Modeling of Three-Tank System with Nonlinear Valves Based on Hybrid System Approach

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Abstract- **In this paper we consider three-tank system as a hybrid system with discrete (on/off) valves and then we use Hybrid System's Description Language to acquire a Mixed Logical Dynamical model of the system. In order to obtain a relatively accurate model of three-tank system, instead of using straight line approximation method for linearization of valves in whole space, we use Piecewise Affine method to overcome nonlinearities of valves, so we linearize valves in three different operation stages. By using these piecewise affine models, we produce a mixed logical model of three-tank system with reasonable accuracy which can be used in model based controllers. We provide a comparison between acquired model, hybrid model using straight line approximation method, and nonlinear model of three-tank system through simulation and numerical computation.**

Keywords- Hybrid System; Three-Tank System; Piecewise Affine Method; Mixed Logical Dynamical Model; Hybrid System's Description Language

I. INTRODUCTION

The tank systems are used widely in many articles as a good case study or experimental system, to impose the proposed methods for identification, fault detection, or control purposes [1-7]. There are varieties of tanks system configurations; most attractive configuration is three interacting tanks system, in which system consists three identical tanks that are connected with on/off valves as shown in Figure 1. This configuration has very rich dynamic, good complexity and ready accessibility [1].

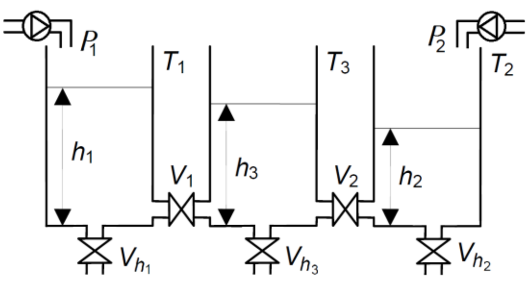


Fig. 1 A typical three-tank interacting connection

In recent years many works done on three tank systems' modeling, here we mention some of notable works which were done on different tank systems' configuration for different purposes.

In [2], Jian Wu and his team employed a model-based approach to achieve the model of system. They first derived the mathematical model of the hybrid system using a hybrid bond graph approach that includes continuous behavior interspersed with discrete mode changes. They considered the pumps in three discrete modes and the valves in two discrete modes. Then all control commands that govern system behavior can be defined as a finite set that includes the valve settings (on and off) and the three pump speeds. At the end they built the three-tank system model as a switching hybrid system (SHS). The shortcoming of their approach is that they consider the pump as discrete input which practically they are continuous inputs.

Naresh Nandola and Sharad Bhartiya in [3] used multiple linearized model to model the nonlinear hybrid system of three spherical interacting tanks system. Each linearized model is a local representation of all locations of hybrid system. They described the nonlinear hybrid system by combining these models using Bayes theorem. Proposed method for modeling the hybrid system causes the optimization problem become a Mixed Integer Nonlinear Programming (MINP) while the mixed logical dynamical model (MLD) optimization problem is a Mixed Integer Quadric Programming (MIQP) which needs more computational effort. The lack of the proposed model is it has a long and complex procedure.

A three cylindrical interacting tank system was used as case study in [4]. Domenico Mignone modeled the system in MLD form of hybrid systems using hybrid system's description language (HYSDEL). The proposed scheme used the MLD model of three tank system and modified it by adding new parameters which were used to detect the fault in system. For overcoming the nonlinearity of the system they used the straight line approximation which causes smaller number of binary variables but at lower accuracy.

Here we use the three tank system showed in Figure 2, which originally has been adopted as a benchmark problem for fault detection algorithms and reconfigurable control [6, 7], to model three-tank system by using piecewise affine method for linearization in order to achieve a model with reasonable accuracy.

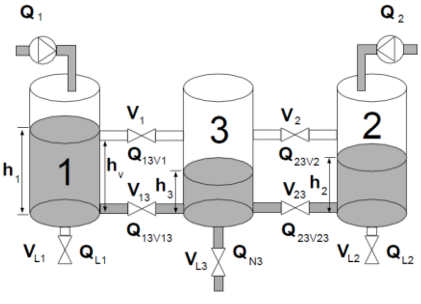


Fig. 2 The three tank system model [4]

The system consists of three identical tanks that are supported with two identical and independent pumps on Tanks' Numbers 1 and 2. The pumps provide the liquid flows Q_1 and Q_2 continuously between 0 to a maximum flow Q_{max}. The tanks interconnected to each other through upper and lower switching valves V_1, V_2, V_{13}, and V_{23} which have only open and close conditions. The liquid levels h_1, h_2, and h_3 in each tanks can be measured with continuous valued level sensors.

In nominal operation, the outflow of system is located at the middle tank; in other hand the valve V_{L3} is open. Other two valves V_{L1} and V_{L2} are close in nominal condition and are used to model failures in system [4, 6, 7].

In this paper, we consider the three tank system as a hybrid system. The modeling is based on Mixed Logical Dynamical model approach for such systems.

The rest of the paper is as follows. In Section 2, we introduce the mathematical model of the three-tank system and use piecewise affine method for linearization. Next, we acquire the MLD model of system in Section 3, also provide simulation and comparison between different methods; all the results are shown in nominal condition. Finally, concluding remarks are drawn in Section 4.

II. MATHEMATICAL MODEL OF THREE-TANK SYSTEM

In this section, we consider the three-tank system's configuration which is depicted in Fig. 2. According to this configuration the continuous-time equations of the system's dynamics are as follow [4]:

$$\dot{h}_1 = \frac{1}{A}(Q_1 - Q_{13V1} - Q_{13V13} - Q_{L1})\,(1-1) \qquad (1.\alpha)$$

$$\dot{h}_2 = \frac{1}{A}(Q_2 - Q_{23V2} - Q_{23V23} - Q_{L2})\,(1-2) \qquad (1.\beta)$$

$$\dot{h}_3 = \frac{1}{A}(Q_{13V1} + Q_{13V13} + Q_{23V2} + Q_{23V23} - Q_{N3})\,(1-3) \qquad (1.\chi)$$

in which, the Q's are flows and A is the cross-sectional area of each tanks.

We postulate that the flow obeys Torricelli's law which expresses the relation between the speed of fluid flowing out of an opening to the height of fluid above the opening. In that case the flow through a lower valve V_{i3} (i=1, 2) is given by:

$$Q_{i3Vi3} = V_{i3} a_z S_{i3} sign(h_i - h_3)\sqrt{|2g(h_i - h_3)|}, i = 1,2 \qquad (2)$$

And similarly, the flow of an upper valve $V_i\,(i=1,2)$ is:

$$\boxed{\begin{aligned} Q_{i3Vi} = V_i a_z S_i sign(\max\{h_v, h_i\} - \max\{h_v, h_3\}) \\ \times\sqrt{|2g(\max\{h_v, h_i\} - \max\{h_v, h_3\})|} \end{aligned}} \qquad (3)$$

The output flow through the valve V_{N3} and fault flows through valves $V_{Li}\,(i=1,2)$ are given as follow:

$$Q_{Li} = V_{Li} a_z S_{Li}\sqrt{2gh_i} \qquad (4-1)$$

$$Q_{N3} = V_{N3} a_z S_{N3}\sqrt{2gh_3} \qquad (4-2)$$

In order to achieve the MLD model of system by using HYSDL, first we should remove nonlinear terms ($sign(x)\sqrt{|x|}$) from Equations (2)-(4). For this reason as showed in Figure 3, instead of using the straight line approximation method we use Piecewise Affine (PWA) approach and divide the space into three separate parts a_1, a_2, a_3. Then we approximate nonlinear term with a linear equation in each spaces (for Equation (4) space is divided into 2 parts) [8].

According to the PWA method and Figure 3, the linear equations of nonlinear terms in each space are as follow:

$$y = \begin{cases} \frac{n\times x_{max}}{(n-1)}(\sqrt{x_{max}} - \sqrt{\frac{x_{max}}{n}})x - \frac{1}{(n-1)}(\sqrt{x_{max}} - \sqrt{\frac{x_{max}}{n}}) + \sqrt{\frac{x_{max}}{n}}, & x \in a_1 \\ (\sqrt{\frac{n}{x_{max}}})x, & x \in a_2 \\ \frac{n\times x_{max}}{(1-n)}(\sqrt{\frac{x_{max}}{n}} - \sqrt{x_{max}})x + \frac{1}{(1-n)}(\sqrt{\frac{x_{max}}{n}} - \sqrt{x_{max}}) - \sqrt{\frac{x_{max}}{n}}, & x \in a_3 \end{cases} \qquad (5)$$

where n is an arbitrary number between 0 and x_{max}; however by choosing a proper value for n, we can increase the accuracy of the MLD model.

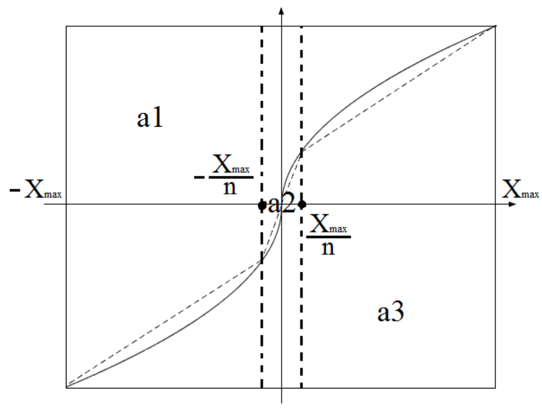


Fig. 3 PWA's linearization scheme: nonlinear term $sign(x)\sqrt{|x|}$ (solid line), approximation of $sign(x)\sqrt{|x|}$ (dash-line), space's borders in $\pm\frac{x_{max}}{n}$ (bold dash-line)

After linearizing nonlinear equations, we should acquire the discrete-time equations of system as follow:

$$x(k+1) = (T_S H + 1)x(k) + T_S G \qquad (6)$$

where T_S is the sampling time. H and G are proper time-invariant matrixes.

In next section according to the linear discrete-time equations of three-tank system, which were obtained in this section, and using HYSDEL we will acquire the MLD model of system.

III. MIXED LOGICAL DYNAMICAL MODEL OF THREE-TANK SYSTEM

There are many different methods for modeling of hybrid system, namely Piecewise affine, Mixed logical dynamical, Linear complementarity, and Max-min-plus-scaling [9].

Among these models, MLD method is more common and has less complexity than other methods which models hybrid system using two linear Equations (7-1, 7-2) and one linear Inequality (7-3). MLD approach has the following structure [10]:

$$x(k+1) = Ax(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k) \qquad (7\text{–}1)$$

$$y(k) = Cx(k) + D_1 u(k) + D_2 \delta(k) + D_3 z(k) \qquad (7\text{–}2)$$

$$E_2 \delta(k) + E_3 z(k) \le E_1 u(k) + E_4 x(k) + E_5 \qquad (7\text{–}3)$$

where $x = [x_c \quad x_l]^T$, $x_c \in R^{nc}, x_l \in \{0,1\}^{nl}$ is the system state, $y = [y_c \quad y_l]^T, y_c \in R^{pc}, y_l \in \{0,1\}^{pl}$ and $u = [u_c \quad u_l]^T$, $u_c \in R^{mc}, u_l \in \{0,1\}^{ml}$ are the output and the input signal, respectively. $\delta \in \{0,1\}^{r_l}$ denotes the logical auxiliary variables, and $z \in R^{rc}$ denotes the continuous auxiliary variables. The indexes c and l represent the type of the variables and respectively mean continuous and logical. A, B_i, C, D_i and E_i are proper and time-invariant matrixes.

The MLD Model (6) of three-tank system is obtained by HYSDEL [11, 12]. The parameters of system and the constraints which are imposed on MLD model are given in the Table 1.

TABLE I MODEL PARAMETERS OF THE THREE-TANK SYSTEM [4]

symbol	Value (MKS)	Meaning
A	0.0154	tank cross-section
a_z	1	flow correction term
S_h	2.10e-5	cross-section of valve V_i
g	9.81	gravity constant
h_v	0.3	height of valves V_1, V_2
$h_{\max}$	0.62	maximum water level in each tank
$Q_{i\max}$	10e-4	maximum inflow through pump
T_s	5	sampling time

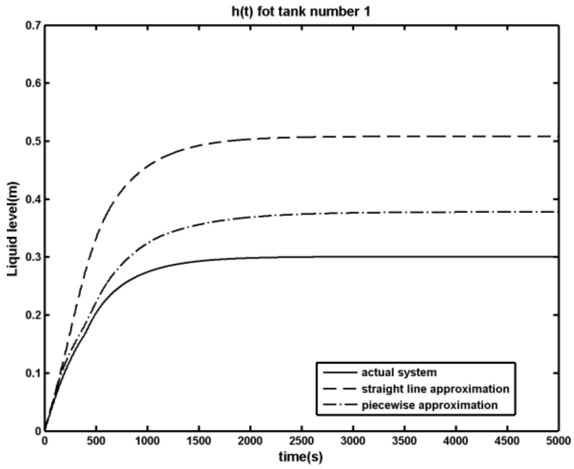
The MLD model of the system by using PWA method for linearization has the following properties:

- 3 states (3 continuous: $h_1(k)$, $h_2(k)$, $h_3(k)$ -0 binary), 6 inputs (2 continuous: $Q_1(k)$, $Q_2(k)$ -4 binary: $V_i(k)$, $V_{i3}(k)$, $i = 1,2$).
- 24 continuous auxiliary variables, 30 binary auxiliary variables, 178 mixed-integer linear inequalities.
- Sampling time is 5s.

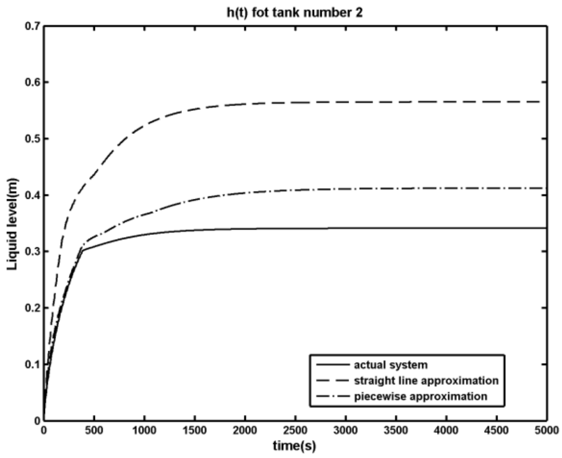
For a comparison, the MLD model of the system by using straight line approximation has the following properties:

- 3 states (3 continuous: $h_1(k)$, $h_2(k)$, $h_3(k)$ -0 binary), 6 inputs (2 continuous: $Q_1(k)$, $Q_2(k)$ -4 binary: $V_i(k)$, $V_{i3}(k)$, $i = 1,2$).
- 11 continuous auxiliary variables, 9 binary auxiliary variables, 78 mixed-integer linear inequalities.
- Sampling time is 5s.

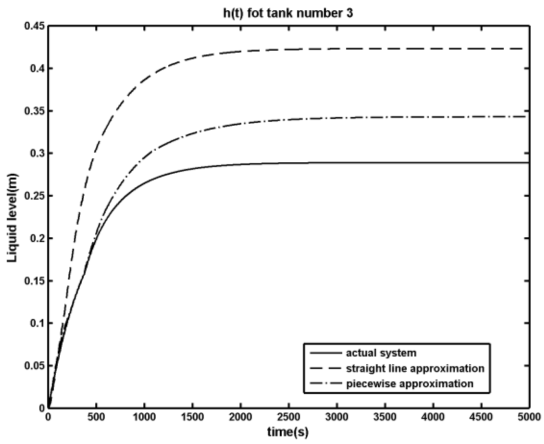
In following, we provide a comparison between these two models and actual system under similar conditions. The results of simulation are shown in Figure 4.



(a) liquid level in tank Number 1



(b) liquid level in tank Number 2



(c) liquid level in tank Number 3

Fig. 4 Comparsion between MLD model using straight line approximation (dash-line), MLD model using piecewise affine (dash-dot-line), and actual system (solid-line)

Now for a numerical comparison between accuracy of the two MLD models, the coefficient of determination is used to show the fitness of each model. The coefficient of determination ranges from 0 to 1 which a zero value means the model has no relation with the actual system, and a one value means that the model describes the actual system perfectly; in other hand the biggest coefficient of determination is better. The most general definition of the coefficient of determination is shown in Equation (8) [13].

$$R^2 \equiv 1 - \frac{SS_{err}}{SS_{err} - SS_{reg}} \tag{8}$$

where R^2 is the coefficient of determination. SS_{err} and SS_{reg} are the residual sum of squares and the explained sum of squares, respectively, which have following equations:

$$SS_{err} = \sum_i (y_i - \hat{y}_i)^2 \tag{9–1}$$

$$SS_{reg} = \sum_i (\hat{y}_i - \bar{y})^2 \tag{9–2}$$

where $\hat{y}_i$ and y_i are actual system data and estimated data from the MLD model, respectively. $\bar{y}$ is the mean of the actual system data.

The results of computation of the coefficient of determination for both MLD models are provided in Table 2.

TABLE II COMPARISON BETWEEN THE COEFFICIENT OF DETERMINATION OF TWO MLD MODELS

	PWA	**Straight line**	**Improvement percentage**
h_1	0.7146	0.5618	27,2
h_2	0.819	0.6301	30
h_3	0.7858	0.591	33

By looking at Figure 4 and Table 2, it is obvious that using piecewise affine approach for linearization has more accuracy than straight line approximation method. So for increase in accuracy of the MLD model of three-tank system we should increase the number of partitions of the piecewise affine approach. However according to the properties of two MLD models which were acquired, we should consider that this increase in partitions' number causes increase in the number of auxiliary variables and mixed-integer linear inequalities and consequently makes more computational effort for using of this MLD model for designing a controller for the three-tank system.

IV. CONCLUSION

As we showed in this paper using MLD model method for modeling three-tank system as a hybrid system is very simple, systematic, and relatively accurate. This accuracy comes from the methods which we use for linearization of nonlinear terms but increase in accuracy also causes increase in computational effort for designing a proper controller based on the MLD model. So we should make a balance between accuracy and computational cost according to our purposes and equipments.

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