dot{x}_c$ verifies:

$$k_r - m\beta_2|\dot{x}_c| \le 0 \Leftrightarrow |\dot{x}_c| \ge \frac{k_r}{m\beta_2} \tag{17}$$

then $\dot{V} \le 0$ and the BIBO-stability of the closed-loop Systems (5), (6) and (14) is concluded. For small cart velocities $\dot{x}_c(t)$,the Lyapunov stability is not guaranteed, but (17) depends on design parameters and can be as small as

[1] Internally stable means that the closed-loop system is asymptotically stable when $w(t)$.

wanted (always limited by experimental devices). In that case, the robustness is verified experimentally.

IV. EXPERIMENTAL SET-UP

To test the obtained controllers against disturbance and faults, their effectiveness are studied experimentally. To compare the performance of the dynamic-LMIcontrol (6) versus the nonlinear control including Swing-Up (12), and adaptive Term (14), two scenarios are implemented: (a) response in front of external perturbation without faults; and (b) response from perturbation when a sensor mails.

Shake Table II is an instrumental shake table developed by Quanser Inc. [3]. The system is comprised of a shake table, a universal power module, a data acquisition card (DAC) along with its external terminal board, and a PC running control software. The PC sends and receives signals through the DAC using WinCon. Designed to simulate earthquakes and evaluate the performance of active mass dampers, the shake table consists of a 1 Hp brushless servo motor driving a lead screw. The lead screw drives a circulating ball nut which is coupled to the 18” °— 18” table (see Fig. 1). The table itself slides on low friction linear ball bearings on 2 ground-hardened shafts. It can drive a 15 Kg. mass at 2.5 g. Maximum travel is ±7cm. In this paper, the external perturbation is a sine chirp wave (Quanser Chirp block in Fig. 2) with increasing frequency from 0.1Hz to 0.7Hz and target time 20s (5s for the first experiment), the total time of the experiments.

Using Matlab’s Robust Control Toolbox [14] to compute (6), the performance index $\gamma = 40.3$ is obtained and the control matrices are:

$$A_k = \begin{bmatrix} -57.99 & -4.89 & -27.66 & 303.85 & -212.17 & 2.12 \\ 0.18 & -3.24 & -1.32 & 52.52 & 4.54 & -0.29 \\ -41.11 & -16.74 & -28.22 & 392.42 & -162.13 & -8.47 \\ 247.39 & 26.78 & 137.73 & -1655.66 & 967.55 & 4.66 \\ 245.41 & 49.72 & 129.82 & -1935.56 & 786.63 & 4.01 \\ 800.71 & 463.07 & 537.60 & -1120.36 & 2231.03 & -99.36 \end{bmatrix}$$

$$B_k = \begin{bmatrix} -0.09 & 1.90 & -0.28 \\ 0.06 & -0.29 & -0.19 \\ -2.87 & 2.50 & -3.33 \\ 14.54 & 1.55 & 16.70 \\ -32.72 & -9.90 & 17.81 \\ 24.16 & -160.28 & 76.91 \end{bmatrix} \qquad (18)$$

$$C_k = [13.13 \quad 1.06 \quad 8.41 \quad -68.68 \quad 72.10 \quad -0.84]$$

To define swing-up controller (11) – (12) and adaptive controller (14), we need $\dot{x}_c$. But we do not have velocities measurements, so we use [15] to approximate $\dot{x}_c$ in (7):

$$\frac{s}{\varepsilon s + 1} \qquad (19)$$

with $\varepsilon = 0.01$. So, the swing-up parameter δ in (12) is found by experimentation:

$$u_s = -0.5\,\mathrm{sgn}(\dot{x}_c) \qquad (20)$$

The adaptive design Parameters (14) are defined as α 10, k_r 20, β_1 β_2 5:

$$\begin{aligned} u_a(t) &= -\delta(t)\,\mathrm{sgn}(\dot{x}_c(t) + \phi(t)) \\ \dot{\delta}(t) &= -10\log(1+\delta(t)) + 20\frac{1+\delta(t)}{1+\log(1+\delta(t))}\left|\dot{x}_c(t)\right| \\ \dot{\phi}(t) &= -5\,\phi(t) - 5\,\dot{x}_c(t) \end{aligned} \qquad (21)$$

To compare the performance of these controllers, an energy index is introduced:

$$J = \frac{\sum \|z(t)\|_2}{\sum \|w(t)\|_2} \qquad (22)$$

This index J is evaluated for all the time interval, in terms of the $z(t)$ $[x_c(t)$ $\ddot{x}_{f2}(t)$ $u(t)]^T$ and external disturbance $w(t)$. Bigger is the value of J, worst is the control performance.

V. EXPERIMENTAL VALIDATION

In this section, we make experiments on a flexible two levels structure [3] to validate the controls defined previously.

A. Comparison of Control Strategy: Nonfaulty System

First of all, let’s make a comparison of the three control strategies: (a) LMI-robust control (6) with (18), (b) chattering control $u_s(t)$ (20), and (c) adaptive control $u_a(t)$ (21). Moreover, we consider also a combination of them: (d) LMI with chattering control $u_{ls}(t)$ (11), and (e) LMI with adaptive control $u_{la}(t)$ (14).

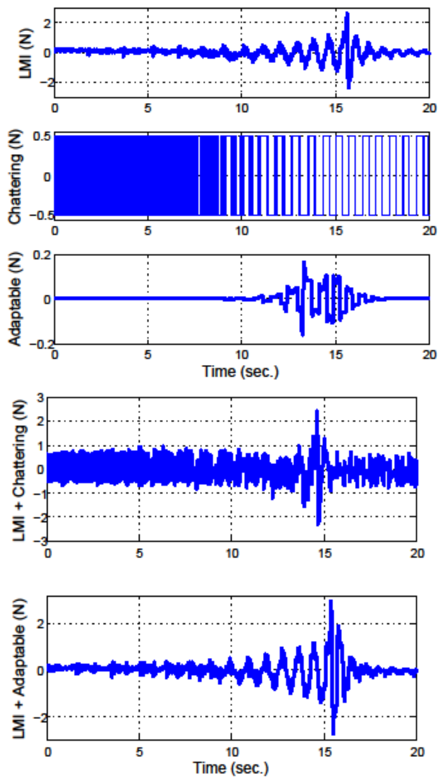


Fig. 3 Time history plot of control laws, under Chirp perturbation

To make this study, we consider the non-faulty system, and only external perturbation is introduced to the system. A priori, Lyapunov stability is proved for LMI control and chattering control. In the case of adaptive control, it appears a restriction: Condition (17) must be imposed. The LMI control law uses measurements $y(t)=\left[x_c(t)\quad \ddot{x}_{f1}(t)\quad \ddot{x}_{f2}(t)\right]^T$ as input, but chattering and adaptive controls only use $\dot{x}_c(t)$ that is not available (we use (22) to evaluate it).

Fig. 2 plots the time history plot of the five strategies, where the adaptive one shows the best performance. The swing-up law presents a chattering effect, due to residual noise. Fig. 3 pictures the time evolution of performance index J (20), where the energy index is worst in the adaptable law (14), suggesting to use the LMI control (6). Fig. 4 presents the time history plot of floor two accelerations where the use of LMI and adaptable combination improves appreciably the behavior.

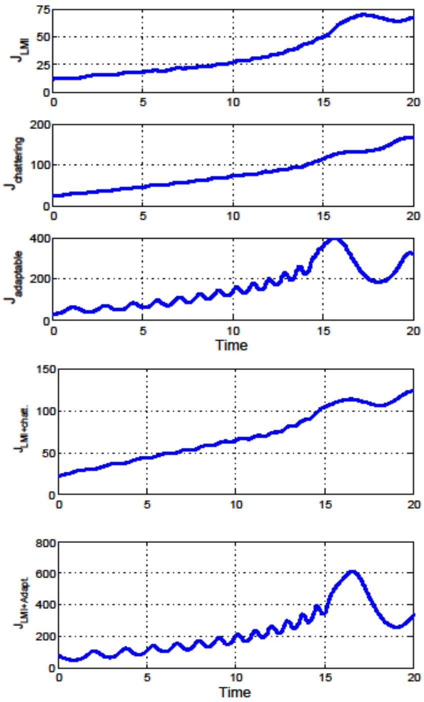
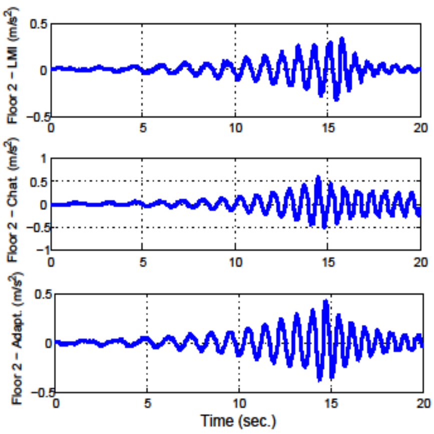


Fig. 4 Time history plot of performance level *J* (20), under Chirp perturbation



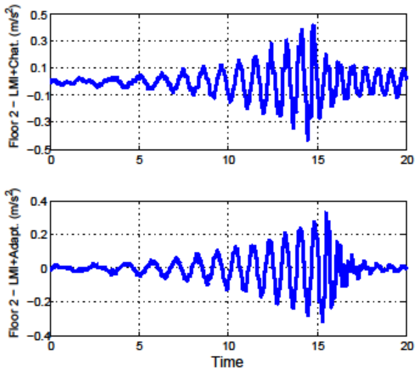


Fig. 5 Floor two acceleration, under Chirp perturbation

So, we have to decide which controller consider. From theoric point of view, only in dynamic-LMI and swing-up controller cases the Lyapunov stability is proved. From Fig. 2 and 3, the swing-up is discarded due to the worst performance. In Fig. 2, the behavior of adaptive law is better than the dynamic-LMI control, but the level performance *J* (22), as pictured in Fig. 3. So, to decide, let's consider the time history plot of the floors accelerations (Fig. 4 and 5): the combination of dynamic-LMI with adaptive law seems to perform correctly.

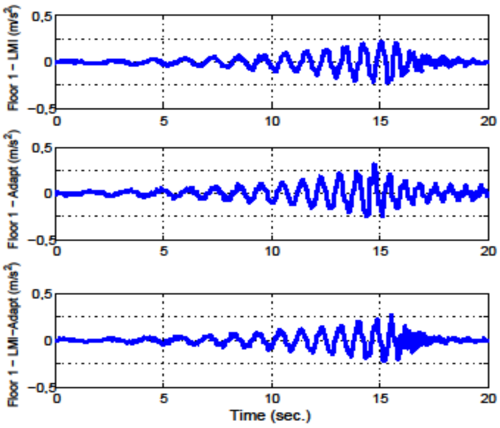


Fig. 6 Floor one acceleration, under Chirp perturbation

Fig. 6 pictures the time history plot of adaptive parameter $\delta(t)$ defined in (21), without system faults. It can be appreciated that the adaptive law is activated under the presence of the perturbation, and returns to zero position when it disappears.

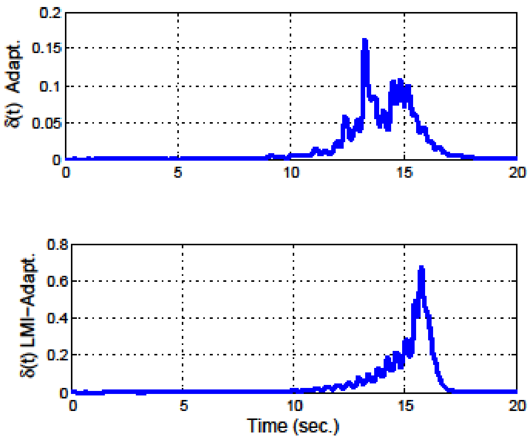


Fig. 7 Time history plot of adaptive parameter $\delta(t)$ defined in (21)

B. *Experiments with Sensor Faults*

We disconnect the sensor from Floors 1 and 2. Fig. 5 pictures the control time history plot of the two control cases under study: dynamic-LMI (5) with (18), and adaptive law combined with LMI control (14) with (21). Again, the experiment takes 20 seconds. The control performance showed in Fig. 7 demonstrates that the adaptive control improves the control performance versus the dynamic-LMI control (6), but the Lyapunov stability is not proved for small cart velocities (17).

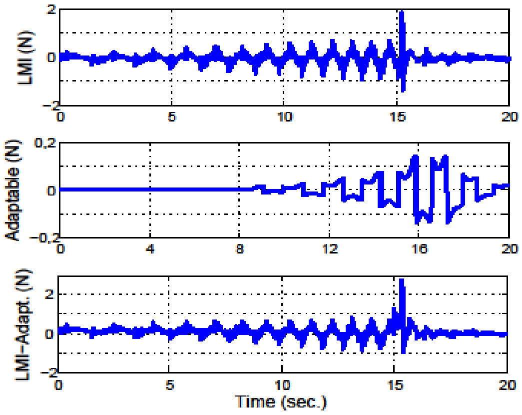


Fig. 8 LMI-control (18), Adaptive control (21) and combined LMI-adaptive control (14), when sensors fault occur

VI. CONCLUSIONS

In this paper, we develop three different controllers for active mass damper systems subject to sinusoidal perturbation: a robust control based on H_∞ theory and designed using linear matrix inequalities (LMI); a swing-up controller depending only on velocity measurements; and nonlinear adaptive controller. According with experiments, where a flexible two levels building with active mass damper and seismically exited was employed, the adaptive law improved controller performance when an external disturbance appears, or when a fault occurs. The adaptive law has the handicap that Lyapunov stability is not proved for small cart velocities. So, designer has to choose between a dynamic H_∞ robust control based on LMI techniques, and an adaptive control that improves the control performance.

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