Control of a Magnetorheological Fluid Vibration Damping System via Feedback Linearization and Suboptimal Design

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Abstract- **The development of control systems for a two degree-of-freedom vibration suppression system using a magnetorheological (MR) fluid damper is the subject of this paper. It is assumed that system encounters impulsive disturbance forces. The objective is to use the (MR) fluid damper for the position control and vibration suppression of the payload. The control force is generated by regulating the electric current to the damper. Two control systems, based on (i) the dynamic inversion (feedback linearization) method and (ii) the state-dependent Riccati equation (SDRE) approach, for the position control of the payload and vibration suppression are derived. The dynamic inversion method yields an asymptotically stable linear second-order position error dynamics of the payload, and accomplishes vibration suppression. The SDRE design approach provides a sub-optimal control law which accomplishes asymptotic stabilization of the origin in the state space. The SDRE method considers control constraint in the design process, and uses a nonlinear quadratic performance index for minimization. Simulation results are obtained in the presence of impulsive force on the system. It is shown that in the closed-loop system, both the control systems are effective in the position regulation and vibration suppression in the system.**

Keywords- Magnetorheological Fluid; Damping; Vibration Suppression; Feedback Linearization; Sub-Optimal

I. INTRODUCTION

System components under shock and vibration are usually undesirable and the isolation or suppression of these undesirable phenomena is the main concern for enhancement of lifetime of shock and vibration prone systems. The existing techniques primarily use passive and active devices for shock isolation and vibration suppression. Passive devices featuring elastomeric materials, and hydraulic and frictional dampers provide design simplicity and low cost. However, performance limitations are inevitable because stiffness and damping elements are not controllable in response to external environments. Additional improvement in desired performance can be obtained by using active measures by means of external actuators. Active devices have the ability of suppressing disturbances in wider frequency range. However, systems employing active devices have instability problems due to unmodeled dynamics and nonlinearities as well as actuator and sensor failures because they have the ability to inject energy into the target systems. Also, active methods require large power sources, many sensors, servo valves and sophisticated control logic. A compromise between passive and active devices has been developed in the form of semi-active devices. Semi-active control systems combine the best features of both the passive and active control systems, offering the reliability of passive devices, yet maintaining the versatility and adaptability of fully active devices [1]. Recent work by several researchers has indicated that semi-active control systems, when appropriately implemented, achieve significantly better results than passive control systems; and may even outperform fully active control systems, demonstrating significant potential for controlling shock and vibration of mechanical systems to a wide variety of dynamic loading conditions [1-6].

Magnetorheological materials, including magnetorheological fluids, magnetorheological elastomer (MRE) and magnetorheological form (MRF) are promising smart materials in engineering due to their real time controllable mechanical property to the applied magnetic field. MRF is liquid and operates in the post-yield regime, while MRE is solid and operates in the pre-yield regime. Normally, they are composed of ferrous particles at the scale of several microns, and carrier with low permeability.

Magnetorheological (MR) fluids consist of solid particles suspended in a carrier liquid. When a magnetic field is applied, the particles are polarized and form chains between electrodes. The fluid becomes semi-solid and exhibits viscoplastic behavior. The transition from liquid to rheological equilibrium is attained in the order of milliseconds resulting in construction of MR dampers with high bandwidth [7]. This feature provides simple, quiet, rapid response interfaces between electronic controls and mechanical systems. When compared to electrorheological (ER) fluids, MR fluids have superior propoerties, including an order of magnitude higher yield stress, typically 50-100 kPa, lower power requirement and a much wider temperature range of operation.

MR elastomer is cured under strong magnetic field. That is, the mixture of ferrous particles and liquid-state elastomer solidifies gradually under strong magnetic field. Therefore, the particle-formed chain-like structures parallel to the applied magnetic field during the curing process are embedded in the elastomer matrix. If the magnetic field is applied parallel to such chain-like structures (this is a requirement for the application of MRE), owing to the magnetic interaction between the particles, the modulus or stiffness of MRE will be changed.

MR/ER dampers exhibit highly nonlinear and hysteretic force-velocity response which is the main hindrance for the design of effective control strategies. So in order to achieve desired control performance an accurate damper model should be used. Different techniques have been developed in literature to model the behavior of the MR/ER dampers. Basically, two types of models have been investigated: non-parametric and parametric models. In [8], a nonparametric approach is presented to model a small ER damper that operates under shear mode by assuming that the damper force could be written in terms of Chebychev polynomials. This approach is extended to model the ER damper in [9, 10]. A neural network model to emulate the dynamic behavior of MR dampers is developed by [11]. However, the non-parametric damper models are quite complicated. A simple Bingham plastic model [12, 13] gives a good description of postyield force behavior of ER or MR damper and accurately accounts for energy dissipation and force versus displacement characteristics. However, the transition from preyield to postyield is discontinuous and the hysteretic behavior cannot be described. The hysteretic Bingham plastic model [14, 15], which can accurately capture the postyield and hysteretic preyield force of MR damper, was proposed through the modification of Bingham plastic model. The hysteresis biviscous model [16-18] is composed of several piecewise continuous models, so it is complicated, but well describes the hysteresis behavior of MR dampers. The nonlinear viscoelastic plastic model [19-21] has several parameters associated with preyield and postyield mechanisms, and shows good accuracy in predicting the damping force of the MR damper. The Bouc-Wen model [1, 7] for ER/MR dampers consists of strong nonlinear differential equations. However, this model is accurate in predicting the damping force in both preyield and postyield regions. The polynomial model proposed in [22] captures the field dependent hysteretic behavior of MR dampers. The hydro-mechanical model proposed in [23] utilizes a differential equation, but accounts for physical parameters such as inertia, damping, yield force and compliances associated with MR or ER dampers.

Although the discovery of ER and MR fluids dates back to the 1940s, only recently have they been applied to engineering applications. To date, a number of ER fluid dampers have been investigated for civil engineering structural vibration control applications [10-12, 24, 25]. In [26], the semi-active control of a vibrating system is achieved by means of an ER damper using Lyapunov stability theory. To accommodate state measurement errors the proposed control scheme is combined with fuzzy control concept. In [27], two control schemes, one based on minimizing the rate of change of energy of the body and the other based on considerations of Lyapunov stability theory are proposed for attenuation of undesirable vibrations. A full car suspension system featuring ER dampers was proposed and its feedback control performance was presented via hardware in-the-loop simulation in [28]. In order to obtain a favorable control performance of the ER suspension system subjected to parameter uncertainties and external disturbances, a sliding mode controller is designed.

A number of experimental studies have been conducted to evaluate the usefulness of MR dampers for vibration reduction under wind and earthquakes. In Refs. [1-3, 29-31], MR dampers are used to reduce the seismic vibration of model building structures. Refs. [4-6] incorporated an MR damper with a base isolation system such that the isolation system would be effective under both strong and moderate earthquakes. A Lyapunov based controller is designed to protect large civil structures using MR dampers [32]. Skyhook and sliding mode controllers for semi-active MR damper shock isolation systems have been proposed in [33]. In [34], a sliding mode controller, robust against parameter variations and external disturbances, was formulated to attenuate the acceleration and displacement of the landing gear system. A semi-active controller based on Lyapunov design [35] for vibration suppression has the disadvantage of excessive chattering of the control input. So far, most of the published researches for MR fluid-based systems are largely concentrated on the modeling issues of nonlinear hysteric phenomenon of the system using experimental data. In this research work, the main focus is directed to the semi-active feedback control design of the MR damper system.

The contribution of this paper lies in the design of two control systems for a two degree-of-freedom magnetorheological (MR) fluid vibration suppression system. The control force is generated by regulating the electric current to the damper. It is assumed that the system is subjected to impulsive disturbance forces. The objective is to use the MR fluid damper for the position control of the payload and vibration suppression in the system. Two control systems, based on (i) the dynamic inversion (feedback linearization) method and (ii) the state-dependent Riccati equation (SDRE) approach, for the control of the payload, perturbed by the external impulsive disturbance forces, are derived. The feedback linearizing control system yields an asymptotically stable linear second-order position error dynamics of the payload if the control input is unconstrained, and accomplishes vibration suppression in the system. But later control constraint is introduced for simulation. Then based on the state-dependent Riccati equation (SDRE) approach, a sub-optimal control system is derived. In the SDRE design process, control input constraint is introduced to maintain the input current within its feasible range. For the design of the controller, a judiciously chosen nonlinear quadratic performance index in an extended state space is considered. A suboptimal control law is obtained by solving the state-dependent Riccati equation. In the closed-loop system, the designed controller achieves asymptotic stabilization of the origin in the state space. Simulation results are obtained for the system perturbed by impulsive disturbance force. It is shown that in the closed-loop system, both the control systems are effective in the position regulation of the payload and vibration suppression with limited input current.

The organization of this paper is as follows. Section 2 provides the mathematical model of the two degree-of-freedom vibration suppression system including the magnetorheological (MR) fluid damper. Section 3 presents the feedback linearizing control system, and a sub-optimal controller based on the SDRE method is designed in Section

4. Finally, simulation results are presented in Section 5, and conclusions are provided in Section 6.

II. MATHEMATICAL MODEL

The mathematical model of the MR damper considered in this paper is based on the hysteretic Bingham model [14, 15]. Of all the models proposed in literature, the hysteretic Bingham model results show a very small force error and are simple to formulate. Fig. 1 shows the two degree-of-freedom shock isolation system including the MR damper.

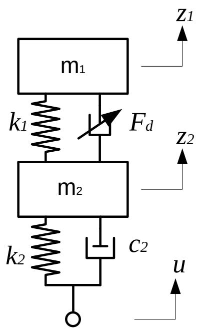

Fig. 1 MR damper based shock isolation system

The equations of motion of the system are given by

$$\begin{aligned} m_1\ddot{z}_1 &= -k_1(z_1 - z_2) - F_d(z) \\ m_2\ddot{z}_2 &= -k_1(z_2 - z_1) - k_2(z_2 - u) - c_2(\dot{z}_2 - \dot{u}) + F_d(z) \end{aligned} \quad (1)$$

where z_1, z_2 denote the positions of the payload mass (m_1) and mass (m_2) from equilibrium position; k_i is the spring constant ($i=1,2$); c_2 is the damping coefficient, u and $\dot{u}$ are treated as disturbance inputs and $F_d(z)$ is the MR damper force which is of the form

$$\begin{aligned} F_d(z) &= c_{po}\Delta\dot{z} + k\Delta z + F_n \tanh[\lambda_1\lambda_2\Delta z + \lambda_2\Delta\dot{z}] \\ F_n(t) &= \alpha + \beta I_c(t) \end{aligned} \quad (2)$$

where $\alpha > 0$, $\beta > 0$, $\Delta z = z_1 - z_2$, $\Delta\dot{z} = \dot{z}_1 - \dot{z}_2$, $\lambda_i > 0$, $c_{po} > 0$, $k > 0$. In the chosen hysteretic Bingham Model (2), it is seen that the MR damper has linear viscous damping as well as it is a nonlinear function of Δz and $\Delta\dot{z}$. $I_c(t)$ is the current which is the control input variable.

Define the state vector $x = [z_1, \dot{z}_1, z_2, \dot{z}_2]^T \in R^4$. Then a state variable representation of (1) and (2) can be compactly written in the form

$$\dot{x} = Ax + B\upsilon + d(t) \quad (3)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k_1 - k}{m_1} & \dfrac{-c_{po}}{m_1} & \dfrac{k_1 + k}{m_1} & \dfrac{c_{po}}{m_1} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_1 + k}{m_2} & \dfrac{c_{po}}{m_2} & \dfrac{-k_1 - k_2 - k}{m_2} & \dfrac{-c_{po} - c_2}{m_2} \end{bmatrix}$$

$$\begin{aligned} B &= [0, (1/m_1), 0, -(1/m_2)]^T \\ C &= [\lambda_1\lambda_2, \lambda_2, -\lambda_1\lambda_2, -\lambda_2] \\ d &= [0, 0, 0, (k_2 u + c_2\dot{u})/m_2]^T \end{aligned} \quad (4)$$

and the nonlinear function υ is

$$\upsilon(I_c(t), x) = -(\alpha + \beta I_c(t)) \tanh[\lambda_1\lambda_2\Delta z + \lambda_2\Delta\dot{z}] \quad (5)$$

The constant matrix A is easily obtained from (1) and (2) by comparison. For the parameters of the system (to be given later), the matrix A is Hurwitz.

Substituting (5) in (1), one obtains a state variable representation of the system given by

$$\dot{x} = f(x) + g(x)I_c + d(t) \quad (6)$$

where

$$\begin{aligned} f(x) &= Ax - B\alpha \tanh(Cx) \\ g(x) &= -B\beta \tanh(Cx) \end{aligned}$$

It is assumed that the current $I_c(t)$ satisfies the inequality

$$0 \le I_c(t) \le I_{cm} \quad (7)$$

where I_{cm} is the maximum value of the current.

We are interested in the design of control systems for the position control and vibration suppression of the payload, caused by impulsive disturbing forces.

III. FEEDBACK LINEARIZING CONTROL

In this section, based on the dynamic inversion (feedback linearization) approach, a control system for the position control of the payload and stabilization of the system is derived [36-37]. First, the derivation of the control law is summarized for a class of single-input single-output systems and then it is extended to the MR damper vibration control system. For the design, it is assumed that the control input (electric current) satisfies (7) for all time.

A. Dynamic Inversion of SISO System

Consider the single-output single-input (SISO) system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &\triangleq h(x) \end{aligned} \quad (8)$$

where $x \in R^n$ (with R^n a smooth manifold), $u \in R$ is the control input, $w \in R$ is the controlled output variable, f and g are smooth vector fields, and h is a smooth nonlinear scalar function. In this case, smooth will mean C^r with r sufficiently large. Differentiating y with respect to time, we get

$$\begin{aligned} \dot{y} &= \partial h(x)\partial x(f(x) + g(x)u) \\ &\triangleq L_f h(x) + L_g h(x)u \end{aligned} \quad (9)$$

where $L_f h(x): R^n \to R$ and $L_g h(x): R^n \to R$ are the Lie derivatives of h with respect to f and g , respectively. If $L_g h(x)$ is bounded away from zero for all

x, the closed-loop system including the state feedback law given by

$$u=\frac{1}{L_g h}(-L_f h+u_a) \tag{10}$$

becomes a linear system from u_a to y given by

$$\dot{y}=u_a \tag{11}$$

where u_a is a new input.

If the function $L_g h(x)$ is zero for all x, we differentiate (9) again to obtain

$$\ddot{y}=L_f^2 h(x)+L_g L_f h(x)u \tag{12}$$

In (12), one has $L_f^2 h(x)=L_f(L_f h)(x)$ and $L_g L_f h(x)=L_g(L_f)h(x)$. Now, if $L_g L_f h(x)$ is bounded away from zero for all x, then the control law given by

$$u=\frac{1}{L_g L_f h(x)}(-L_f^2 h+u_a) \tag{13}$$

yields the input-output system

$$\ddot{y}=u_a \tag{14}$$

More generally, if γ is the smallest integer for which $L_g L_f^i h(x)=0$ for all x and $i=0,\ldots,\gamma-2$ and $L_g L_f^{\gamma-1}h(x)$ is bounded away from zero, then the control law is given by

$$u=\frac{1}{L_g L_f^{\gamma-1}h(x)}(-L_f^{\gamma}h+u_a) \tag{15}$$

yields

$$y^{(\gamma)}=u_a \tag{16}$$

The parameter γ is the relative degree of the output y. Note that the Control Law (15) linearizes the input (u)−output (y) map, and u_a may be chosen to stabilize the System (16).

B. *Inverse Control of MR damper System*

In this subsection, the control of the MR damper system is considered. (Preliminary inverse control results based on this subsection was presented at 18th International Conference on Systems Engineering, 2005.) In this derivation it is considered that the system experiences an impulsive force at t=0. As such, the initial condition of the system is instantaneously perturbed; therefore, for $t\geq 0$ it behaves as a system given by (6) with d(t)=0.

It is of interest to control the position z_1 of the payload. For $t>0$, (6) reduces to

$$\dot{x}=f(x)+g(x)I_c \tag{17}$$

For the design, a controlled output variable y is selected as

$$y=[1,0,0,0]x=h(x) \tag{18}$$

For the derivation of the control law, one differentiates y (t) along the solution of (17) till the control input appears. Differentiating y successively gives

$$\dot{y}=L_f x_1=x_2$$
$$\ddot{y}=L_f^2 x_1+L_g L_f x_1 I_c \tag{19}$$

where

$$L_f^2 x_1=[-k_1\Delta z-c_{po}\Delta\dot{z}-k\Delta z-\alpha\tanh(Cx)]/m_1$$
$$L_g L_f x_1=-\beta\tanh(Cx)/m_1 \tag{20}$$

Note that $L_g L_f x_1$ is zero if $\tanh(Cx)$ is zero; that is, if x lies in the region Ω_e,

where

$$\Omega_e=\{x\in R^4 : Cx=0\} \tag{21}$$

As such for $x\in\Omega_e$, the control force $F_n\tanh(Cx)$ vanishes, and the system (3) simplifies to

$$\dot{x}=Ax \tag{22}$$

for $t>0$.

Noting that A is a Hurwitz matrix, the trajectory $x(t)$ decays exponentially as long as $x(t)$ remains in Ω_e.

The output $y=x_1$ has its relative degree 2. In view of (19) a feedback linearizing control input is chosen as

$$I_c(t)=(L_g L_f x_1)^{-1}[-L_f^2 x_1-p_2 x_2-p_1 x_1] \tag{23}$$
$$\triangleq \mu(x)$$

where $p_2>0$ and $p_1>0$ are feedback gains.

Note that the control input is well defined for x outside Ω_e. Substituting (23) in (19) gives

$$\ddot{y}+p_2\dot{y}+p_1 y=0 \tag{24}$$

For the positive values of p_i, it follows that $y=x_1\to 0$ as t $\to\infty$. One can shape the transient response of x_1 by the proper selection of the feedback gains.

It is seen from (21) that as the trajectory x tends to Ω_e, the function $L_g L_f x_1$ tends to zero; and therefore, I_c (t) in (23) tends to infinity. For practical reasons, it is assumed that the current $I_c(t)$ satisfies the inequality given in (7) ($I_c(t) \le I_{cm}$).Therefore, the control law (23) is modified as follows:

$$I_c(t) = \begin{cases} 0, & \mu(x) < 0 \\ \mu(x), & 0 \le \mu(x) \le I_{cm} \\ I_{cm}, & I_{cm} < \mu(x) \end{cases} \quad (25)$$

It will be seen later that despite the use of saturating control input (25), x_1 is regulated to zero and vibration in the system is completely suppressed. (Preliminary inverse control results based on Subsection 3.2 was presented at 18th International Conference on Systems Engineering [38])

IV. SUB-OPTIMAL CONTROL LAW

This section presents the derivation of the sub-optimal control law using the state-dependent Riccati equation (SDRE) method [39-40]. In this derivation also, it is considered that the system experiences an impulsive force at *t=0*.

For the derivation of the control law, the system is expressed as a linear-like system for which the system matrices are functions of the state vector *x*. It will be convenient to express the input current as

$$I_c = \left(I_{cm}/2\right) + u_c$$
$$|u_c| \le \left(I_{cm}/2\right) \triangleq u_{cm} \quad (26)$$

where u_c is now treated as a control input. Then for $t>0$, (6) gives

$$\dot{x} = Ax - B\left[\alpha + \left(\beta I_{cm}/2\right)\right]\tanh(Cx) - \beta B \tanh(Cx) u_c \quad (27)$$

which can be expressed as

$$\dot{x} = \left[A - B\left\{\alpha + 0.5\beta I_{cm}\right\}\right] \tanh(\mathrm{Cx})\ (\mathrm{Cx})^{-1} C - \beta B \tanh(Cx) u_c$$
$$\triangleq A_s(x)x + B_s(x)u_c \quad (28)$$

where the matrices A_s and B_s are defined in (28). Note that system (28) appears as a linear-like system, and it is well defined for all *x*, because limit of tanh(*Cx*) $(Cx)^{-1}$ exists as *x* tends to zero.

The SDRE method is suitable for the design of a stabilizer even when there is a hard constraint on the input u_c. Following [39-40], the bounded control problem is transformed to an equivalent nonlinear regulator problem by introducing a slack variable x_s which satisfies

$$\dot{x}_s = u_n \quad (29)$$

where u_n is a new control input and u_c takes the form of a saturation sin function given by

$$u_c = sat\sin\left(u_{cm}, x_s\right) \quad (30)$$

where one defines

$$sat\sin\left(u_{cm}, x_s\right) = \begin{cases} u_{cm}, & x_s > \pi/2 \\ u_{cm}\sin(x_s), & -\pi/2 \le x_s \le \pi/2 \\ -u_{cm}, & x_s < -\pi/2 \end{cases} \quad (31)$$

Defining an augmented state vector as $x_a = \left(x^T, x_s\right)^T \in R^5$, the Systems (28)-(31) can be written in a compact form as

$$\dot{x}_a = A_a(x_a)x_a + B_a u_n \quad (32)$$

where the nonlinear matrix $A_a(x_a)$ and the constant matrix B_a are

$$A_a = \begin{bmatrix} A_s & B_s \dfrac{sat\sin\left(u_{cm}, x_s\right)}{x_s} \\ 0_{1\times4} & 0 \end{bmatrix}, \quad B_a = \begin{bmatrix} 0_{1\times4} \\ 1 \end{bmatrix} \quad (33)$$

Note that the System (32) is still a linear-like system and is well defined.

Consider an optimal control problem in which for the Nonlinear System (32), a nonlinear quadratic performance index of the form

$$J_a = \frac{1}{2}\int_0^\infty \left(x_a^T Q_a x_a + \varepsilon u_n^2\right) dt \quad (34)$$

$$Q_a = \begin{bmatrix} Q & 0_{4\times1} \\ 0_{1\times4} & Rq_q \end{bmatrix} \quad (35)$$

where

$$q_q = \begin{cases} \dfrac{\left[sat\sin\left(u_{cm}, x_s\right)\right]^2}{x_s^2} & |x_s| \le \pi/2 \\ \dfrac{u_{cm}^2}{(\pi/2)^2} & |x_s| > \pi/2 \end{cases} \quad (36)$$

is to be minimized, where Q is a semi-positive definite symmetric matrix and $R>0$. The weighting matrix $Q_a(x_a)$ and the scalar parameter $\varepsilon > 0$ are chosen properly for obtaining desirable responses in the closed-loop system. Now instead of deriving an optimal control law by solving the Hamilton-Jacobi equation, for simplicity, a suboptimal control law is designed using the SDRE method.

Consider a region of interest $\Omega_d \in R^5$ of the state space surrounding the origin $x_a = 0$. For the existence of a solution using the SDRE method, the following assumption is made.

Assumption1: The pair $\left(A_a\left(x_a\right), B_a\right)$ is point-wise stabilizable at each $x_a \in \Omega_d$. Now for obtaining a suboptimal solution using the SDRE method, one solves the state-dependent Riccati equation given by

$$A_a^T\left(x_a\right)P\left(x_a\right) + P\left(x_a\right)A_a\left(x_a\right)$$
$$-P\left(x_a\right)B_a\varepsilon^{-1}B_a^T P\left(x_a\right) + Q_a\left(x_a\right) = 0 \quad (37)$$

to obtain a symmetric positive definite solution for $P\left(x_a\right)$. Then the nonlinear feedback control law is given by

$$u_n\left(x_a\right) = -\varepsilon^{-1}B_a^T P\left(x_a\right)x_a \quad (38)$$

Readers may refer to [39-40] for the properties and capabilities of the SDRE method. It is interesting to note that the suboptimal law satisfies

$$\frac{dH(x_a,\lambda)}{du_n}=0, \tag{39}$$

where the Hamiltonian of the nonlinear optimal control problem is

$$H(x_a,\lambda)=\frac{1}{2}\left[x_a^T Q_a(x_a)x_a+\varepsilon^{-1}u_n^2\right] + \lambda^T\left[A_a(x_a)x_a+B_a u_n\right] \tag{40}$$

and $\lambda \in R^5$ is the co-state or the Lagrange multiplier.

Substituting the Control Law (38) in (32) gives the closed-loop system

$$\dot{x}_a=\left[A_a(x_a)-B_a\varepsilon^{-1}B_a^T P(x_a)\right]x_a \triangleq A_c(x_a)x_a \tag{41}$$

The closed-loop matrix $A_c(x_a)$ is guaranteed to be Hurwitz at every $x_a \in \Omega$ from the Riccati equation theory. Since the elements of $A_a(x_a)$ are smooth functions, expanding $A_c(x_a)$ about $x_a=0$, and using mean value theorem, one can show that the equilibrium point $x_a=0$ of (41) is asymptotically stable. The performance of the closed-loop system depends on the weighting matrix $Q_a(x_a)$ and the parameter ε.

V. SIMULATION RESULTS

In this section, the simulation results are presented. Parameters of the hysteretic Bingham model of [23] are: α=132, β=171, c_{p0}=5000 (Nsec/m), k=4000(N/m), λ_1=0.7, and λ_2=5000. The mechanical parameters are m_1=745 (kg), m_2=65(kg), k_1=52,000(N/m), k_2=156,000 (N/m), and c_2=530 (Nsec/m). For the impact force applied to m_2, a velocity shock of 10 (m/s) is treated as an initial velocity for m_2.

A. Inverse Control

First responses of the closed-loop System (1) including the saturating inverse Control Law (25) are presented. Figs. 2 and 3 show the responses for the choice of the feedback gains p_1=16 and p_2=5.65. The saturation value I_{cm} of I_c is selected to be 2 A (amperes). In Fig. 2, responses for fixed values of current I_c=0 and I_c=2 A are also plotted for comparison. It is observed that after initial transient, the state vector converges to the origin in about two seconds. The control input $I_c(t)$ saturates as well vanishes over certain intervals of time. It is seen that the responses with the inverse control law has overshoot of smaller values compared to those of the system with I_c=0; and the settling time is larger if I_c=2 A. The damper forces remains within 500 N for the chosen saturation level of the current. It is observed that the acceleration of the payload mass (m_1) (denoted in the figure as a_1) is smaller compared to the acceleration (denoted as a_2) of m_2.

In order to examine the effect of a different choice of the feedback gains, simulation is done using larger values as p_1= 100 and p_2=14.14. Responses are shown in Figs. 4 and 5. Although, the state vector tends to zero with saturating Control Law (25), the transient responses give larger overshoots over a longer period. This indicates that higher gains are not preferable.

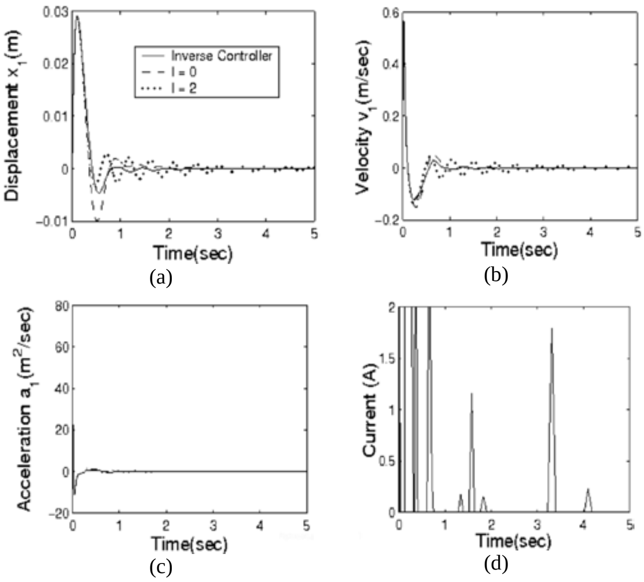

Fig. 2 Inverse control: (a) x_1, (b) v_1, (c) acceleration a_1, and (d) control input I_c

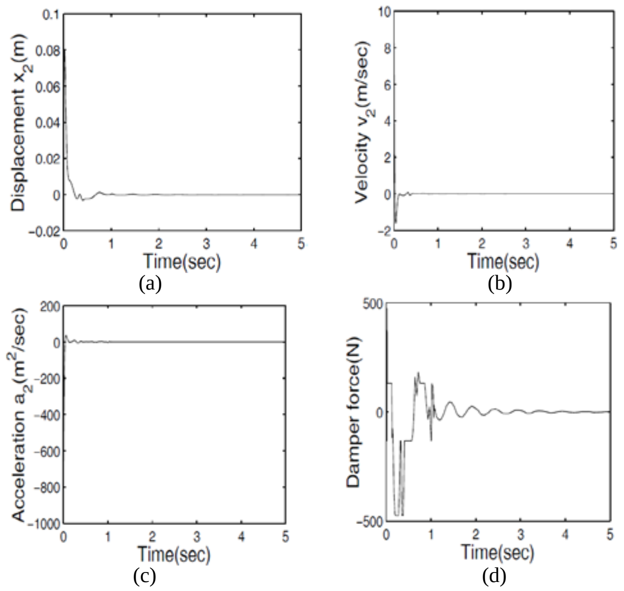

Fig. 3 Inverse control: (a) x_2, (b) v_2, (c) acceleration a_2, and (d) control input F_d

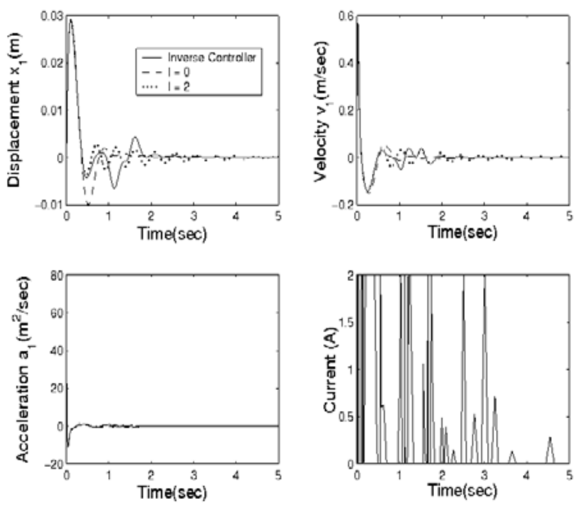

Fig. 4 Inverse control, effect of the choice of p_1 and p_2: (a) x_1, (b) v_1, (c) acceleration a_1, and (d) control input I_c

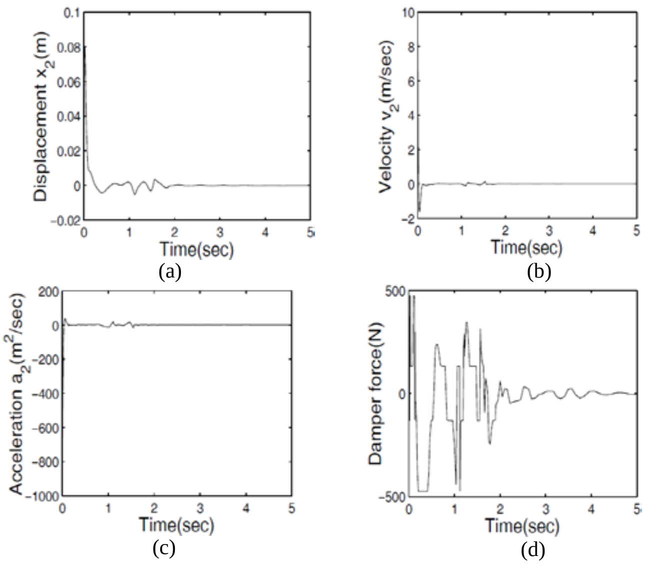

Fig. 5 Inverse control, effect of the choice of p_1 and p_2: (a) x_2, (b) v_2, (c) acceleration a_2, and (d) control input F_d

C. Sub-Optimal Control

Now responses using the sub-optimal Control Law (38) are obtained. For the simulation, the value of I_{cm} is taken as 2A. The weighting matrix Q is chosen as a 4×4 null matrix with its two nonzero diagonal elements q_{ii} set to 10^6, i=1, 3 , where q_{ii} is the diagonal element in the *ith* row. The weighting parameter R is 2 and ε is chosen to be 0.01. The nonlinear function q_q used in the performance index is given in (36). The responses are shown in Figs. 6 and 7. The responses for fixed values of the current (I_c=0 and I_c=2A) are also shown in Fig. 6. It is seen that the state vector x tends to zero and the control input remains within 2 A as expected. It is again observed that the overshoot in the displacement using the SDRE control law is smaller compared to those obtained with I_c=0, and the response for fixed I_c =2 A takes longer time for convergence. Again the acceleration a_2 of m_2 is larger than the transmitted acceleration a_1 of m_1.

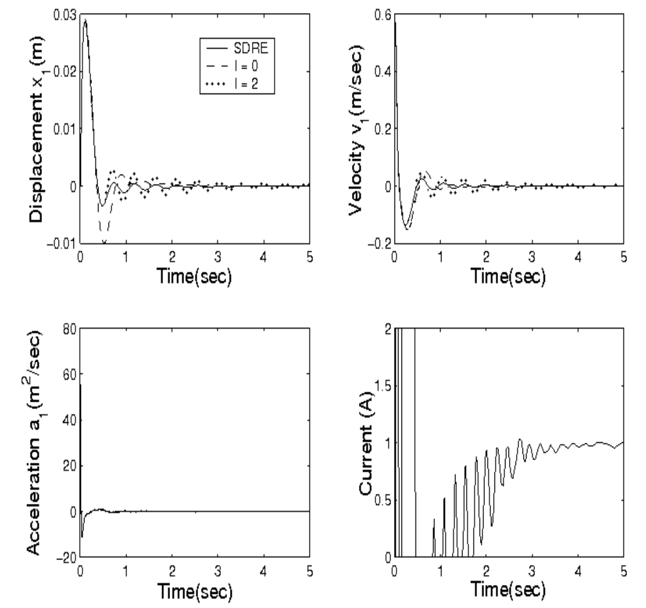

Fig. 6 Sub-optimal Control: (a) x_1, (b) v_1, (c) acceleration a_1, and (d) control input I_c

To examine the effect of the choice of the performance index on responses, simulation is done using a different weighting matrix Q. Now the chosen values are q_{11}=q_{33}=10. These are smaller than the values used in the previous case. The remaining parameters used for Figs. 6 and 7 are retained for simulation. The responses are shown in Figs. 8 and 9. It is observed that the responses are somewhat similar to those obtained in Figs. 6 and 7. Again it is seen that the transient response of the payload position is better compared to the responses obtained using fixed values I_c=0 and I_c=2.

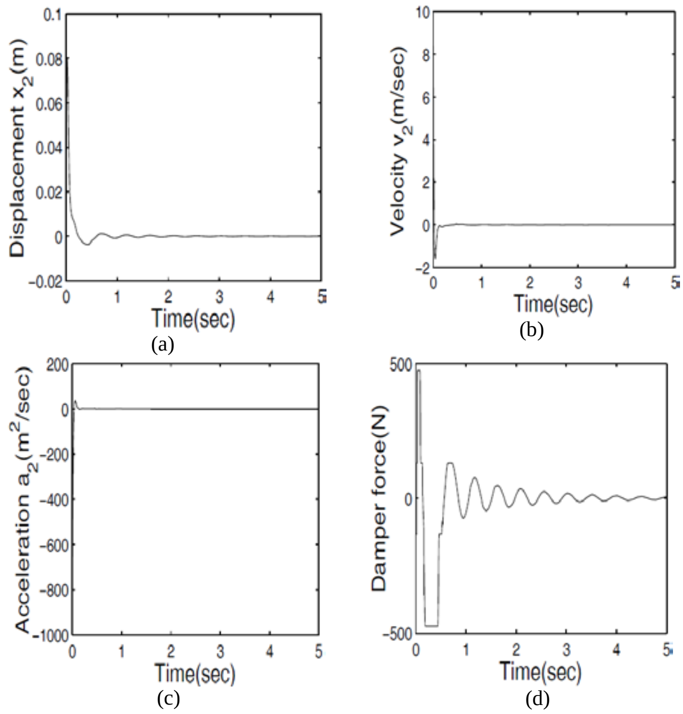

Fig. 7 Sub-optimal Control: (a) x_2, (b) v_2, (c) acceleration a_2, and (d) control input F_d

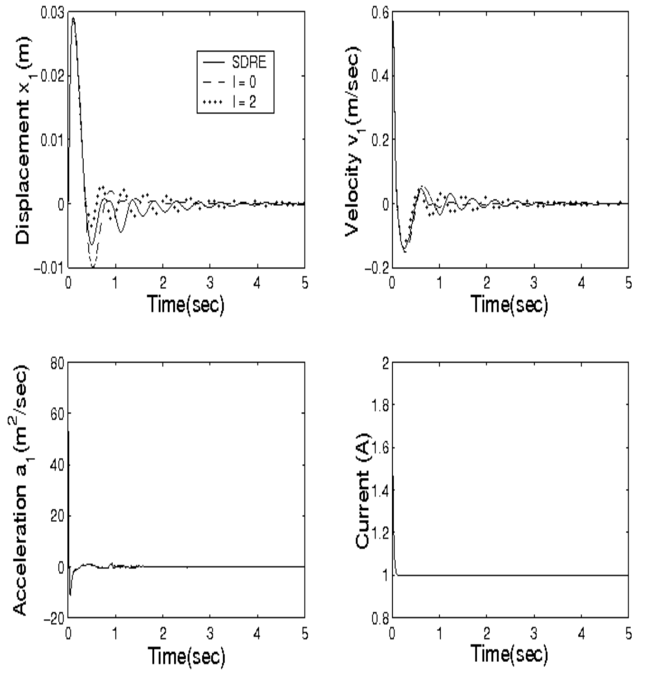

Fig. 8 Sub-optimal Control, Effect of Q: (a) x_1, (b) v_1, (c) acceleration a_1, and (d) control input I_c

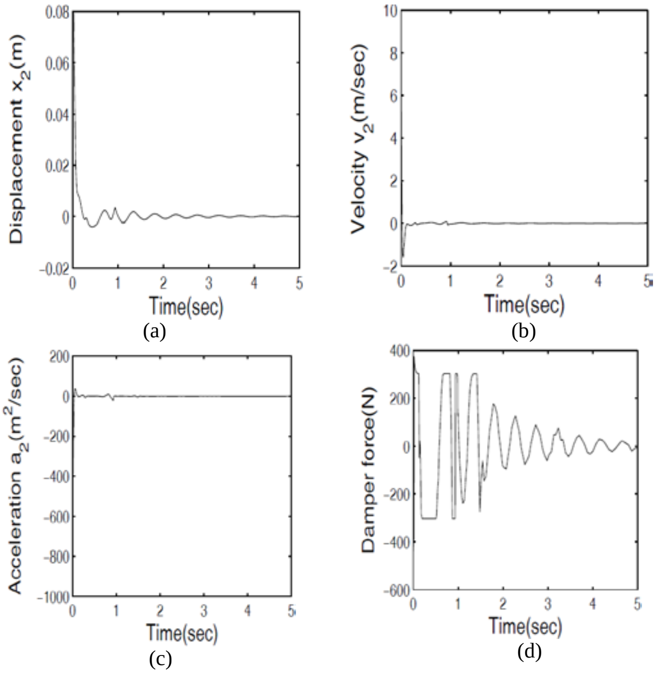

Fig. 9 Sub-optimal Control, Effect of Q: (a) x_2, (b) v_2, (c) acceleration a_2, and (d) control input F_d

VI. CONCLUSIONS

In this paper, the control of a two degree-of-freedom magnetorheological damper system was considered. Hysteretic Bingham model was used for the representation of MR damper. It was assumed that the system encounters impulsive forces. The objective was to control the position of the payload and suppression of vibration using the current as control input variable. Two control systems, namely (i) an inverse control law and (ii) a suboptimal control law based on the SDRE technique, were designed. The input current was clamped to meet the magnitude limits for simulation with the inverse control law. But the constraint on the current was imposed in the design process of the SDRE method. Extensive simulation was done. These results showed that the derived control systems are effective in position control and vibration suppression of the payload. Furthermore, it was observed that the designed controllers give better transient performance compared to the one obtained with fixed values of the current input. Moreover, there exists enough flexibility in the choice of feedback parameters to shape the transient responses.

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