Guaranteed Anti-Sway Operation of an Overhead Crane: A Cascaded Backstepping Approach

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***Abstract*- This paper presents a novel real-time robust nonlinear control law exploring a cascaded backstepping control approach to deal with the benchmark control problem of anti-sway operation of an overhead crane with restricted travel. The overall control algorithm has been partitioned into two separate controllers: one for position control of the crane, and the other for the anti-sway operation of the payload in spite of the motion constraints and external disturbances. Here, the cascaded backstepping method has been explored to ensure the guaranteed anti-sway operation of the overhead crane. The effectiveness of the proposed control law has been verified by experimental studies in hard real-time. The experimental results show that the proposed control laws are quite efficacious in achieving the overall control objectives.**

Keywords- Real-Time Control; Cascaded Backstepping; Underactuated System; Lyapunov Based Stability; Overhead Crane; Stabilization Zone and Swinging Zone

I. INTRODUCTION

Design of a robust control law for uncertain and perturbed nonlinear system is a challenging problem for control system engineers. This actually triggered widespread research towards the development of robust nonlinear control algorithms during the last decade. Consequently, three powerful robust nonlinear control methodologies like robust adaptive control, sliding mode control, and backstepping control have emerged. Backstepping is Lyapunov method based versatile robust control design approach for nonlinear systems and it ensures the convergence of the regulated variables to zero [1-3]. The main idea of the backstepping method is that the overall dynamic system is partitioned into two series cascaded subsystems. Therefore, the states of the first subsystem are the control variables for the second. In backstepping approach, at first, the desired control input for the second subsystem is computed and then the control input for the first subsystem is computed to realize the desired state, which is the desired control input for the second subsystem. However, in case of multivariable control problem, quite often the system structure is not in lower triangular form (strict feedback form or semi strict feedback form). If it is possible to represent the state space structure of such system in *block strict feedback form* by means of some mathematical manipulation, it is possible to address the multivariable control problems using backstepping technique. This technique is also known as *cascaded backstepping* [4-5].

The control problem of a crane is a good example of inherently nonlinear underactuated dynamical system [6-12] and has been an interesting benchmark problem in the nonlinear control engineering. It has drawn attention of the control system professionals for the last few decades, therefore, it has been widely used as a testing platform for the development of the advanced control algorithms . It is quite interesting to note that the position control problem of the crane and anti-sway operation of the payload is a multivariable nonlinear control problem and the system structure is not in lower triangular form. Therefore, in this paper we have proposed a novel real-time robust nonlinear control law employing a cascaded backstepping control approach anti-sway operation of the crane. As the movement of the cart is vital for the position control problem, similarly the swing of the payload is restricted due to safety reason. In the present control problem we have one control input variable and two output variables (angular position of the payload in space and the position of the crane on the rail). Moreover, as our control problem has one control input and the number of degree-of-freedom to be controlled are two; the control methodology proposed in this work is similar to the control problem of an under-actuated mechanical system [6-14].

Here, we have exploited two zone control theory [15-16] after partitioning the entire problem domain into the stable tracking zone and the anti-swing zone. We have used two different control laws: one for the anti swing zone and the other for the stable tracking zone. The controller for the anti-sway zone stabilizes the angular motion of payload within a restricted zone and after that the controller initiates the stable-tracking zone control algorithm which ensures proper position control of the crane. The effectiveness of the proposed control laws has been established by experimental studies in hard real-time. The experimental results show that this proposed two-stage controller is able to maintain the anti-sway operation The rest of the paper is organized as follows. In Section II we briefly describe about the physical system and its approximate mathematical model. Section III describes the actual design backstepping controller for the anti-swing zone. The results of real time experiments have been presented in Section IV. Section V concludes the work.

II. THE PHYSICAL SYSTEM AND ITS APPROXIMATE MODEL

The picture of the physical system, which is actually our experimental set up of the cart- pole system, is shown in Fig. 1. It is the Digital Pendulum System manufactured by Feedback Instruments Ltd, UK, bearing model number 33-005-PCI. A dc motor is performing the role of the actuator. We can control the movement of the cart by controlling the pulse input of the dc motor.. Advantech PCI1711 card has been used for A/D and D/A conversion of the measurement and control signals respectively. The state equation of the physical system is given below (here we are using Newtonian method of modeling)

Fig. 1 Experimental setup

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = x_4$$
$$\dot{x}_3 = \frac{a(F - T - \mu x_4^2 \sin x_2) + l \cos x_2 \mu g \sin x_2}{J + \mu l \sin^2 x_2} \qquad (1)$$
$$\dot{x}_4 = \frac{l(F - T - \mu x_4^2 \sin x_2) + \mu g \sin x_2}{J + \mu l \sin^2 x_2}$$

Where, $a = l^2 + \dfrac{J}{M+m}$ and $\mu = l(M+m)$. The state vector of the system is consists of four state variables: $X = [x_1, x_2, x_3, x_4]^T$, where x_1 is the cart position, x_2 is the payload angle between its vertical upright position and the axis of the centre of mass of the payload and measured counter-clockwise from the cart, x_3 is the cart velocity, and x_4 is the angular velocity of the pendulum. Here, the control objective is not only position control of the crane but also assuring a proper anti-swing operation of the payload.

III. DESIGN OF THE CASCADED BACKSTEPPING CONTROLLER

Choice of control variables plays a crucial role in the design of the control system. In this application we want to control the two output variables viz. angle of the pendulum in the space and the position of the cart on the rail simultaneously with a single control input signal (u) applied on the cart. Here, we are utilizing the concept of *cascaded backstepping* to address this problem. One can judicially select a regulated variable as a function of all four state variables, in which the angle of the pendulum and angular velocity (i.e., x_2 and x_4) constitute the dynamics of the input subsystem block, and the displacement and velocity of the cart (i.e., x_1 and x_3) represent the dynamics of the outer block. Now, if it is possible to design a controller which ensures the convergence of the regulated variable towards zero then it is possible to control all the four state variables in desired manner. Now we can employ the two zone control technique, where we will design one control law with enhanced steady state performance for the stable tracking zone and another control law for the anti-swinging zone (Fig. 2). Now there are numerous control techniques available for stable tracking zone of the over head crane which has been explored by the several researchers in their research articles [3, 5 & 6]. Now main goal of this research is to divide the total operating region of a control in two separate zones and develop a novel controller for anti-sway operation of the payload in Antiswing zone using the concept of cascaded backstepping.

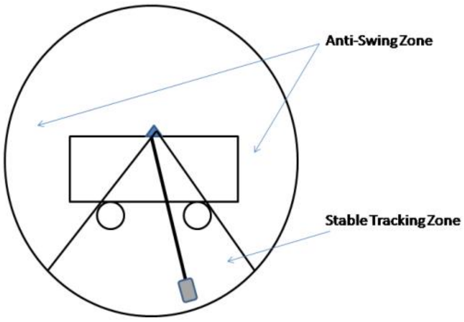

Fig. 2 Two zone control concept

A. *Controller Design for Anti-Swing Zone*

In practical experiment the controller for stable tracking zone is able to maintain the stability of the overall system within 0.3 radians. The controller which is designed for stable tracking zone cannot ensure the proper anti-sway operation in anti-swing zone. For the anti-swing zone, we have designed a nonlinear cascaded backstepping controller that can initiate proper anti-sway operation of the payload such that the angular movement of the payload should restricted inside the stable tracking zone. In case of anti-sway operation the prime concern is the angular deviation of the payload hence we choose angular displacement of the payload as our primary regulated variable

$$. \; z_3 = x_2 - k_2\left(x_1 + l x_3 \cos x_2 - a x_4\right) \qquad (2)$$

K_2 is a positive constant, to be chosen at the time of design. The derivative of z_3 is computed as

$$\dot{z}_3 = \dot{x}_2 - k_2\left(\dot{x}_1 + l\dot{x}_3 \cos \dot{x}_2 - a\dot{x}_4\right) \qquad (3)$$

The stabilizing function α_2 for the swinging operation is chosen as:

$$\alpha_2 = -c_3 z_3 + k_3\left(x_3 + \frac{\mu g l^2 \cos x_2^2 \sin x_2 - \mu a g \sin x_2}{J + \mu l \sin^2 x_2}\right) \qquad (4)$$

Here, c_3 is a positive design constant. So the dynamic error variable z_4 in this case can be defined as

$$z_4 = (1 + k_3 l x_3 \sin x_2) x_4 - \alpha_2 \quad (5)$$

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$$z_4 = (1 + k_3 l x_3 \sin x_2) x_4 - \alpha_2 \quad (6)$$

Now let us calculate the dynamics of the regulated variable z_4 as follows:

$$\dot{z}_4 = (1 + k_3 l x_3 \sin x_2)\dot{x}_4 + [1 + k_3 l \dot{x}_3 \sin x_2 + (1 + k_3 l x_3 \cos x_2 \dot{x}_2)] x_4 - \dot{\alpha}_2 \quad (7)$$

The actual control input u appears in the last equation and after simplification we get the following expression for u:

$$u = \Gamma_1(\bar{X})x_1 + \Gamma_2(\bar{X})x_2 + \Gamma_3(\bar{X})x_3 + \Gamma_4(\bar{X})x_4 + \Gamma_5(\bar{X}) \quad (8)$$

Where

$$\Gamma_1(X) = k_2 d_4 h_1 h_7, \; \Gamma_2(X) = -d_4 h_1 h_7$$

$$\Gamma_3(X) = h_7 [k_2 l h_1 d_4 h_3 - afk_2 + h_6 flh_3 + flh_3 + d_3 k_2 h_1 - k_2 l h_1 h_4 h_5 - k_2 \mu g l h_2^2 + k_2 \mu l^2 h_4 h_5 h_2]$$

$$\Gamma_4(X) = h_7 [-d_3 h_1 - h_1 d_3 h_6 - a d_4 h_1 k_2 + afh_6 + k_2 a \mu h_2^2 h_5 l - k_2 \mu g l^2 h_2 h_4 + k_2 \mu g l^2 h_3^3 - 2\mu^2 l^3 h_1^{-1} h_3 h_4^2 + 2ak_2 \mu^2 g l h_1^{-1} h_2 h_4 - k_2 a \mu g h_3 + k_2 \mu g l^2 h_2 h_4]$$

$$\Gamma_5(X) = h_7 [\mu l h_4 h_5 - \mu g h_2 - d_3 k_2 \mu a g h_2 + k_2 \mu g l h_4 - k_2 \mu a h_2 h_5 + k_2 \mu g l^2 d_3 h_4 h_3]$$

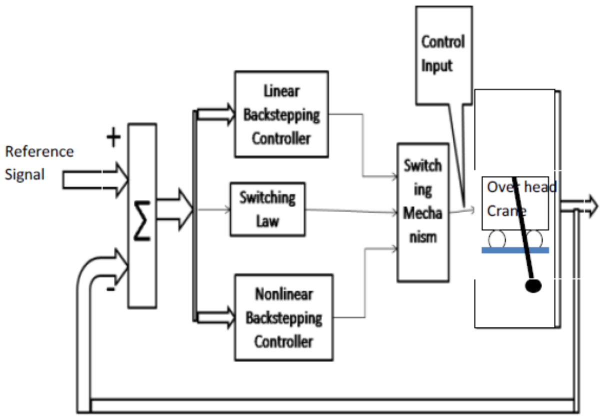

Fig. 3 Block diagram of the controller

Where $h_1 = J + \mu l \sin^2 x_2$, $h_2 = \sin x_2$, $h_3 = \cos x_2$, $h_3 = \cos x_2$, $h_4 = \sin x_2 \cos x_2$, $h_5 = x_4^2$, $h_6 = k_2 l h_2 x_3$ and $d_3 = c_3 + c_4$, $d_4 = c_3 c_4 + 1$ and c_3, c_4 are positive design constant. The block diagram of the composite controller has been shown in Fig. 3.

B. *Stability Analysis*

For stability analysis of the proposed controller let us first consider the Lyapunov function V as shown below.

$$V = \frac{1}{2} z_3^2 + \frac{1}{2} z_4^2 \quad (9)$$

Now using control law of (8) it can be shown that the derivative of the above Lyapunov function become:

$$\dot{V} = -c_3 z_3^2 - c_4 z_4^2 \quad (10)$$

The fact that $\dot{V}_1 \le 0$ from (10) implies that $V(t) \le V(0)$ and therefore z_3 and z_4 are bounded.

Now define the following new function

$$N(t) = c_1 z_3^2 + c_2 z_4^2 \quad (10)$$

Now, integrating (21) gives

$$\begin{aligned} V(t) &= V(z_3(0), z_4(0)) + \int_0^t \dot{V}(\tau) d\tau \\ &= V(z_3(0), z_4(0)) - \int_0^t N(\tau) d\tau \end{aligned} \quad (11)$$

Thus,

$$\int_0^t N(\tau) d\tau = V(z_3(0), z_4(0)) - V(t) \quad (12)$$

Considering $\dot{V} \le 0$ and $V(t) > 0$ the following results can be easily derived:

$$\lim_{t \to \infty} \int_0^t N(\tau) d\tau < \infty \quad (13)$$

To use Barbalat's lemma, let us check the uniform continuity of $\dot{V}(t)$. The derivative of $\dot{V}(t)$ is

$$\ddot{V}(t) = 2[c_1 z_3 \dot{z}_3 + c_2 z_4 \dot{z}_4] \quad (14)$$

This shows that $\ddot{V}(t)$ is bounded, because z_3 & z_4 are bounded. Therefore $\dot{V}(t)$ is uniformly continuous. Through Barbalat's lemma, it can be shown that z_3 and z_4 converge to zero as $t \to \infty$. For anti-swing zone we can pursue stability analysis in similar manner.

Now according to the definition of z_3 (Equation (2)) it can be concluded that the boundedness of z_3 depends on the state variables of the crane. Similarly definition of z_4 reveals the fact that z_4 is the difference between virtual control and the stabilizing function. Consequently, asymptotic convergence of z_3 and z_4 to zero automatically implies the asymptotic convergence of all the state variables to their desired values.

IV. EXPERIMENTAL RESULTS

We have validated the proposed control algorithm in the Digital Pendulum System manufactured by Feedback Instruments Ltd, UK, bearing model number 33-005-PCI. The total experimental setup is shown in Fig. 1. The experiment has been performed for 100 seconds. The following design parameters are chosen for the swing up and stabilization controller: k_1=1, k_2 = 0.1, c=0.05, d_3=20 and d_4=100. As already mentioned, we have used Matlab® Simulink environment and Real-time Windows Target to implement and test the proposed control algorithm in real-time. The control algorithm has been developed as a Simulink model and then it is passed through the "Build" operation to create all executable files that are necessary for the real-time implementation. To demonstrate the efficacy of the proposed ant-sway control algorithm the payload was perturbed using an external disturbance signal (applied on t=44 sec); but it has also been observed that the controller bring back the payload within its stable tracking zone within a few seconds. The dynamic responses of the payload are shown in Fig. 4(a)–(b) respectively for angle and angular velocity.

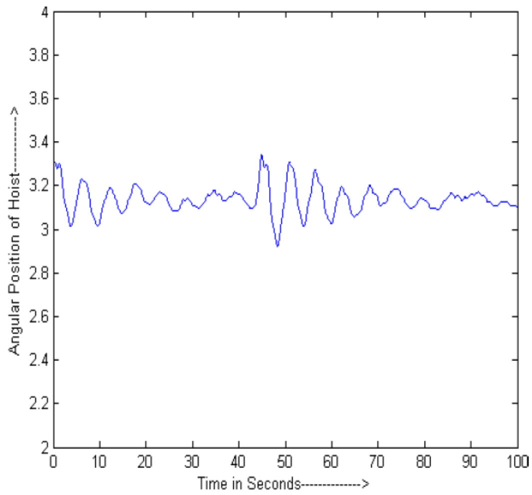

Fig. 4(a) Angular variation of payload

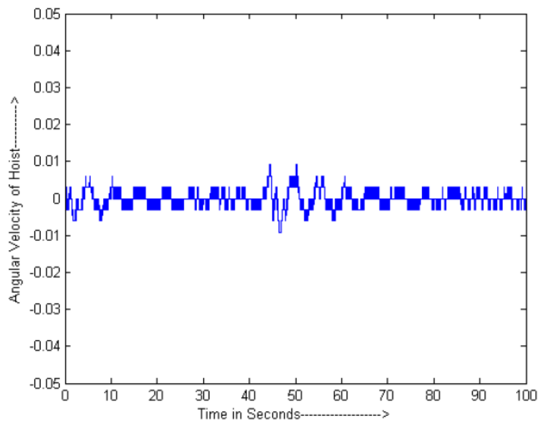

Fig. 4(b) Angular velocity of payload

The cart position and the velocity of the cart are respectively shown in Fig. 5(a)-(b). Figure 6 shown the variation of Input voltage with time, it can be observed that the voltage level never reached its predefined safety limits.

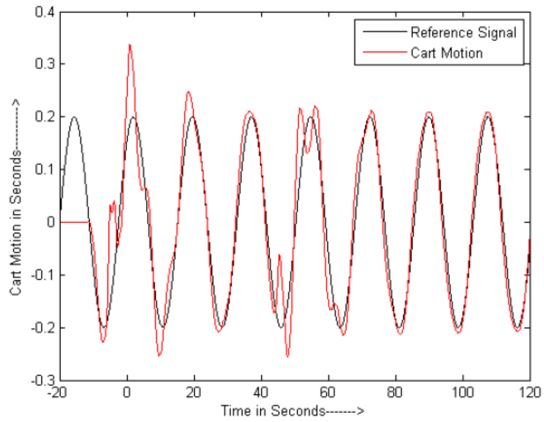

Fig. 5(a) Tracking performance of the overhead crane

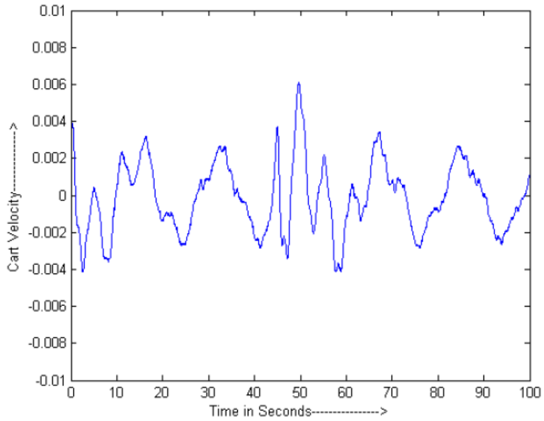

Fig. 5(b) Translational velocity of the overhead crane

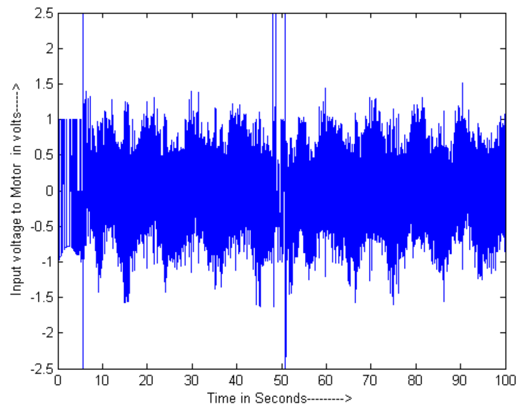

Fig. 6 Variation of voltage with time

V. CONCLUSIONS

Control of a crane is a popular benchmark problem for the development and test of nonlinear control law. In this paper we have presented a complete control strategy for an overhead crane control problem after partitioning the entire problem domain into two zones: anti-swing zone and stable tracking zone. However, the controller designed for stable tracking zone cannot cover the anti-swing zone. Hence, a nonlinear cascaded backstepping controller has been designed subsequently, which is based on Lyapunov theory. This controller takes care of proper anti-swing operation of the payload. Moreover, the controller can initiate the anti-swinging operation of the payload in such a manner that the movement of the payload becomes restricted inside the stable tracking zone. The experimental results reveal that the proposed comprehensive controller exhibits an excellent tracking ability and anti-swing operation of the crane even when it is subjected to some external impact disturbances.

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