Investigation stability of Rikitake system

Yousof Gholipour[1*], Mahmood Mola[2]

[1]Department of Electrical Eng., Faculty of Eng., Mehriz Branch. Islamic Azad University. Mehriz.Iran.

[2]Department of Electrical Eng., Faculty of Eng., Science and Research Branch. Islamic Azad University. Tehran, Iran.

[1]y.gholipour@yahoo.com;[2]mhmola@ieee.org

Abstract- **The Rikitake dynamical system is a model which attempts to explain the irregular polarity switching of the geomagnetic field. The system exhibits Lorenz-type chaos and orbiting around two unstable fixed points. The study showed that the system is experiencing a chaotic behavior at certain value of the control parameter. We investigated stability of this system by changing resistance of wires. When the wire resistance to change, will change value of a –that is one mainly parameter of stability- we hold constant other values to find the best conditions of stability. After per change of a, numerical simulations to illustrate the effect of a, are presented and at the end conclusions and comparing the states together are obtained.**

Keywords- Chaos; Rikitake System; Sinusoid; Stability

I. INTRODUCTION

First of all let us discuss the physics behind Rikitake system. Figure 1 shows the Rikitake system.

A common torque of magnitude G is applied to both conducting disks D1 and D2. Both disks rotate in the same sense, one with an angular velocity of ω_1 around the axis of rotation A1and the other with angular velocity ω_2 around the axis of rotation A2. A current I1 circulates the current loop L1 where the current loop is coaxial with D2 with axis A2 and L1 is located below D2, and causes a magnetic field B2 to pass through the rotating disk D2. Loop L1 is connected to Disk D1 and its axis of rotation A1by conducting brushes and the current along A1 is upward and in D1 it is radial outward. Similarly a current I2 circulates the current loop L2 where the current loop is coaxial with D1 with axis A1 and L2 is located above D1, and causes a magnetic field B1 to pass through the rotating disk D1. Loop L2 is connected to Disk D2 and its axis of rotation A2 by conducting brushes and the current along A2 is upward and in D2 it is radial inward. Since D1 is rotating B1 caused by L2 will create an induced emf(**e**lectro**m**agnetic **f**ield) between the center of D1 and its rim causing an induced inward current I1 to occur in the opposite direction to I1 where the total current becomes less than the original current I1. Similarly since D2 is rotating B2 caused by L1 will create an induced emf between the center of D2 and its rim causing an induced outward current I2 to occur in the opposite direction to I2 where the total current becomes less than the original current I2. This process continues until we achieve current reversal in both loops which causes a reversal in the total magnetic field. Under particular initial conditions this process becomes chaotic.[3]

in this paper we suppose that the equilibrium point $E_+(x_0,y_0,z_0)$ is (1,1,1) we change value of a & u while a & u are positive(a>0&u>0) and observe stability of Rikitake system that this work has not investigated in the previous articles.Many researchers have discussed the dynamics of Rikitake system. This system is a mathematical model obtained from a simple mechanical system used by Rikitake [1] to study the reversals of the Earth's magnetic field. Historically E. C. Bullard has extensively discussed the behavior of earth's magnetic field and its simulation with dynamos. He first discussed the magnetic field within the earth, then the similar behavior between a set of homogeneous dynamos and terrestrial magnetism and in 1955 discussed the stability of a homopolar dynamo. Liu Xiao-Jun, et.al. [5] analyzed the dynamics of Rikitake two-disk dynamo to explain the reversals of the Earth's magnetic field. They concluded that the chaotic behavior of the system can be used to simulate the reversals of the geomagnetic field. The Rikitake chaotic attractor was studied by several authors. T. McMillen [6] and Mohammad Javidi,et.al.[4] has studied the shape and dynamics of the Rikitake attractor. J. Llibre .et.al [7] used the Poincare compactification to study the dynamics of the Rikitake system at infinity. Chien- Chih Chen et al [10] have studied the stochastic resonance in the periodically forced Rikitake dynamo. In the past decade, many researchers start working on controlling the chaotic behaviors. Harb and Harb [11] have designed a nonlinear controller to control the chaotic behavior in the phase-locked loop by means of nonlinear control. Ahmad Harb[3] have designed a controller to control the unstable chaotic oscillations by means of back stepping method. The synchronization for chaotic of Rikitake system was studied by several authors. Mohamad Ali khan[12] and Carlos Aguilar-Ibañez[9] and U.E. Vincent[8].

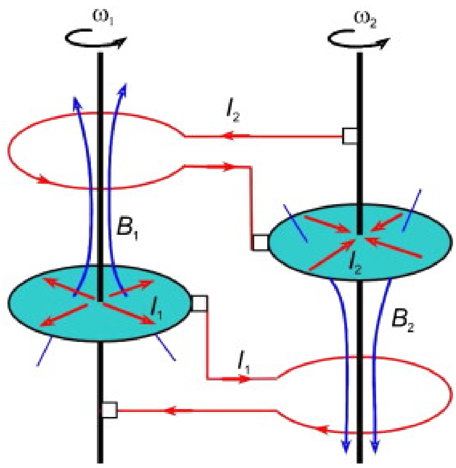

Fig 1:Rikitake two disk dynamo

II. MATHEMATICAL MODEL OF RIKITAKE SYSTEM

The Rikitake system consists of two conducting rotating disks (see Fig 1). These disks are connected into two coils. The current in each coil feeds the magnetic field of the other. The self inductance (L) and resistance (*R*) are the same in each circuit. An external constant mechanical torque (G) for each circuit is applied on the axis to rotate with an angular velocity.

The original differential equations derived by Rikitake are:

$$\begin{cases} L_1\dfrac{dI_1}{dt}+R_1I_1=\omega_1MI_2 \\ L_2\dfrac{dI_2}{dt}+R_2I_2=\omega_2NI_1 \\ C_1\dfrac{d\omega_1}{dt}=G_1-MI_1I_2 \\ C_2\dfrac{d\omega_2}{dt}=G_2-NI_1I_2 \end{cases} \qquad (1)$$

Where $\omega_1 MI_2$ and $\omega_2 MI_1$ are the rotation voltages V1 and V2.where L, R are the self-inductance and resistance of the coil, the electric currents I, ω, C, G are the electric currents, the angular velocity , momentum of inertia, and the driving force, M, N are the mutual inductance between the coils and the disks.

Now we consider a further simplification by $L1=L2$, $R1=R2$, $M=N$, $C1=C2$, $G1=G2$ and set

$$I_1=\sqrt{\frac{G}{M}}x,\ I_2=\sqrt{\frac{G}{M}}y,\ \omega_1=\sqrt{\frac{GL}{CM}}z$$
$$\omega_2=\sqrt{\frac{GL}{CM}}(z-a),\ t=\sqrt{\frac{CL}{GM}}t',\ v=R\sqrt{\frac{C}{LGM}} \qquad (2)$$

Where, constant parameter a,u>0.

The system mathematical model can be written as follows:

$$\begin{cases} x'=-ux+yz \\ y'=-y+(z-a)x \\ z'=1-xy \end{cases} \qquad (3)$$

Where $(x, y, z) \in R^3$ are the state variables and a > 0, u > 0 are parameters. Note that system (3) is a quadratic system in R^3. The choice of the parameters a > 0 and u > 0 reflects a physical meaning in the Rikitake model.

It is well known that system (3) has two equilibrium points

$$\mathbf{E_+ = (x_0, y_0, z_0)\ ;\ E_- = (-x_0, -y_0, z_0)}$$

In order to study the stability of $E_\pm$, it is only sufficient to study the stability of the equilibrium point E_+.

$$\begin{cases} x_0=\sqrt{\dfrac{a+\sqrt{a^2+4u^2}}{2u}} \\ y_0=\sqrt{\dfrac{2u}{a+\sqrt{a^2+4u^2}}} \\ z_0=\dfrac{a+\sqrt{a^2+4u^2}}{2u} \end{cases} \qquad (4)$$

Where:

$$a=R\sqrt{\frac{LC}{GM}} \quad \& \quad u=(\omega_1-\omega_2)\sqrt{\frac{CM}{GL}} \qquad (5)$$

III. STABILITY OF RIKITAKE SYSTEM

In this section we investigate stability of Rikitake system and suppose that the equilibrium point $E_+(x_0,y_0,z_0)$ is (1,1,1) we change value of a & u while a & u are positive(a>0&u>0) and observe stability of Rikitake system.

We know that the formula of resistance is:

$$R=\rho\frac{L}{A}=\rho\frac{L}{\pi r^2} \qquad (6)$$

We changed Radius of wire and obtained, R on base of (5) we have four positions:

1- u >a
2- 0<u&a<1
3- u=a
4- u<a

Now display the numerical solution of Rikitake system. For all positions The initial conditions (x_0,y_0,z_0) = (1,1,1) have been used and set steps h=0.001. We plotted the system behavior for four positions around X, Y and Z axes, and because of more clarification in four positions we chose special values in our figures.

1- u > a

The parameters have been set to u >a and. The numerical solution has been approximated from t = 0 to t = 100

For plot we set a=1 &u=4.5.

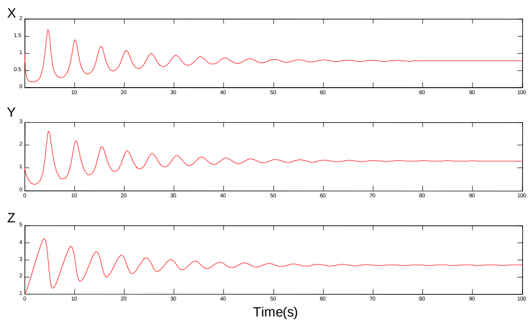

Fig. 2 the time series X(t),Y(t),Z(t) for u>a

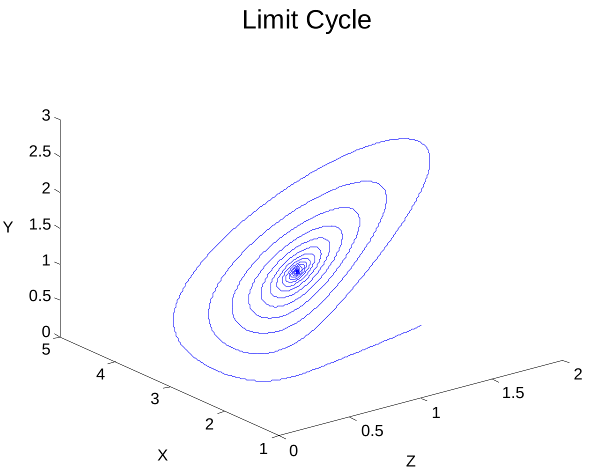

Fig. 3 the limit cycle for u>a

We see that, with increase the Proportion of u to a, the stability of system will be increase and we will have a stable system after some fluctuation. This system is a chaotic system but with this conditions we will have a stable system. From limit cycle we can conclude that, this system is stable. In fig. 2 we see that, the system quickly reached a stable status.

2- 0< u & a <1

Now we set parameters 0< u & a <1. The numerical solution has been approximated from t = 0 to t = 100.

For plot we set a=0.8 &u=0.72.

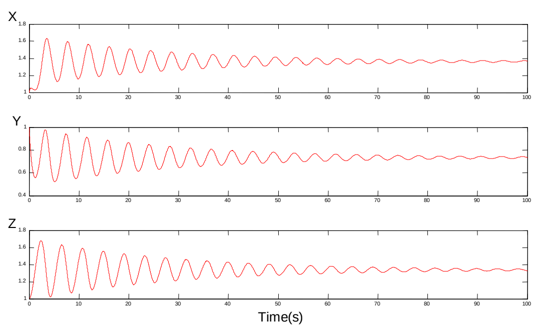

Fig. 4 the time series X(t),Y(t),Z(t) for 0<u&a<1

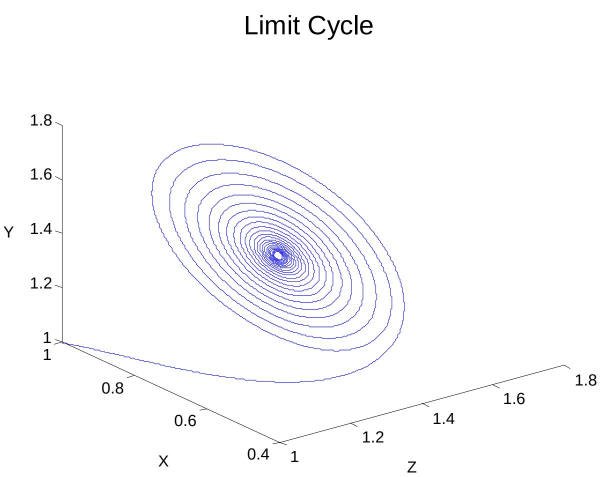

Fig. 5 the limit cycle for 0<u&a<1

When we set 0<a<1 , 0<u<1 the system will be stable. Note that in previous state, the system becomes stable faster than this state. From limit cycle we can conclude that, this system will be stable with a little delay. In fig. 4 we see that, the system is stable with delay.

3- u = a

Now set parameters u=a. The numerical solution has been approximated from t = 0 to t = 1000.

For plot we set a=3 &u=3.

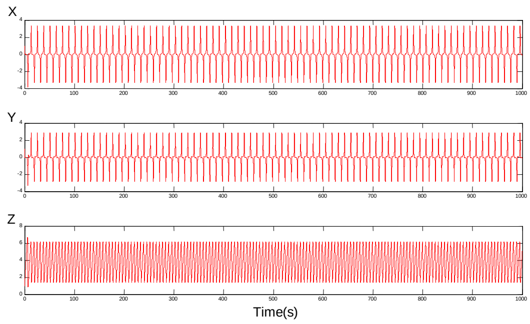

Fig. 6 the time series X(t),Y(t),Z(t) for u=a

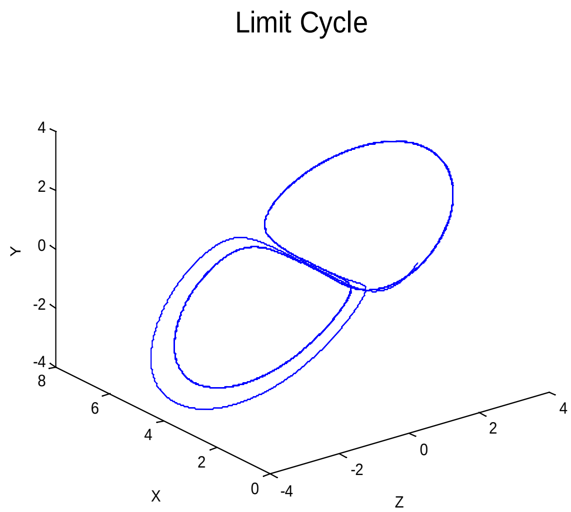

Fig. 7 the limit cycle for u=a

When we set values of u&a equal together the system will be sinusoid and it doesn't stable or chaotic. From limit cycle we can see that, the system will not be stable and it has a sinusoid behavior. In fig. 6 X, Y, Z axes shows that this system is not stable and it is sinusoidal system.

4- u < a

Now set parameters u<a. The numerical solution has been approximated from t = 0 to t = 200.

For plot we set a=3 &u=1.2.

When we set u<a, the system will be chaotic and it doesn't stable or sinusoid. From X, Y, Z axes and limit cycle we can see that, the system has a chaotic behavior.

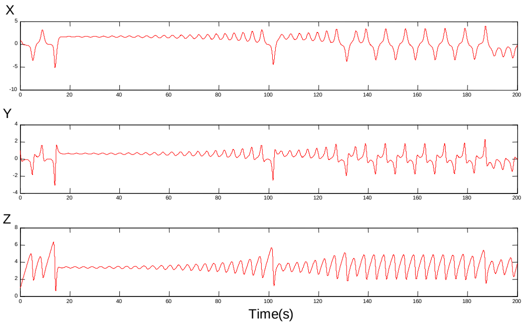

Fig. 8 the time series X(t),Y(t),Z(t) for u<a

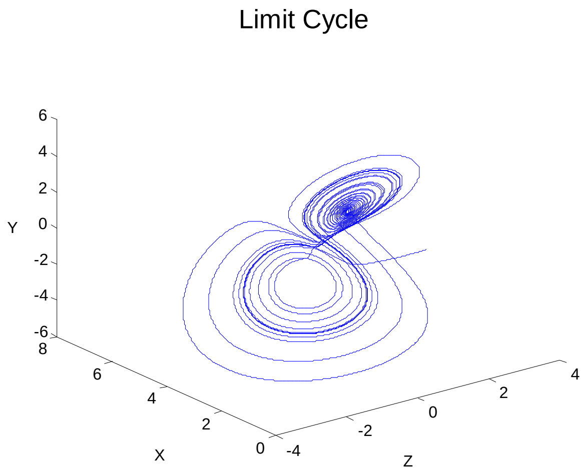

Fig. 9 the limit cycle for u<a

IV. CONCLUSIONS

This paper investigated stability of Rikitake system. Firstly discussed the physics behind Rikitake system. After that presented the mathematical model of the Rikitake system consists of three nonlinear differential equations, which found to be the same as the mathematical model of the well known Lorenz system. The study showed that the system has intrinsic chaotic behavior at certain value. we suppose that the equilibrium point $E_+(x_0,y_0,z_0)$ is (1,1,1) and changed value of R that it caused different a & u and observed stability of Rikitake system, finally we concluded from numerical simulations that when u >a and 0<u&a<1 the system will be stable, and other states isn't stable. If want to have a stable system we should adjust some conditions of Rikitake system like resistance of wire.

ACKNOWLEDGMENT

We acknowledge our friends Mr. Oliaei and Mr.Pourqorban for useful suggestions and review of this paper.

REFERENCES

Denis de Carvalho Braga and Fabio Scalco Dias and Luis Fernando Mello. On the stability of the equilibria of the Rikitake system. Physics Letters, 374(2010): 4316-4320.

Renjun Xu, Rikitake Model of Geomagnetic Reversal, Chaos @ Winter 2008.

Ahmad Harb, Nabil Ayoub, Nonlinear Control of Chaotic Rikitake Two-Disk Dynamo, International Journal of Nonlinear Science ,Vol.15(2013) No.1,pp.45-50.

Mohammad Javidi, Nemat Nyamorad, Numerical Chaotic Behavior of the Fractional Rikitake System, World Journal of Modelling and Simulation, Vol. 9 (2013) No. 2, pp. 120-129.

Liu Xiao-jun, Li Xian-feng, Chang Ying-xiang, Zhang Jian-gang. Chaos and Chaos Synchronism of the RikitakeTwo-Disk Dynamo.Fourth International Conference on Natural Computation, IEEE computer Society,DOI10.1109/ICNC.2008.706:613-617.

T. McMillen. The shape and dynamics of the Rikitake attractor. The Nonlinear Jour., 1(1999):1-10.

J. Llibre, M. Messias. Global dynamics of the Rikitake system.Physica D, 238(2009):241-252.

U.E. Vincent, R. Guo, Finite-time synchronization for a class of chaotic and hyperchaotic systems via adaptive feedback controller, Physics Letters A 375 (2011) 2322–2326.

Carlos Aguilar-Ibañez, Rafael Martinez-Guerra,Ricardo Aguilar-López, Juan L. Mata-Machuca, Synchronization and parameter estimations of an uncertain Rikitake system, Physics Letters A 374 (2010) 3625–3628.

C.-C. Chen, C.-Y. Tseng. A study of stochastic resonance in the periodically forced Rikitake dynamo. Terr. Atmos.Ocean. Sci., 18(4)(2007):671-680.

Ahmad Harb, Bassam Harb. Chaos control of third-order phase-locked loops using backstepping nonlinear controller.Chaos, Solitons & Fractals, 20(4)(2004).

Mohammad Ali Khan,Different Synchronization Schemes for chaotic Rikitake Systems ,Journal of Advanced Computer Science and Technology, 1 (3) (2012) 167-175 .