Global Chaos Synchronization for WINDMI and Coullet Chaotic Systems Using Active Control

Sundarapandian Vaidyanathan [*1], Suresh Rasappan [2]

Research and Development Centre [*1], Department of Mathematics[2], Vel Tech Dr. RR & Dr. SR Technical University
#42, Avadi- Alamathi Road, Avadi, Chennai-600062. Tamilnadu, INDIA

[*1] sundarvtu@gmail.com; [2] mrpsuresh83@gmail.com

***Abstract*- In this paper, global chaos synchronization problem is investigated for WINDMI (J.C. Sportt, 2003) and Coullet (P. Coullet, et al., 1979) chaotic systems using active feedback control. Our theorems on synchronization for WINDMI and Coullet chaotic systems are established using Lyapunov stability theory. The controller design can be divided into two steps: the first one needs the derivation of control Lyapunov function and the second step involves using existing control Lyapunov function to synchronize the chaotic system. The active control method is effective and convenient to synchronize the chaotic systems. Mainly this technique gives the flexibility to construct a control law. Numerical simulations are also given to illustrate and validate the synchronization results derived in this paper.**

Keywords- Chaos; Synchronization; Active Control; WINDMi system; Coullet System

I. INTRODUCTION

Dynamics systems described by nonlinear differential equations can be strongly sensitive to initial conditions. This phenomenon is known as deterministic chaos, which means that the mathematical description of the system is deterministic but behaviour of the system is unpredictable. The synchronization of chaotic system was first researched by Yamada and Fujisaka [1] with subsequent work by Pecora and Carroll [2], [3]. The synchronization of chaos is one way of explaining sensitive dependence on initial conditions [4], [5]. The problem of chaos synchronization is to design a coupling between the two systems such that the chaotic time evaluation becomes ideal. The output of the response system asymptotically follows the output of the drive system i.e. the output of the drive system controls the response system.

The synchronization for chaotic systems has been widespread to the scope, such as generalized synchronization [6], phase synchronization [7], lag synchronization, projective synchronization [8], generalized projective synchronization [9, 10, 11, 12] and even anti-synchronization. The property of anti-synchronization establishes a predominating phenomenon in symmetrical oscillators, in which the state vectors have the same absolute values but opposite signs [13, 14, 15]. A variety of schemes for ensuring the control and synchronization of such systems have been demonstrated based on their potential applications in various fields including chaos generator design, secure communication [16, 17], physical systems [18], chemical reaction [19], ecological systems [20], information science [21], energy resource systems [22], ghostburster neurons [23], bi-axial magnet models [24], neuronal models [25, 26], IR epidemic models with impulsive vaccination [27] and predicting the influence of solar wind to celestial bodies [28], etc. So far a variety of impressive approaches have been proposed for the synchronization of the chaotic systems such as the OGY method [29], sampled feedback synchronization method [30], time delay feedback method [31], adaptive design method [32, 33, 34], sliding mode control method [35, 36, 37], active control method [38, 39] and backstepping control design [40, 41] etc.

In this paper, active control design approach is proposed. The controller design can be divided into two steps. The first one needs the derivation of control Lyapunov function and the second step involves using an existing control Lyapunov function to synchronize the chaotic systems. This approach is systematic and guarantees the global synchronization of the WINDMI (J. C. Sportt, [42]) and Coullet (P. Coullet et. al., [43]) chaotic systems. This paper is organized as follows. In Section II, the methodology of chaotic synchronization by active control method is given. In Section IV, the chaos synchronization of two identical WINDMI chaotic systems is discussed. In Section V, the chaos synchronization of two identical Coullet chaotic systems is discussed. In Section VI, the chaos synchronization of WINDMI and Coullet chaotic systems is discussed. Section VII gives the conclusions of this paper.

II. PROBLEM STATEMENT AND OUR METHODOLOGY

In general, the two dynamic systems in synchronization are called the master and slave system respectively. A well designed controller will make the trajectory of slave system track the trajectory of the master system.

Consider the master system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \rightarrow R^n$ is the nonlinear part of the system. The system (1) is considered as the *master* or *drive* system.

Consider the slave system with the controller $[u_1, u_2, u_3, ...u_n]^T$ described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state vector of the slave system, B is the $n \times n$ matrix of the system parameters, $g: R^n \to R^n$ is the nonlinear part of the slave system and $u \in R^n$ is the active controller of the slave system. If $A = B$ and $f = g$, then x and y are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two different chaotic systems. The Chaotic Systems (1) and (2) depend not only on state variables but also on time t and the parameters.

We define the synchronization error as

$$e = y - x \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = By - Ax + g(y) - f(x) + u \tag{4}$$

The synchronization error system controls a controlled chaotic system with control input $[u_1, u_2, u_3, \ldots u_n]$. Thus the active feedback control so as to stabilize the Error Dynamics (4) for all initial conditions $e(0) \in R^n$, i.e. $\lim_{t\to\infty} \|e(t)\| = 0$ for all initial conditions $e(0) \in R^n$.

Theorem: The Chaotic Systems (1) and (2) are globally exponentially synchronized with active control $u = \mu(x, y, e)$, where $e = y - x$ is an error and $\mu = R^n \to R^n$ is continuous vector function with x, y and e as its arguments.

Proof: The active control design uses Lyapunov function methodology for establishing the synchronization of Master System (1) and Slave System (2). By the Lyapunov function methodology, a candidate Lyapunov function is taken as

$$V(e) = e^T P e \tag{5}$$

where P is a $n \times n$ positive definite matrix. Note that $V: R^n \to R^n$ is a positive definite function by construction. It is assumed that the parameters of the master and slave systems are known and that the states of both systems (1) and (2) are measurable. If a controller u can be found such that

$$\dot{V}(e) = -e^T Q e \tag{6}$$

where Q is a positive definite matrix, then $\dot{V}: R^n \to R^n$ is a negative definite function. Hence, by Lyapunov stability theory [44], the Error Dynamics (4) is globally exponentially stable and hence the Condition (5) will be satisfied for all initial conditions $e(0) \in R^n$. then the states of the Master System (1) and the slave system (2) are globally exponentially synchronized. ■

III. SYSTEM DESCRIPTION

A. *The WINDMI System*

The WINDMI (J.C. Sportt, [42]) system is a complex driven-damped dynamical system. The WINDMI system describes as the energy flow through the solar wind magnetosphere- ionosphere system. The dynamics of the chaotic WINDMI system is described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_3 - x_2 + b - e^{x_1} \end{aligned} \tag{7}$$

where x_1, x_2, x_3 are state variables and a, b are positive real constants. The WINDMI System (7) is chaotic when the parameter values $a = 0.7$ and $b = 2.5$ and the chaotic attractor as shown in Figure 1.

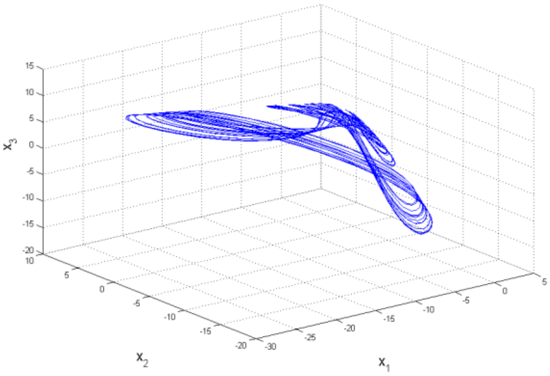

Fig. 1 Chaotic attractor of WINDMI system

B. *The Coullet System*

The Coullet (P. Coullet et al, [43]) chaotic system, proposed by Coullet and Arneodo. The Coullet chaotic system is one of the paradigms of chaotic system and it includes a simple cubic part and three positive parameters. The dynamics of the chaotic Coullet system is described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - cx_3 - x_1^3 \end{aligned} \tag{8}$$

where x_1, x_2, x_3 are state variables and a, b, c are positive real constants. For the Coullet Chaotic System (8), the parameter values are taken as those which result in chaotic behaviour of the system. When $a = 0.7$, b=3.5 and c=1, the chaotic attractor as shown in Figure 2.

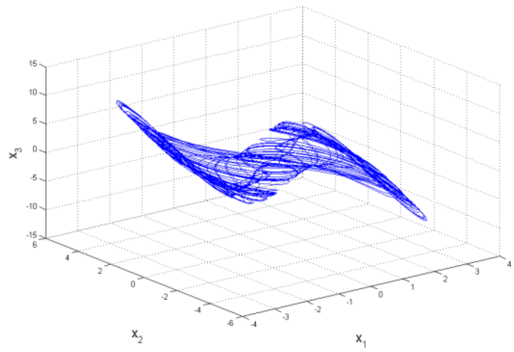

Fig. 2 The chaotic attractor of coullet system

IV. SYNCHRONIZATION OF TWO IDENTICAL WINDMI SYSTEMS USING ACTIVE CONTROL

In this section, the active control method is applied for the synchronization of two identical WINDMI (J.C. Sportt, 2003) chaotic systems [42]. The WINDMI system is taken as the master system, which is described by the equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_3 - x_2 + b - e^{x_1} \end{aligned} \tag{9}$$

where x_1, x_2, x_3 are state variables and a, b are positive real constants.

The WINDMI system (2007) is also taken as the slave system, which is described by the equations

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_3 + u_2 \\ \dot{y}_3 &= -ay_3 - y_2 + b - e^{y_1} + u_3 \end{aligned} \tag{10}$$

where $u = [u_1, u_2, u_3]^T$ is the active controller to be designed so as to synchronize the states of the identical WINDMI Systems (9) and (10).

The synchronization error is defined by

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3 \tag{11}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1 \\ \dot{e}_2 &= e_3 + u_2 \\ \dot{e}_3 &= -ae_3 - e_2 - e^{y_1} + e^{x_1} + u_3 \end{aligned} \tag{12}$$

Theorem 1. *The identical WINDMI Chaotic Systems (9) and (10) are exponentially and globally synchronized for any initial conditions with the active controller* u *defined by*

$$\begin{aligned} u_1 &= -e_2 - k_1 e_1 \\ u_2 &= -e_3 - k_2 e_2 \\ u_3 &= ae_3 + e_2 + e^{y_1} - e^{x_1} - k_3 e_3 \end{aligned}$$

Proof. We introduce the active feedback control to design the controller u, as long as these feedback stabilize System (10) converge to zero as the time $t \to \infty$.

The candidate Lyapunov function is taken as

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{13}$$

which is positive definite on R^3. A simple calculation gives

$$\begin{aligned} \dot{V}(e) &= e_1 e_2 + e_1 u_1 + e_2 e_3 \\ &\quad + e_2 u_2 + e_3(-ae_3 - e_2 - e^{y_1} + e^{x_1}) + e_3 u_3 \end{aligned} \tag{14}$$

We define

$$u_1 = u_{1a} + u_{1b}, \; u_2 = u_{2a} + u_{2b}, \; u_3 = u_{3a} + u_{3b} \tag{15}$$

We choose

$$\begin{aligned} u_{1a} &= -e_2, \; u_{1b} = -k_1 e_1 \\ u_{2a} &= -e_3, \; u_{2b} = -k_2 e_2 \\ u_{3a} &= ae_3 + e_2 + e^{y_1} - e^{x_1}, \; u_{3b} = -k_3 e_3 \end{aligned} \tag{16}$$

Substitution of (16) into (15) yields

$$\begin{aligned} u_1 &= -e_2 - k_1 e_1 \\ u_2 &= -e_3 - k_2 e_2 \\ u_3 &= ae_3 + e_2 + e^{y_1} - e^{x_1} - k_3 e_3 \end{aligned} \tag{17}$$

Substitution of (17) into (14) yields

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{18}$$

which is a negative definite function on R^3 since $k_1, k_2, k_3 > 0$.

Thus, by Lyapunov stability theory [44], then the Error Dynamics (12) is globally exponentially stable. ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB. For the WINDMI Chaotic System (9), the parameter values are taken as those which result in chaotic behavior of the system.

When $a = 0.7$ and $b = 2.5$ and the chaotic attractor as shown in Figure 1.

The initial values of the Master System (9) are taken as

$x_1(0) = 0.641,\; x_2(0) = 0.518$ and $x_3(0) = 0.925$,

while the initial values of the slave system (10) are taken as

$y_1(0) = 0.125,\; y_2(0) = 0.834$ and $y_3(0) = 0.153$

Figure 3 shows that the synchronization between the states of the Master System (9) and the Slave System (10) and Figure 4 shows that the synchronization error between the states of the Master System (9) and the Slave System (10).

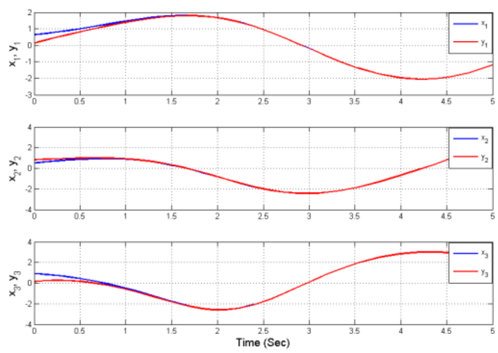

Fig. 3 Synchronization of the identical WINDMI systems

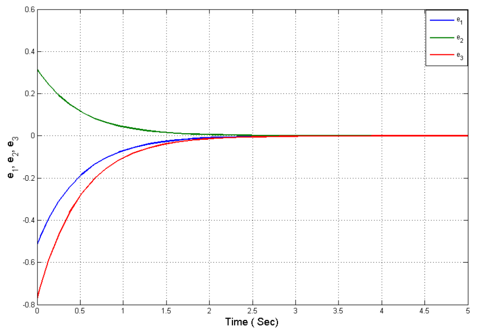

Fig. 4 Synchronization error of the identical WINDMI systems

V. SYNCHRONIZATION OF TWO IDENTICAL COULLET SYSTEMS USING ACTIVE CONTROL

In this section, the active control method is applied for the synchronization of two identical Coullet (P. Coullet et., al., 1979) chaotic systems [43].

The Coullet system (2007) is taken as the master system, which is described by the equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - cx_3 - x_1^3 \end{aligned} \quad (19)$$

where x_1, x_2, x_3 are state variables and a, b, c are positive real constants.

The Coullet system (2007) is also taken as the slave system, which is described by the equations

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_3 + u_2 \\ \dot{y}_3 &= ay_1 - by_2 - cy_3 - y_1^3 + u_3 \end{aligned} \quad (20)$$

where $u = [u_1, u_2, u_3]^T$ is the active controller to be designed so as to synchronize the states of the identical Coullet Systems (19) and (20).

The synchronization error is defined by

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3 \quad (21)$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1 \\ \dot{e}_2 &= e_3 + u_2 \\ \dot{e}_3 &= ae_1 - be_2 - ce_3 - y_1^3 + x_1^3 + u_3 \end{aligned} \quad (22)$$

Theorem 2. *The identical Coullet Chaotic Systems (19) and (20) are exponentially and globally synchronized for any initial conditions with the active controller* u *defined by.*

$$\begin{aligned} u_1 &= -e_2 - k_1 e_1 \\ u_2 &= -e_3 - k_2 e_2 \\ u_3 &= -ae_1 + be_2 + ce_3 + y_1^3 - x_1^3 - k_3 e_3 \end{aligned}$$

Proof. We introduce the active feedback control to design the controller u, as long as these feedback stabilize System (22) converge to zero as the time $t \to \infty$.

The candidate Lyapunov function is taken as

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \quad (23)$$

A simple calculation gives

$$\begin{aligned} \dot{V}(e) &= e_1 e_2 + e_1 u_1 + e_2 e_3 \\ &\quad + e_2 u_2 + e_3(ae_1 - be_2 - ce_3 - y_1^3 + x_1^3) + e_3 u_3 \end{aligned} \quad (24)$$

We define

$$u_1 = u_{1a} + u_{1b}, \; u_2 = u_{2a} + u_{2b}, \; u_3 = u_{3a} + u_{3b} \quad (25)$$

We choose

$$\begin{aligned} &u_{1a} = -e_2, \; u_{1b} = -k_1 e_1 \\ &u_{2a} = -e_3, \; u_{2b} = -k_2 e_2 \\ &u_{3a} = -ae_1 + be_2 + ce_3 + y_1^3 - x_1^3, \; u_{3b} = -k_3 e_3 \end{aligned} \quad (26)$$

Substitution of (26) into (25) yields

$$\begin{aligned} u_1 &= -e_2 - k_1 e_1 \\ u_2 &= -e_3 - k_2 e_2 \\ u_3 &= -ae_1 + be_2 + ce_3 + y_1^3 - x_1^3 - k_3 e_3 \end{aligned} \quad (27)$$

Substitution of (27) into (24) yields

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (28)$$

which is a negative definite function on R^3 since $k_1, k_2, k_3 > 0$

Thus, by Lyapunov stability theory [44], then the Error Dynamics (22) is globally exponentially stable. ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB. For the Coullet Chaotic System (19), the parameter values are taken as those which result in chaotic behaviour of the system. When $a = 0.7$, b=3.5 and c=1 , the chaotic attractor as shown in Figure 2.

The initial values of the master system (19) are taken as

$$x_1(0) = 0.125, x_2(0) = 0.625 \text{ and } x_3(0) = 0.925,$$

while the initial values of the slave system (20) are taken as

$$y_1(0) = 0.945, y_2(0) = 0.032 \text{ and } y_3(0) = 0.112$$

Figure 5 shows that the synchronization between the states of the Master System (19) and the Slave System (20) and Figure 6 shows that the synchronization error between the states of the Master System (19) and the Slave System (20).

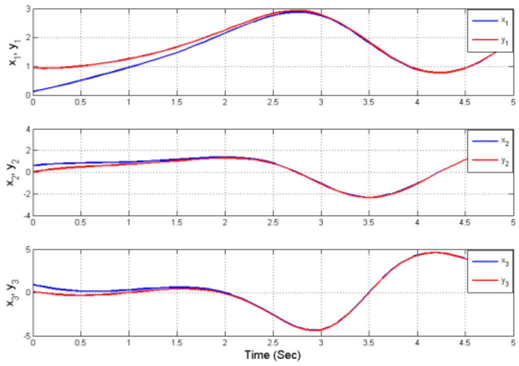

Fig. 5 Synchronization of the identical coullet systems

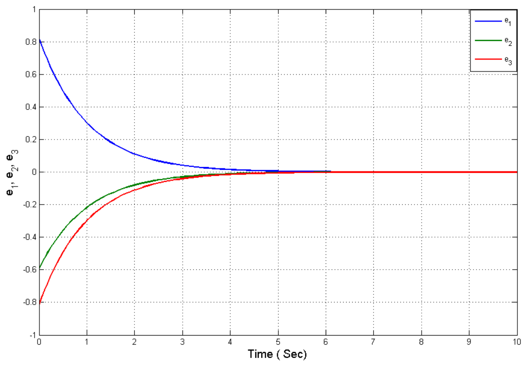

Fig. 6 Synchronization error of the identical coullet systems

VI. SYNCHRONIZATION OF WINDMI AND COULLET CHAOTIC SYSTEMS

In this section, the active control method is applied for the synchronization of two different chaotic systems described by WINDMI system [42] as the *master* or *drive* system and the Coullet system [43] as the *slave* or *response* system.

The dynamics of the WINDMI system, taken as the master system, is described by

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= -ax_3 - x_2 + b - e^{x_1}\end{aligned} \qquad (29)$$

where a and b are positive real constants.

The dynamics of the Coullet system, taken as the slave system, is described by

$$\begin{aligned}\dot{y}_1 &= y_2 + u_1\\ \dot{y}_2 &= y_3 + u_2\\ \dot{y}_3 &= \alpha y_1 - \beta y_2 - \gamma y_3 - y_1^3 + u_3\end{aligned} \qquad (30)$$

wher $\alpha,\ \beta,\ \gamma$ are positive constants and $u = [u_1, u_2, u_3]^T$ is the active controller to be designed so as to synchronize the states of the different Chaotic Systems (29) and (30).

The synchronization error is defined by

$$e_1 = y_1 - x_1,\quad e_2 = y_2 - x_2,\quad e_3 = y_3 - x_3$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= e_2 + u_1\\ \dot{e}_2 &= e_3 + u_2\\ \dot{e}_3 &= \alpha y_1 - \beta y_2 - \gamma y_3 - y_1^3 + ax_3 + x_2 - b + e^{x_1} + u_3\end{aligned} \qquad (31)$$

Theorem 3. *The WINDMI Chaotic System (27) and the Coullet Chaotic System (28) are exponentially and globally synchronized for any initial conditions with the active controller* u *defined by*

$$\begin{aligned}u_1 &= -e_2 - k_1 e_1\\ u_2 &= -e_3 - k_2 e_2\\ u_3 &= -\alpha y_1 + \beta y_2 + \gamma y_3 + y_1^3 - ax_3 - x_2 + b - e^{x_1} - k_3 e_3\end{aligned}$$

Proof. We introduce the active feedback control to design the controller u, as long as these feedback stabilize system (29) converge to zero as the time $t \to \infty$.

The candidate Lyapunov function is taken as

$$V(e) = \frac{1}{2}e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \qquad (32)$$

A simple calculation gives

$$\begin{aligned}\dot{V}(e) = {} & e_1e_2 + e_1u_1 + e_2e_3 + e_2u_2\\ & + e_3(\alpha y_1 - \beta y_2 - \gamma y_3 - y_1^3 + ax_3\\ & + x_2 - b + e^{x_1} + u_3)\end{aligned} \qquad (33)$$

We define

$$u_1 = u_{1a} + u_{1b},\ u_2 = u_{2a} + u_{2b},\ u_3 = u_{3a} + u_{3b} \qquad (34)$$

We choose

$$\begin{aligned}& u_{1a} = -e_2,\ u_{1b} = -k_1 e_1\\ & u_{2a} = -e_3,\ u_{2b} = -k_2 e_2\\ & u_{3a} = -\alpha y_1 + \beta y_2 + \gamma y_3 + y_1^3 - ax_3 - x_2 + b - e^{x_1},\\ & u_{3b} = -k_3 e_3\end{aligned} \qquad (35)$$

Substitution of (35) into (34) yields

$$\begin{aligned}u_1 &= -e_2 - k_1 e_1\\ u_2 &= -e_3 - k_2 e_2\\ u_3 &= -\alpha y_1 + \beta y_2 + \gamma y_3 + y_1^3 - ax_3 - x_2 + b - e^{x_1} - k_3 e_3\end{aligned} \qquad (36)$$

Substitution of (36) into (33) yields

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (37)$$

which is a negative definite function on R^3 since $k_1,\ k_2,\ k_3 > 0$

Thus, by Lyapunov stability theory [44], then the Error Dynamics (31) is globally exponentially stable. ■

Numerical Simulation:

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB.

For the WINDMI Chaotic System (29), the parameter values are taken as those which result in chaotic behavior of the system. When $a = 0.7$ and $b = 2.5$ and the chaotic attractor as shown in Figure 1 and the Coullet Chaotic System (30), the parameter values are taken as those which result in chaotic behavior of the system. When $\alpha = 0.7,\ \beta = 3.5$ and $\gamma = 1$, the chaotic attractor as shown in Figure 2.

The initial values of the Master System (29) are taken as

$$x_1(0) = 0.567,\ x_2(0) = 0.876 \text{ and } x_3(0) = 0.234,$$

while the initial values of the slave system (28) are taken as

$$y_1(0) = 0.984,\ y_2(0) = 0.327 \text{ and } y_3(0) = 0.790.$$

Figure 7 shows that the synchronization between the states of the Master System (29) and the Slave System (30) and Figure 8 shows that the synchronization error between the states of the Master System (29) and the Slave System (30).

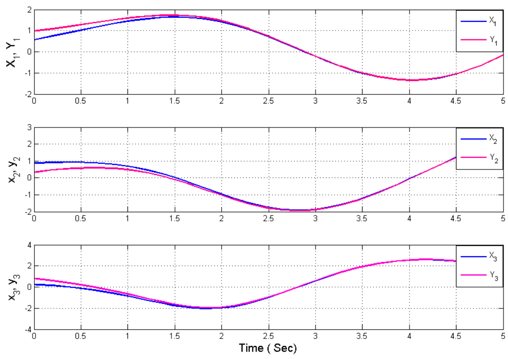

Fig. 7 Synchronization of the WINDMI and Coullet systems

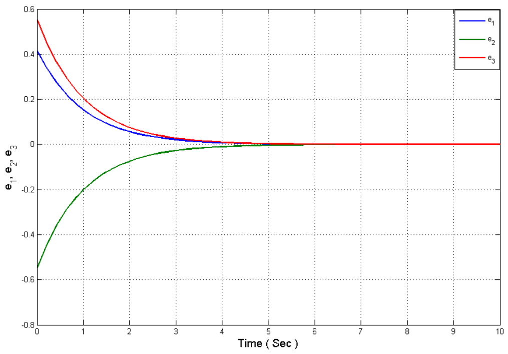

Fig. 8 Synchronization error of the WINDMI and Coullet systems

VII. CONCLUSIONS

In this paper, active control method has been applied to achieve global chaos synchronization for WINDMI and Coullet chaotic systems. Since the Lyapunov exponents are not required for these calculations, the active control design is very effective and convenient to achieve global chaos synchronization. Numerical simulations have been given to illustrate and validate the effectiveness of the active control based synchronization schemes of the WINDMI and Coullet chaotic systems.

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Dr. V. Sundarapandian earned his Doctor of Science degree in Electrical and Systems Engineering from Washington University, St. Louis, USA) in 1996.

He is the Professor and Dean at the Research and Development Centre in Vel Tech Dr. RR & Dr. SR Technical University, Chennai, Tamil Nadu, India.

He is the Editor-in-Chief of several International Journals on Control Engineering, Computer Science, Information Technology and Computational Science. His research interests are Linear and Nonlinear Control Systems, Nonlinear Dynamical Systems and Bifurcations, Chaos and Control, Optimal Control, Soft Computing, Mathematical Modeling. He has delivered many Key Note Lectures on Nonlinear Control Systems, Modern Control Theory, Dynamical Systems and Chaos, Scientific Computing, MATLAB and SCILAB.

Mr. R. Suresh is currently pursuing Ph.D. degree in Mathematics from Vel Tech Dr. RR & Dr. SR Technical University, Tamil Nadu, India under the guidance of Dr. V. Sundarapandian. He is working as a Lecturer in Department of Mathematics at Vel Tech Dr. RR & Dr. SR Technical University since 2008. His research interests are Chaos and Control.