Sliding Mode Compensation for Model Uncertainty, Payload Variation and Actuator Dynamics for Inverse Dynamics Velocity Control of Direct Drive Robot Manipulator

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Abstract- **The robot manipulator joint actuator dynamics and the payload variation are major factors that severely affect the overall dynamics and hence the tracking performance of the direct drives manipulator. This paper proposes a robust and computationally simple approach aimed at compensating the tracking error due to model uncertainty, actuator dynamics, and payload variation for the inverse dynamics velocity control of direct drive robot manipulator. The proposed scheme employs a simple Proportional-Integral (PI) type sliding mode control scheme for the design of the compensation control signal. In this scheme, the sliding mode compensation control input can be calculated from the nominal model of the manipulator, provided the bound on the model uncertainty can be estimated. Thus, for the proposed control method, unlike the classical inverse dynamics controller, calculation of the parameters of the dynamic model very accurately in real-time is not a serious requirement. The effectiveness of the proposed control algorithm has been validated in simulation studies considering the model of a 3-DOF direct drive robot manipulator with different loading conditions.**

Keywords- Sliding Mode Compensation; Inverse Dynamics; Velocity Control; Direct Drive Manipulator; Robust Tracking

I. INTRODUCTION

Velocity control is an important technique of kinematic motion control of robotic manipulator. For example, during the end effectors pose control problem, the motion control of the robot can be conveniently carried out through a controller that is centered on the robot kinematics [1-16]. However, this kinematics based robot control can be effective only when the velocity errors of robot joint vanish asymptotically [2, 3]. For this type of problems, velocity controller may be incorporated in the robot control system to drive each robot joint so that actual joint velocity accurately tracks a desired velocity profile in the robot joint space. Few velocity control methodologies of robot manipulator have already appeared in the literature due to the intrinsic worth of the velocity control technique along with Kinematic control in conjunction with joint velocity control [1^7- 21]. In [18], the manipulators have inner joint velocity loop for each electromechanical axis in addition to outer position loop that is based on kinematics. Velocity field control [21, 22] is another advanced method to control end effectors velocity in operational space where use of velocity controller is observed. For contour tracking tasks of unknown object, velocity feedback in hybrid force/velocity control [19], sometimes with friction [20], is used.

On the other hand, joint velocity control is also an important issue of force control of robot manipulator [13]. Position control would not be appropriate in force control as very small error can adversely affect the work space control. The inverse dynamics controller [1-3] and PD controller with compensation [2, 3] are found to be prevalent among the velocity control of robot manipulator. The idea behind these velocity controllers has been originated from the individual velocity regulators of the corresponding electrical motors where feed-forward compensation is added to handle the disturbances due to manipulator nonlinear dynamics [2, 3].

As the direct drive manipulators are supposed to operate at quite high speeds and should accurately position the end effector, the control algorithms that can address both robot dynamics as well as proper control of velocity profiles of the robot joints would be useful. In this paper, we have proposed a novel approach of velocity control of direct drive manipulator based on inverse dynamics control [1]. Inverse dynamics velocity control methodology has been adopted in this paper because it decouples the complex nonlinear system of manipulator arm into a linear closed loop system. The tracking performance of this velocity controller exclusively depends on the cancellation of the nonlinear dynamics of the robot by the control law itself which is designed on the basis of manipulator dynamic model [1-3]. However, this controller is found to have some limitations in real time implementation, because model parameters have to be computed accurately at each sampling interval. In practice, it is difficult to know the parameters of the manipulator model very exactly due to presence of uncertainties. Therefore, the classical inverse dynamics control algorithm cannot be computed accurately to nullify the nonlinear couplings. One possible solution to this problem could be to design the inverse dynamics control law considering the nominal model of the manipulator and then add a robust feedback compensation term to compensate for the parametric uncertainties. Among different robust control methodologies, the sliding mode control [4-6], which is based on the variable structure system (VSS) theory [11, 14], is well known for its robustness against model and parametric uncertainties in feedback control. In the sliding mode control scheme, tracking error is forced to slide on a predetermined sliding surface [14, 15] by designing a discontinuous control input.

In this paper, we have formulated a modified inverse dynamics velocity controller with sliding mode compensation so that the compensated system is not affected by any modelling uncertainties and the system becomes robust to parameter variations. In our design, the bounds of the unknown parameters, which are needed for calculating the switching gains, are not required to be known very exactly. In this investigation, the bounds are estimated [6] as a function of the state of the dynamics and the tracking error. The proposed controller has been designed for direct drive manipulator with electric motor actuated joints. In the inverse dynamics velocity control scheme [1-3] the electrical dynamics of actuator is normally is ignored for the simplification of calculation. However, these inaccuracies in the modelling may result into a significant tracking error which will creep into the system and which may not vanish asymptotically. In this investigation, we have also considered the electrical model of the actuator while determining the manipulator model. It is certainly a pragmatic approach to consider the manipulator with voltage controlled dc motor [7] compared to torque controlled dc motor where electrical dynamics are neglected. To the best of the knowledge of the authors, sliding mode compensated velocity control of electric motors actuated direct drive manipulators considering actuator dynamics has never been addressed in the literature.

Kinematic control of robot manipulator is quite convenient when the end-effector pose has to be controlled for performing the desired task by the motion control of the manipulator [1-21]. However, this control technique can be effective only when the velocity errors of robot joint vanish asymptotically [2, 3]. For this type of problems, velocity controller may be incorporated in the robot control system to drive each robot joint so that actual joint velocity accurately tracks a desired velocity profile in the robot joint space. Few velocity control methodologies of robot manipulator have already appeared in the literature due to the intrinsic merit of the velocity control technique [18-21]. As transmission systems like gear train introduces steady-state errors, direct drive manipulators are quite useful in kinematic control applications where accurate positioning and pose control of the end-effector is required. However, the direct drive manipulators operate at quite high speeds and they are expected to position the end effectors with acceptable level of accuracy. Therefore, the velocity control algorithms should be able to address the effect of the robot dynamics due to high-speed operation of the manipulator and at the same time exhibit proper tracking control of the velocity profiles of the robot joints. Inverse dynamics [22] based control of the manipulator can address the effects of dynamics due to high speed operation and can manifest satisfactory tracking performance provided the model of the plant is known quite accurately. Moreover, the inverse dynamics velocity control methodology decouples the complex nonlinear system of manipulator arm into a linear closed loop system. The tracking performance of this velocity controller exclusively depends on the cancellation of the nonlinear dynamics of the robot by the control law itself, which is designed on the basis of manipulator dynamic model [1, 3, 18]. However, this controller is found to have some limitations in real-time implementation, because model parameters have to be computed accurately at each sampling interval. In practice, it is difficult to know the parameters of the manipulator model very exactly due to presence of uncertainties. Therefore, the classical inverse dynamics control algorithm cannot be computed accurately to nullify the nonlinear couplings. One possible solution to this problem could be to design the inverse dynamics control law considering the nominal model of the manipulator and then add a robust feedback compensation term to compensate for the parametric uncertainties. Among different robust control methodologies, the sliding mode control [4, 6, 23, 25, 26] which is based on the variable structure system (VSS) theory [11,14, 24-27], is well known for its robustness against model and parametric uncertainties in feedback control. In the sliding mode control scheme, tracking error is forced to slide on a predetermined sliding surface [14, 15] by designing a discontinuous control input.

In this paper, we have formulated a modified inverse dynamics velocity controller after incorporating sliding mode compensation so that the compensated system becomes robust against the effects of model parameter uncertainties and payload variations. Here, we have considered the effect of payload as disturbance to the manipulator control system. The results of the simulation studies, with 3kg payload, exhibit that the proposed sliding mode based compensation technique results a very simple and robust inverse dynamics velocity control scheme for the direct drive manipulator control scheme. To the best of the knowledge of the authors, sliding mode compensated velocity control of direct drive manipulators considering actuator dynamics has never been addressed in the literature.

The rest of this article is organized as follows. The problem formulation has been discussed in section II. In section III the robot dynamics and its structure properties are reviewed. Section IV presents Inverse Dynamics Velocity Control algorithm. Sliding mode compensation of the same controller with bound estimation and the stability analysis of proposed controller is presented in section V. The elimination of chattering is discussed in section VI. Finally, the simulation results are shown in Section VII.

II. PROBLEM FORMULATION

Inverse dynamics velocity controller [2, 3] which is designed considering the dynamic model of the manipulator, incorporates a non-linear control input and decouples the nonlinear robot dynamics, thus makes the robot manipulator a linear closed loop system. However, the inverse dynamics control methodology requires various parameters of the dynamic model of the manipulator to be calculated very precisely during each sampling interval when implemented in digital environment; otherwise the output tracking error cannot be assured to converge to zero [1] because of the inexact cancellation of nonlinearities. Unfortunately, these requirements of the inverse dynamics controller are difficult to satisfy in practice because of the huge computational burden. The uncertainties are arising out of the effects of several factors, such as, uncompensated viscous friction, Coulomb friction, discrete controller implementation, estimation of joint velocity via numerical differentiation of the joint position [2]. Another main source of parameter variation is pay-load change [4,5,10] because the inertia matrix changes with pay load variation. The mathematical model of manipulator also differs from the real system as mathematical model is obtained considering some approximations and rounding off. Moreover, actuator dynamics are often simplified or even neglected while determining the manipulator model [1]. These inaccuracies in modeling hinder the design of such a nonlinear control input of the inverse dynamics velocity control scheme which can nullify the nonlinearities present in the system and decouple the system as linear closed loop system during implementation. This motivated us to design a robust compensator considering the nominal model of the manipulator for the inverse dynamics velocity control, so that it can mitigate these problems and yield better performance even in face of parameter uncertainties. A sliding mode compensation [4-6] based on Variable Structure Control can effectively suppress the impacts of nonlinearities caused by parametric uncertainties. The design of the sliding mode compensating input requires only the estimates of the bounds of model uncertainties instead of the accurate knowledge of the system dynamics. In sliding mode compensation, a sliding surface of the system states is defined *a priori* and the joint velocity errors are compelled to slide onto it during the subsequent time provided the bounds of the uncertainties[6] are well estimated.

III. MATHEMATICAL MODELLING OF DIRECT DRIVE MANIPULATOR

Dynamics of an n-link serial robot manipulator [2, 3] can be written as:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + g(\boldsymbol{q}) = \boldsymbol{\tau} \qquad (1)$$

where, $\mathbf{q}$ is $n \times 1$ joint displacement vector, $\dot{\boldsymbol{q}}$ is $n \times 1$ joint velocity vector, $M(\boldsymbol{q})$ is $n \times n$ manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of centripetal and Coriolis torques and $g(\boldsymbol{q})$ is $n \times 1$ vector of gravitational torque due to gravity.

The time derivative of the inertia matrix $M(q)$ [1] satisfies

$$\boldsymbol{x}^T \left[\frac{1}{2}\,\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right] \boldsymbol{x} = 0 \qquad \forall\, \boldsymbol{x},\boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n \qquad (2)$$

$$\text{and } \dot{M}(\boldsymbol{q}) = C(\boldsymbol{q},\dot{\boldsymbol{q}}) + C^T(\boldsymbol{q},\dot{\boldsymbol{q}}) \qquad \forall\, \boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n \qquad (3)$$

IV. INVERSE DYNAMIC VELOCITY CONTROLLER

We consider the following velocity controller [2] based on the inverse dynamics control [1]:

$$\boldsymbol{\tau} = M(\boldsymbol{q})\boldsymbol{v} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + g(\boldsymbol{q}) \qquad (4)$$

$$\text{where, } \quad \boldsymbol{v} = -K_v\dot{\boldsymbol{q}} - K_i\boldsymbol{\psi} + \boldsymbol{r} \qquad (5)$$

$$\text{and } \quad \boldsymbol{r} = \ddot{\boldsymbol{q}}_{\boldsymbol{d}} + K_v\dot{\boldsymbol{q}}_{\boldsymbol{d}} + K_i\boldsymbol{\psi}_{\boldsymbol{d}} \qquad (6)$$

We define $\dot{\boldsymbol{\psi}} = \dot{\boldsymbol{q}}$ and $\dot{\boldsymbol{\psi}}_{\boldsymbol{d}} = \dot{\boldsymbol{q}}_{\boldsymbol{d}}$**.** Inverse Dynamics velocity controller [2] presented hare has been inspired from the inverse dynamics control scheme of reference [1]. However, integral of desired velocity and actual velocity of the robot arm have been used in place of desired position and actual position of the robot arm respectively [2, 3].

Substituting (4), (5), and (6) in the dynamic equation of n-link manipulator defined by (1), we get

$$[\boldsymbol{v} - \ddot{\boldsymbol{q}}] = 0 \;\Rightarrow \ddot{\tilde{\boldsymbol{q}}} + K_v\dot{\tilde{\boldsymbol{q}}} + K_i\boldsymbol{\xi} = 0 \qquad (7)$$

For a choice [1] of $K_v = diag(2\omega_1, \ldots, 2\omega_n)$ and $K_i = diag(\omega_1{}^2, \ldots, \omega_n{}^2)$ the above second order system becomes a critically damped system. Now let us denote $\dot{\tilde{\boldsymbol{q}}}$ and $\ddot{\tilde{\boldsymbol{q}}}$ as the velocity error and derivative of velocity error respectively, and $\boldsymbol{\xi}$ as the integral of velocity error $\dot{\tilde{\boldsymbol{q}}}$**:**

$$\ddot{\tilde{\boldsymbol{q}}} = \ddot{\boldsymbol{q}}_{\boldsymbol{d}} - \ddot{\boldsymbol{q}} \qquad (8)$$

$$\dot{\tilde{\boldsymbol{q}}} = \dot{\boldsymbol{q}}_{\boldsymbol{d}} - \dot{\boldsymbol{q}} \qquad (9)$$

$$\dot{\boldsymbol{\xi}} = \dot{\tilde{\boldsymbol{q}}} \qquad (10)$$

where, $\boldsymbol{\xi} = \boldsymbol{\psi}_{\boldsymbol{d}} - \boldsymbol{\psi}$**.** Now, **t**he objective of velocity control is to drive the robot arm in such a way that the error $\dot{\tilde{\boldsymbol{q}}}$ becomes zero asymptotically, i.e.,

$$\lim_{t\to\infty} \dot{\tilde{\boldsymbol{q}}}(t) = \lim_{t\to\infty}[\dot{\boldsymbol{q}}_{\boldsymbol{d}}(t) - \dot{\boldsymbol{q}}(t)] = 0 \qquad (11)$$

The schematic diagram of inverse dynamics velocity control scheme is shown in Fig.1. The notations used in the schematic diagram of Fig.1 are as follows. V is the voltage signal, L is the armature inductance and R is armature resistance of the motor, i_a is the armature current, τ_m is the torque generated by motor, K_b is the back emf constant, K_m is the torque constant and B is the viscous friction coefficient.

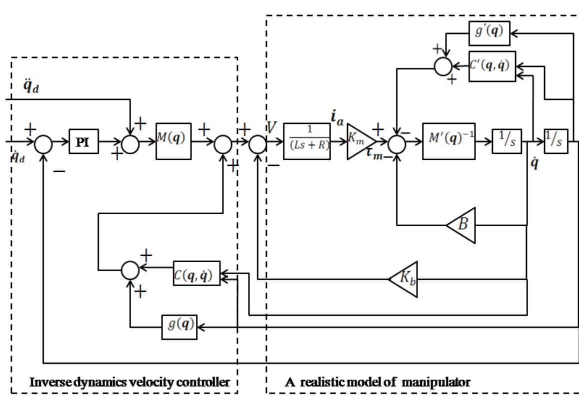

Fig1. Implementation of inverse dynamics velocity control

V. UNCERTAINTY DURING IMPLEMENTATION

Here we have taken the dynamic model of a direct drive manipulator with electrical motor actuated joints. The electric motor drives (actuators) of this model is considered to be driven by the electrical signal. The overall dynamic model also considers the dynamics of the electric motors. We can re-write equation (1) after incorporating the uncertainty components that arise during implementation as:

$$\ddot{\boldsymbol{q}} = M'(\boldsymbol{q})^{-1}\left[K_m \frac{(V-K_b\dot{\boldsymbol{q}})}{(Ls+R)} - C'(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} - B\dot{\boldsymbol{q}} - g'(\boldsymbol{q})\right] \quad (12)$$

where, $M'(\boldsymbol{q})$, $C'(\boldsymbol{q},\dot{\boldsymbol{q}})$ and $g'(\boldsymbol{q})$ are related to manipulator dynamics containing unknown parameter variation [4] as given below.

$$M'(\boldsymbol{q}) = \{M(\boldsymbol{q},\boldsymbol{\phi}) + \delta M(\boldsymbol{q},\delta\boldsymbol{\phi})\} \quad (13)$$

$$C'(\boldsymbol{q},\dot{\boldsymbol{q}}) = \{\, C(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\phi}) \;+ \delta C(\boldsymbol{q},\dot{\boldsymbol{q}},\delta\boldsymbol{\phi})\} \quad (14)$$

$$g'(\boldsymbol{q}) = \{g(\boldsymbol{q},\boldsymbol{\phi}) + \delta g(\boldsymbol{q},\delta\boldsymbol{\phi})\} \quad (15)$$

Where $\delta\boldsymbol{\phi}$ indicates the variation of unknown system parameter $\boldsymbol{\phi}$ and $\delta M(\cdot)$, $\delta C(\cdot)$ and $\delta g(\cdot)$ represent changes caused by $\delta\boldsymbol{\phi}$ with respect to nominal values of $M(\boldsymbol{q},\boldsymbol{\phi})$, $C(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\phi})$ and $g(\boldsymbol{q},\boldsymbol{\phi})$respectively.

$$K_m(V - K_b\dot{\boldsymbol{q}}) = (Ls + R)[M'(\boldsymbol{q})\ddot{\boldsymbol{q}} + C'(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \mathrm{B}\dot{\boldsymbol{q}} + g'(\boldsymbol{q})]$$

$$V = \frac{(Ls+R)}{K_m}[M'(\boldsymbol{q})\ddot{\boldsymbol{q}} + C'(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + B\dot{\boldsymbol{q}} + g'(\boldsymbol{q})] + K_b\dot{\boldsymbol{q}}$$

$$V = \frac{L}{K_m}[M'(\boldsymbol{q})\dddot{\boldsymbol{q}} + \dot{M}'(\boldsymbol{q})\ddot{\boldsymbol{q}} + B\ddot{\boldsymbol{q}} + C'(\boldsymbol{q},\dot{\boldsymbol{q}})\ddot{\boldsymbol{q}} + \dot{g}'(\boldsymbol{q})] + \frac{R}{K_m}[M'(\boldsymbol{q})\ddot{\boldsymbol{q}} + C'(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \mathrm{B}\dot{\boldsymbol{q}} + g'(\boldsymbol{q})] + K_b\dot{\boldsymbol{q}}$$

$$\Rightarrow V = \left[\frac{L}{K_m}\left(\dot{M}'(\boldsymbol{q}) + C'(\boldsymbol{q},\dot{\boldsymbol{q}}) + B\right) + \frac{R}{K_m}M'(\boldsymbol{q})\right]\ddot{\boldsymbol{q}} + \left[\frac{R}{K_m}(C'(\boldsymbol{q},\dot{\boldsymbol{q}}) + B) + K_b\right]\dot{\boldsymbol{q}} + \frac{R}{K_m}g'(\boldsymbol{q}) + \left[\frac{L}{K_m}\left(M'(\boldsymbol{q})\dddot{\boldsymbol{q}} + \dot{g}'(\boldsymbol{q})\right)\right] \quad (16)$$

We can write the above equation as:

$$V = M''(\boldsymbol{q})\ddot{\boldsymbol{q}} + C''(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g''(\boldsymbol{q}) + d(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \quad (17)$$

where,

$$M''(\boldsymbol{q}) = \left[\frac{L}{K_m}\left(\dot{M}'(\boldsymbol{q}) + C'(\boldsymbol{q},\dot{\boldsymbol{q}}) + B\right) + \frac{R}{K_m}M'(\boldsymbol{q})\right] \quad (18)$$

$$C''(\boldsymbol{q},\dot{\boldsymbol{q}}) = \left[\frac{R}{K_m}(C'(\boldsymbol{q},\dot{\boldsymbol{q}}) + B) + K_b\right] \quad (19)$$

$$g''(\boldsymbol{q}) = \frac{R}{K_m}g'(\boldsymbol{q}) \quad (20)$$

$$d(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) = \left[\frac{L}{K_m}\left(M'(\boldsymbol{q})\dddot{\boldsymbol{q}} + \dot{g}'(\boldsymbol{q})\right)\right] \quad (21)$$

We can represent the inertia, centripetal and Coriolis matrices and also gravitational vector as the summation of corresponding nominal model matrices and perturbed matrices [4, 11]. The modified form of equation (17) can be expressed as:

$$V = \{M(\boldsymbol{q}) + \Delta M(\boldsymbol{q})\}\ddot{\boldsymbol{q}} + \{\, C(\boldsymbol{q},\dot{\boldsymbol{q}}) \;+ \Delta C(\boldsymbol{q},\dot{\boldsymbol{q}})\}\dot{\boldsymbol{q}} + \{g(\boldsymbol{q}) + \Delta g(\boldsymbol{q})\} + d(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \quad (22)$$

Here, $M''(\boldsymbol{q}) = \{M(\boldsymbol{q}) + \Delta M(\boldsymbol{q})\}$ (23)

$$C''(\boldsymbol{q},\dot{\boldsymbol{q}}) = \{\, C(\boldsymbol{q},\dot{\boldsymbol{q}}) \;+ \Delta C(\boldsymbol{q},\dot{\boldsymbol{q}})\} \quad (24)$$

and, $g''(\boldsymbol{q}) = \{g(\boldsymbol{q}) + \Delta g(\boldsymbol{q})\}$ (25)

Here, $M(\boldsymbol{q})$, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and $g(\boldsymbol{q})$ are related to nominal model and $\Delta M(\cdot)$, $\Delta C(\cdot)$ and $\Delta g(\cdot)$ represent model uncertainty. Here, the control input is a voltage signal. After deriving the expression of control input from (4) and then substituting it in the dynamic equation (22) we get,

$$\Rightarrow M(\boldsymbol{q})[\boldsymbol{v} - \ddot{\boldsymbol{q}}] = \Delta M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \Delta C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \Delta g(\boldsymbol{q}) + d(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$$

$$\Rightarrow M(\boldsymbol{q})[\boldsymbol{v} - \ddot{\boldsymbol{q}}] = \mathrm{h}(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$$

$$\Rightarrow [\boldsymbol{v} - \ddot{\boldsymbol{q}}] = M(\boldsymbol{q})^{-1}\mathrm{h}(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$$

$$\Rightarrow [\boldsymbol{v} - \ddot{\boldsymbol{q}}] = f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \quad (26)$$

where,

$$\mathrm{h}(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) = \Delta M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \Delta C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \Delta g(\boldsymbol{q}) + d(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \quad (27)$$

and, $f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) = M(\boldsymbol{q})^{-1}\mathrm{h}(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$. (28)

$f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$ can be regarded as the modeling error or, the model uncertainty.

VI. SLIDING MODE CONTROLLER DESIGN

Comparing equation (7) and (26) it is evident that a nonlinear coupling exists in (26). Now, substituting expression of $\boldsymbol{v}$ from equation (5) and expression of $\boldsymbol{r}$ from equation (6) into equation (26), we get

$$\ddot{\tilde{\boldsymbol{q}}} + K_v\dot{\tilde{\boldsymbol{q}}} + K_i\boldsymbol{\xi} = f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \quad (29)$$

The term $f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$ is a non linear one due to parametric uncertainties, and hinders the system to behave as a linear closed loop system. Now, the influence of $f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$ is to be nullified in equation (29) on application of a proper sliding mode compensating input $\boldsymbol{u}_{sl}$. Therefore, the inverse dynamics velocity control law incorporating sliding mode compensation becomes:

$$\boldsymbol{\tau} = M(\boldsymbol{q})[\boldsymbol{v} + \boldsymbol{u}_{sl}] + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g(\boldsymbol{q}) \quad (30)$$

Substituting (30) on (22) we get

$$[\boldsymbol{v} - \ddot{\boldsymbol{q}}] + \boldsymbol{u}_{sl} = f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \quad (31)$$

$$\Rightarrow [\boldsymbol{r} - K_v\dot{\boldsymbol{q}} - K_i\boldsymbol{\psi} - \ddot{\boldsymbol{q}}] + \boldsymbol{u}_{sl} = f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \quad (32)$$

Equation (31) gets modified as (32) when expression of $\boldsymbol{v}$ from (5) is substituted in (31).

A. Bound Estimation

Generally uncertainty is considered to be bounded. But, for $f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$ which entails viscous friction, the above consideration does not holds good as they are functions of system states. Therefore, uncertainties may exceed any constant bound [6] if the system becomes unstable. We assume the uncertainty as

$$\|f\| \le \alpha_1 + \beta_1\|\dot{\tilde{\boldsymbol{q}}}\| + \gamma_1\|\boldsymbol{\xi}\| \quad (33)$$

where $\alpha_1 > 0, \beta_1 > 0$ *and* $\gamma_1 > 0$ are some constants.

From (28) we get,

$$f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) = M(\boldsymbol{q})^{-1}\mathrm{h}(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$$

$$\Rightarrow f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) = [M(\boldsymbol{q})^{-1}\{\Delta M(\boldsymbol{q})\}\ddot{\boldsymbol{q}} + M(\boldsymbol{q})^{-1}\{\Delta C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \Delta g(\boldsymbol{q})\} + M(\boldsymbol{q})^{-1}d(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})]$$

$$\Rightarrow f(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}) = M^*(\boldsymbol{q})\ddot{\boldsymbol{q}} + C^*(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g^*(\boldsymbol{q}) + d^*(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$$

α_1, β_1 *and* γ_1 are estimated in advance so that they satisfy (33). They are presented as:

$$\left.\begin{aligned} &\alpha_1 > Sup\|f(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q})\| \\ &\beta_1 > Sup\left\|M(\boldsymbol{q})^{-1}\{\tfrac{\partial}{\partial\dot{\tilde{\boldsymbol{q}}}}(\Delta C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}})\}\right\| \\ &\gamma_1 > 0 \end{aligned}\right\} \quad (34)$$

Actually, these three design parameters represent the uncertainty bounds, which have been used during implementation of the controller. Actually, α_1 denotes the infinity norm of the function f_1. β_1 is introduced to robutify the controller against the uncertain modelling error that has been associated with the coriolis matrix. Similarly, γ_1 is introduced to robustify the system against the term associated with vector ξ (i.e. the position error related terms).

B. Selection of Sliding Surface

The sliding surface is described as $\boldsymbol{s} = 0$ where

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{q}}} + C\boldsymbol{\xi} = 0 \quad (35)$$

where $\boldsymbol{s}^T = [s_1, \dots, s_n]$, and $= diag[c_1, \dots, c_n]$, $c_i > 0$

In ideal sliding motion $\boldsymbol{s} = 0$ and $\dot{\boldsymbol{s}} = 0$. This is transformed in the following error system as

$$\frac{d}{dt}[\boldsymbol{\xi}] = -C\boldsymbol{\xi} \quad (36)$$

and

$$\frac{d}{dt}[\dot{\tilde{\boldsymbol{q}}}] = -C\dot{\tilde{\boldsymbol{q}}} \quad (37)$$

Once the system is in sliding mode, the error signal would be decaying exponentially and actual velocity will track the desired velocity.

A vector of self defined reference variable [6] is introduced in this context as given below:

$$\dot{\boldsymbol{q}}_{\boldsymbol{r}} = \dot{\boldsymbol{q}}_{\boldsymbol{d}} + C\boldsymbol{\xi} \quad (38)$$

From equation (38) we can write

$$\ddot{\boldsymbol{q}}_{\boldsymbol{r}} = \ddot{\boldsymbol{q}}_{\boldsymbol{d}} + C\dot{\tilde{\boldsymbol{q}}} \quad (39)$$

Hence, $\boldsymbol{s}$ can also be expressed as

$$\boldsymbol{s} = \dot{\boldsymbol{q}}_{\boldsymbol{r}} - \dot{\boldsymbol{q}} \quad (40)$$

We define,

$$\boldsymbol{s}_{\boldsymbol{1}} = \boldsymbol{\psi}_{\boldsymbol{r}} - \boldsymbol{\psi} \quad (41)$$

And

$$\dot{\boldsymbol{s}}_{\boldsymbol{1}} = \dot{\boldsymbol{\psi}}_r - \dot{\boldsymbol{\psi}} = \dot{\boldsymbol{q}}_r - \dot{\boldsymbol{q}} = \boldsymbol{s} = \dot{\tilde{\boldsymbol{q}}} + C\boldsymbol{\xi} \quad (42)$$

C. Calculation for Compensating Input

For sliding mode compensation, the compensating input $\boldsymbol{u}_{\boldsymbol{sl}}$ is calculated to nullify the influence of nonlinear term $f(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q})$. Hence, the above mentioned self defined reference variable enable the inverse dynamic velocity controller to make the overall closed loop system as a linear system.

As, $\dot{\boldsymbol{q}}_{\boldsymbol{r}}$ satisfies the equation (7), we can write

$$\boldsymbol{v}_{\boldsymbol{r}} - \ddot{\boldsymbol{q}}_{\boldsymbol{r}} = 0$$

$$\Rightarrow \boldsymbol{r} - K_v\dot{\boldsymbol{q}}_{\boldsymbol{r}} - K_i\boldsymbol{\psi}_{\boldsymbol{r}} - \ddot{\boldsymbol{q}}_{\boldsymbol{r}} = 0 \quad (43)$$

where $\boldsymbol{v}_{\boldsymbol{r}} = -K_v\dot{\boldsymbol{q}}_{\boldsymbol{r}} - K_i\boldsymbol{\psi}_{\boldsymbol{r}} + \boldsymbol{r}$. Now, Subtracting (32) from (43) we get,

$$\dot{\boldsymbol{s}} = -K_v\boldsymbol{s} - K_i\boldsymbol{s}_{\boldsymbol{1}} - \boldsymbol{u}_{\boldsymbol{sl}} + f(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}) \quad (44)$$

Here, we have considered the following:

$$\boldsymbol{s} = \dot{\boldsymbol{q}}_{\boldsymbol{r}} - \dot{\boldsymbol{q}} \text{ and } \boldsymbol{s}_{\boldsymbol{1}} = \boldsymbol{\psi}_{\boldsymbol{r}} - \boldsymbol{\psi}.$$

When the system is in ideal sliding motion, then it satisfies: $\boldsymbol{s} = 0$ and $\dot{\boldsymbol{s}} = 0$. The equivalent value of $\boldsymbol{u}_{\boldsymbol{sl}}$ can be obtained from (18) as:

$$\boldsymbol{u}_{\boldsymbol{sleq}} = -K_i\boldsymbol{s}_{\boldsymbol{1}} + f(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}) \quad (45)$$

Finally the sliding mode compensating input can be defined as:

$$\boldsymbol{u}_{\boldsymbol{sl}} = -K_i\boldsymbol{s}_{\boldsymbol{1}} + \Delta\boldsymbol{u}_{\boldsymbol{sl}} \quad (46)$$

where $\Delta\boldsymbol{u}_{\boldsymbol{sl}}$ nullifies the influence of $f(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q})$ and chosen as:

$$\Delta\boldsymbol{u}_{\boldsymbol{sl}} = [\alpha + \beta|\dot{\tilde{\boldsymbol{q}}}| + \gamma|\boldsymbol{\xi}|]sign(\boldsymbol{s}) \quad (47)$$

$\alpha > 0, \beta > 0$ *and* $\gamma > 0$ are constants. They are chosen such that the sliding surface approaches to zero asymptotically.

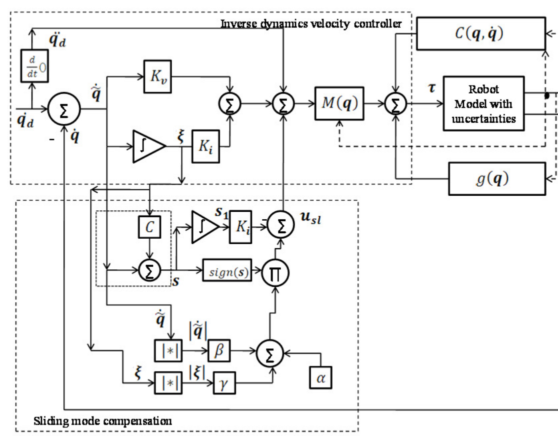

Fig 2. Sliding mode compensation of Inverse dynamics velocity control

Theorem 1: Considering error system defined by (26), with the sliding surface $\mathbf{s} = 0$ described by (35), $\mathbf{s}$ approaches zero asymptotically provided that the control laws (31),(46),(47) and uncertainty bound estimation (33) are considered. Thus the robustness to uncertainties is established. In addition, when the tracking error of the system given by (26) is in sliding mode, the switching surface approaches zero asymptotically and the tracking error converges to the neighborhood of zero as time $t \to \infty$.

Proof: A Lyapunov function candidate is chosen as

$$V(\boldsymbol{s}, \boldsymbol{\xi}) = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{s} + \frac{1}{2}\boldsymbol{\xi}^T P\boldsymbol{\xi} \quad (48)$$

Where $P = pI, p > 0$ is a constant, I is an identity matrix of order n.

Differentiating $V(\boldsymbol{s}, \boldsymbol{\xi})$ with respect to time we get

$$\dot{V}(\boldsymbol{s},\boldsymbol{\xi}) = \boldsymbol{s}^T\dot{\boldsymbol{s}} + \boldsymbol{\xi}^T P\dot{\tilde{\boldsymbol{q}}}$$

Substituting expression of $\dot{\boldsymbol{s}}$ from (44)

$$\dot{V}(\boldsymbol{s},\boldsymbol{\xi}) = \boldsymbol{s}^T\left(-K_v\boldsymbol{s} - K_i\boldsymbol{s_1} - \boldsymbol{u}_{sl} + f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})\right) + \boldsymbol{\xi}^T P\dot{\tilde{\boldsymbol{q}}}$$

$$\dot{V}(\boldsymbol{s},\boldsymbol{\xi}) = -\boldsymbol{s}^T K_v\boldsymbol{s} + \boldsymbol{s}^T\left(-K_i\boldsymbol{s_1} + K_i\boldsymbol{s_1} - \Delta\boldsymbol{u}_{sl} + f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})\right) + \boldsymbol{\xi}^T P\dot{\tilde{\boldsymbol{q}}}$$

$$\dot{V}(\boldsymbol{s},\boldsymbol{\xi}) = -\boldsymbol{s}^T K_v\boldsymbol{s} + \boldsymbol{s}^T\left(-\left[\alpha + \beta|\dot{\tilde{\boldsymbol{q}}}| + \gamma|\boldsymbol{\xi}|\right]sign(\boldsymbol{s}) + f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})\right) + \boldsymbol{\xi}^T P\dot{\tilde{\boldsymbol{q}}} \qquad (49)$$

Considering (33) $\boldsymbol{s}^T f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q})$ can be written as

$$\boldsymbol{s}^T f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \le \|\boldsymbol{s}\|\left(\alpha_1 + \beta_1\|\dot{\tilde{\boldsymbol{q}}}\| + \gamma_1\|\boldsymbol{\xi}\|\right)$$

$$\boldsymbol{s}^T f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \le \alpha_1\|\boldsymbol{s}\| + \beta_1\|\boldsymbol{s}\|\|\dot{\tilde{\boldsymbol{q}}}\| + \gamma_1\|\boldsymbol{s}\|\|\boldsymbol{\xi}\|$$

$$\boldsymbol{s}^T f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \le \alpha_1\|\boldsymbol{s}\| + \beta_1\|\boldsymbol{s}\|(\|\boldsymbol{s}\| + C\|\boldsymbol{\xi}\|) + \gamma_1\|\boldsymbol{s}\|\|\boldsymbol{\xi}\|$$

$$\boldsymbol{s}^T f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q}) \le \alpha_1\|\boldsymbol{s}\| + \beta_1\|\boldsymbol{s}\|^2 + (\lambda_{max}(C)\beta_1 + \gamma_1)\|\boldsymbol{s}\|\|\boldsymbol{\xi}\| \qquad (50)$$

Considering (49) and (50) we can write

$$\begin{aligned}\dot{V}(\boldsymbol{s},\boldsymbol{\xi}) \le \big(&-\|\boldsymbol{s}\|\alpha - \beta\|\boldsymbol{s}\|\|\dot{\tilde{\boldsymbol{q}}}\| - \gamma\|\boldsymbol{s}\|\|\boldsymbol{\xi}\|\big) \\ &+ (\alpha_1\|\boldsymbol{s}\| + \beta_1\|\boldsymbol{s}\|^2 \\ &+ (\lambda_{max}(C)\beta_1 + \gamma_1)\|\boldsymbol{s}\|\|\boldsymbol{\xi}\|) \\ &+ (\|\boldsymbol{\xi}\|P\|\dot{\tilde{\boldsymbol{q}}}\|)\end{aligned}$$

$$\begin{aligned}\Rightarrow \dot{V}(\boldsymbol{s},\boldsymbol{\xi}) \le (&-\|\boldsymbol{s}\|\alpha - \beta\|\boldsymbol{s}\|(\|\boldsymbol{s}\| + C\|\boldsymbol{\xi}\|) - \gamma\|\boldsymbol{s}\|\|\boldsymbol{\xi}\|) \\ &+ (\alpha_1\|\boldsymbol{s}\| + \beta_1\|\boldsymbol{s}\|^2 \\ &+ (\lambda_{max}(C)\beta_1 + \gamma_1)\|\boldsymbol{s}\|\|\boldsymbol{\xi}\|) \\ &+ (\|\boldsymbol{\xi}\|P(\|\boldsymbol{s}\| + C\|\boldsymbol{\xi}\|))\end{aligned}$$

$$\Rightarrow \dot{V}(\boldsymbol{s},\boldsymbol{\xi}) \le -(\beta - \beta_1)\|\boldsymbol{s}\|^2 + (\lambda_{max}(C)\beta_1 - \lambda_{max}(C)\beta + \gamma_1 - \gamma + p)\|\boldsymbol{s}\|\|\boldsymbol{\xi}\| - p\lambda_{max}(C)\|\boldsymbol{\xi}\|^2 - (\alpha - \alpha_1)\|\boldsymbol{s}\|$$

$$\Rightarrow \dot{V}(\boldsymbol{s},\boldsymbol{\xi}) \le -[\|\boldsymbol{s}\| \ \|\boldsymbol{\xi}\|]Q\begin{bmatrix}\|\boldsymbol{s}\| \\ \|\boldsymbol{\xi}\|\end{bmatrix} < 0 \qquad (51)$$

Where $\alpha > \alpha_1$ and β,γ is chosen such that Q is a positive definite matrix. Q is defined as

$$Q = \begin{bmatrix}(\beta - \beta_1) & -\frac{(\lambda_{max}(C)\beta_1 - \lambda_{max}(C)\beta + \gamma_1 - \gamma + p)}{2} \\ -\frac{(\lambda_{max}(C)\beta_1 - \lambda_{max}(C)\beta + \gamma_1 - \gamma + p)}{2} & p\lambda_{max}(C)\end{bmatrix} \qquad (52)$$

Let us consider, $V_1(t) = \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T R\dot{\tilde{\boldsymbol{q}}}$, where R is a positive definite matrix. $V_1(t)$ is lower bounded at zero only for $\dot{\tilde{\boldsymbol{q}}} = 0$. Otherwise, $V_1(t) > 0$.

We get

$$\dot{V}_1(t) = \dot{\tilde{\boldsymbol{q}}}^T R\ddot{\tilde{\boldsymbol{q}}} = \dot{\tilde{\boldsymbol{q}}}^T R(\dot{\boldsymbol{s}} - C\dot{\tilde{\boldsymbol{q}}}) \qquad (53)$$

From the previous analysis we can infer that on the sliding surface $\dot{\boldsymbol{s}} = 0$ which implies

$$\dot{V}_1(t) = -\dot{\tilde{\boldsymbol{q}}}^T RC\dot{\tilde{\boldsymbol{q}}} \qquad (54)$$

Both of R and C matrices are positive definite according to considerations. Thus, $\dot{V}_1(t)$ can be proved to be negative definite.

$$\ddot{V}_1(t) = -2\dot{\tilde{\boldsymbol{q}}}^T RC\ddot{\tilde{\boldsymbol{q}}} = 2\dot{\tilde{\boldsymbol{q}}}^T RCC\dot{\tilde{\boldsymbol{q}}} \qquad (55)$$

From (51) it can be inferred that $\boldsymbol{s}$ is bounded, as $V(\boldsymbol{s},\boldsymbol{\xi}) \le V(0,0)$. $\dot{\tilde{\boldsymbol{q}}}$, being a function of $\boldsymbol{s}$ is also bounded. Hence, $\ddot{V}_1(t)$ is bounded.

As, $V_1(t)$ satisfies all the conditions mentioned by the lemma,

VII. CHATTERING REDUCTION CONSIDERING A CONTINUOUS FUNCTION

The sliding mode compensating input is discontinuous on sliding surface. Therefore, the chattering occurs at a theoretically infinite frequency. The high frequency components of the chattering are undesirable because they may excite un-modeled high-frequency plant dynamics [5, 11]. To overcome this problem, discontinuous sign function is replaced by a continuous function as given below:

$$sign(s_i) \to \frac{s_i}{|s_i| + \delta_i} \qquad (56)$$

where $\delta_i > 0$ and $i = 1,2,\dots n$

This continuous function is similar to the saturation function [10]. This continuous approximation ensures the boundedness [14] in the neighbourhood of sliding surface.

VIII. PERFORMANCE EVALUATION OF PROPOSED CONTROLLER

To evaluate the performance of the proposed controller with sliding mode compensation in a comprehensive simulation environment, we have considered a Simulink model of 3DOF direct drive manipulator compatible in MATLAB ® (Ver.7) environment. The simulation step time (fixed) is considered to be 0.01 ms. The control input is computed based on the nominal model of 3DOF direct drive manipulator. We have implemented the compensated control algorithm on the 3DOF direct drive manipulator model which is very close to a realistic model. The simulation results with the proposed controller are presented in the following sections.

A. Parameters for Simulation:

We have considered the model of the three degrees of freedom direct drive robot manipulator for the comprehensive simulation. Comparing with the equation (1) we can write the elements of the dynamic equation [12] of the three degrees of freedom direct drive robot manipulator as follows.

The elements $M_{ij}(q)$ $(i, j = 1, 2, 3)$ of the inertia matrix are:

$$M_{11}(\boldsymbol{q}) = 4 + 3\cos(q_2) + \cos(q_2 + q_3) + \cos(q_3)$$
$$M_{12}(\boldsymbol{q}) = 1.67 + 1.5\cos(q_2) + 0.5\cos(q_2 + q_3) + \cos(q_3)$$
$$M_{13}(\boldsymbol{q}) = 0.33 + 0.5\cos(q_2 + q_3) + 0.5\cos(q_3)$$
$$M_{21}(\boldsymbol{q}) = M_{12}(\boldsymbol{q})$$
$$M_{22}(\boldsymbol{q}) = 1.67 + \cos(q_3)$$
$$M_{23}(\boldsymbol{q}) = 0.33 + 0.5\cos(q_3)$$
$$M_{31}(\boldsymbol{q}) = M_{13}(\boldsymbol{q})$$
$$M_{32}(\boldsymbol{q}) = M_{23}(\boldsymbol{q})$$
$$M_{33}(\boldsymbol{q}) = 0.33$$

The elements $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ $(i, j = 1, 2, 3)$ of the centrifugal and Coriolis matrix are

$$C_{11}(\boldsymbol{q},\dot{\boldsymbol{q}}) = [-1.5\sin(q_2) - 0.5\sin(q_2 + q_3)]\dot{q}_2 + [-0.5\sin(q_2 + q_3) - 0.5\sin(q_3)]\dot{q}_3$$
$$C_{12}(\boldsymbol{q},\dot{\boldsymbol{q}}) = [-1.5\sin(q_2) - 0.5\sin(q_2 + q_3)](\dot{q}_1 + \dot{q}_2) + [-0.5\sin(q_2 + q_3) - 0.5\sin(q_3)]\dot{q}_3$$
$$C_{13}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -0.5\sin(q_3)\dot{q}_3 + [-0.5\sin(q_2 + q_3) - 0.5\sin(q_3)](\dot{q}_1 + \dot{q}_2)$$
$$C_{21}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -0.5\sin(q_3)\dot{q}_3 + [1.5\sin(q_2) + 0.5\sin(q_2 + q_3)]\dot{q}_1$$

$$C_{22}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -0.5\sin(q_3)\,\dot{q}_3$$
$$C_{23}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -0.5\sin(q_3)\,(\dot{q}_1+\dot{q}_2+\dot{q}_3)$$
$$C_{31}(\boldsymbol{q},\dot{\boldsymbol{q}}) = [0.5\sin(q_2+q_3)]\dot{q}_1 + [0.5\sin(q_3)](\dot{q}_1+\dot{q}_2)$$
$$C_{23}(\boldsymbol{q},\dot{\boldsymbol{q}}) = 0.5\sin(q_3)\,(\dot{q}_1+\dot{q}_2)$$
$$C_{33}(\boldsymbol{q},\dot{\boldsymbol{q}}) = 0$$

The elements of the gravitational torque vector $g(\boldsymbol{q})$ are given by

$$g_{11}(\boldsymbol{q}) = [25\cos(q_1) + 15\cos(q_1+q_2) + 5\cos(q_1+q_2+q_3)]g$$
$$g_{21}(\boldsymbol{q}) = [15\cos(q_1+q_2) + 5\cos(q_1+q_2+q_3)]g$$
$$g_{31}(\boldsymbol{q}) = [5\cos(q_1+q_2+q_3)]\,g$$
$$g = 9.81\,ms^{-2}$$

The controller with sliding mode compensation is verified for reference velocity similar to the velocity given in reference [3]. Reference velocities are presented for each joint respectively.

$$\dot{q_{d1}} = 4.7124t^2e^{-2t^3} + 4.2t^2e^{-2t^3}\sin(6t) + 4.2\left[1-e^{-2t^3}\right]\cos(6t)\ rad/sec$$
$$\dot{q_{d2}} = 6.2832t^2e^{-2t^3} + 7.089t^2e^{-2t^3}\sin(3t) + 3.5448\left[1-e^{-2t^3}\right]\cos(3t)\ rad/sec$$
$$\dot{q_{d3}} = 7.88t^2e^{-2t^3} + 9.88t^2e^{-2t^3}\sin(1.5t) + 2.75\left[1-e^{-2t^3}\right]\cos(1.5t)\ rad/sec$$

Inverse Dynamics Controller is implemented with the following gains[3]

K_i= diag [900, 900] (l/s^2) and K_v = diag [60, 60] (1/s)

And for sliding surface, we have chosen $C = diag\ [60, 60]$.

As far as actuators are concerned, the required parameters are considered from the actuator specification as given in the work of M. M. Fateh. Considering estimated bounds and following the condition stated by (34) α, β and γ are chosen as:

Payload (kg)	Controller Parameter Settings		
3	$\alpha_1 = \alpha_2 = \alpha_3 = 750$	$\beta_1 = \beta_2 = \beta_3 = 1650$	$\gamma_1 = \gamma_2 = \gamma_3 = 7.5$

Value of δ is taken to be equal to 0.001, which is sufficient for reduction of the chattering from the control input. Considering the parametric variation of 50% in inertia matrix and Coriolis matrix of the robot model as referred in [10] as well as the electrical dynamics of actuator, we have made an attempt to construct a comprehensive simulation model of a manipulator which is close to a realistic one compared to the nominal model. The tracking performance of inverse dynamics velocity controller and the same controller with sliding mode compensation has been compared here quantitatively. The supporting figures are represented in the section below.

B. *Simulation Results*

The simulation result of Fig 3. shows the reference velocity input and corresponding output of the three links. The simulation results for the comparison of joint velocity errors of uncompensated and compensated controllers are presented in Fig. 4 and Fig. 5. In all the mentioned simulation studies we have used a payload of 3 Kg.

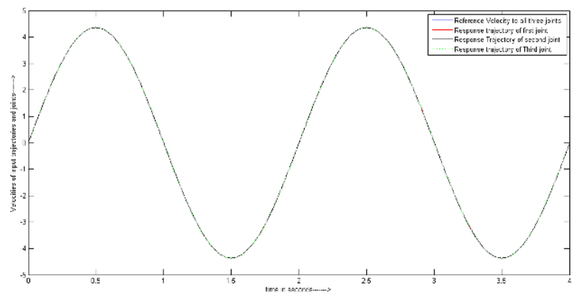

Fig.3. Reference Velocity Input and Response Signal

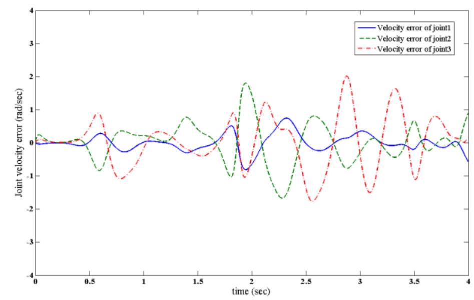

Fig. 4 Tracking Error of the Three DOF Manipulator without Compensating Input (Only Inverse Dynamics Velocity Controller)

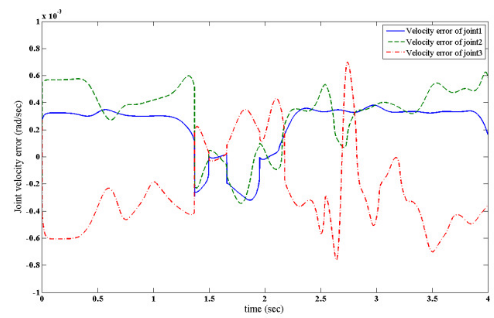

Fig. 5 Tracking Error of the Three DOF Manipulator without Proposed Compensating Controller

C. *Discussion*

The time evolution of the velocity error $\dot{\tilde{\boldsymbol{q}}}$ reflects how well the compensated controller performs over the actual velocity controller. The performance criterion considered is the root mean square (RMS) value of the velocity error truncated Euclidean norm on a trip of time T**;** that is:

$$\mathcal{L}_T^2[\dot{\tilde{\boldsymbol{q}}}] = \sqrt{\frac{1}{T}\int_0^T \left\|\dot{\tilde{\boldsymbol{q}}}(\sigma)\right\|^2 d\sigma} \qquad \text{(rad/sec)} \quad (57)$$

The $\mathcal{L}_T^2$ norm has been previously evoked by J. Moreno and R. Kelly [2, 3] as a criterion for tracking performance.

In practice, the discrete implementation of the criterion (53) leads to:

$$\mathcal{L}_T^2[\dot{\tilde{\boldsymbol{q}}}] = \sqrt{\frac{1}{T}\sum_{k=0}^{T/h}\left\|\dot{\tilde{\boldsymbol{q}}}(\sigma)\right\|^2 h} \qquad \text{(rad/sec)} \qquad (58)$$

where h = 0.01 ms is the sampling period and T= 4 sec is the trip time.

IX. CONCLUSIONS

This paper has dealt with inverse dynamics velocity control scheme for robot manipulator, which is a model based motion control scheme (joint velocity tracking controller). Being a velocity controller, the position error of inverse dynamics control algorithm has been replaced with integral of velocity error. Inverse dynamics control depends on the manipulators dynamics and requires exact dynamic model, which is difficult to calculate. Hence, the performance of inverse dynamics controller is affected due to parametric uncertainties. A non-linear feedback compensation based on the sliding mode control is found to be effective over this problem and it makes the system robust to this model uncertainties. The proposed control scheme does not require the calculation of the accurate dynamic model during every sampling interval; therefore, the computational burden becomes manageable. The bound of the parameter uncertainties is estimated as a function of system states in this paper and the analysis shows that in the presence of the uncertainties, the closed-loop system is still robust in stabilization and performance. Chattering is reduced by using the continuous function instead of Signum function. Simulation results show the validity of the proposed velocity controller with sliding mode compensation. Finally, to conclude, the authors take the liberty to claim that the proposed control law ensures the guaranteed tracking performance of the direct drive manipulator irrespective of its loading condition and offers a control algorithm, which is extremely suitable for real-time applications.

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